Mathematical Modelling and Applied Computing (MMAC).
ISSN 0973-6093 Volume 7, Number 1 (2016), pp. 9-10

http://www.ripublication.com/mmac.htm

CGF inequality and another operator equation

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Abstract: In this paper, we will prove a property of another operator equation via CGF inequality.

Keywords: CGF inequality, Douglas theorem

MSC: 47A63

INTRODUCTION

In this paper, we will prove a property of an operator equation via CGF inequality under the condition of $(p+t)s \geq 2(p+t)s_0 + r$ with $\min\{p,1\}+t \geq r$.

Theorem 1. (CGF Inequality, [Y])Let $p>0, t\geq 0, r\geq 0, s>s_0>0,$ if $A \geq B \geq 0,$ then

$$(A^{r/2}(A^{t/2}B^pA^{t/2})^{s_0}A^{r/2})^{\frac{\delta+r}{(p+t)s_0+r}} \geq (A^{r/2}(A^{t/2}B^pA^{t/2})^{s}A^{r/2})^{\frac{\delta+r}{(p+t)s+r}},$$

where $\delta = \min\{(p+t)s, 2(p+t)s_0 + \min\{\min\{p,1\}+t, r\}\}.$

MAIN RESULT

Theorem 2.

For $p>0, t\geq 0, r\geq 0, s>s_0>0,$ $(p+t)s \geq 2(p+t)s_0 + r$, $\min\{p,1\}+t \geq r$ with $2\dfrac{(p+t)s_0+r}{(p+t)s+r} = \dfrac{1}{n}$, if $A \geq B \geq 0,$ there exists a unique solution X, X>0 with $\|x\|\leq 1$ s.t. $\underbrace{F(s_0)XF^2(s_0)XF^2(s_0)X\cdots F^2(s_0)XF(s_0)}_{\text{there lists } X \text{ for } n \text{ times}} = F(s)$,

where $F(u) = A^{r/2}(A^{t/2}B^pA^{t/2})^{u}A^{r/2}.$

Proof. By Theorem 1, we have $F^2(s_0) \geq F^{1/n}(s)$. By Douglas Theorem in [D], there exists a unique operator T, $\|T\| \leq 1, F^{1/2n}(s) = F(s_0)T = T^* F(s_0)$.

Let X=TT*, then $F^{1/n}(s) = F(s_0)XF(s_0)$. It means that $F(s) = (F(s_0)XF(s_0))^n$, by which we can obtain the result.

ACKNOWLEDGMENTS

Jian Shi (the corresponding author) is supported by Hebei Education Department (Science and Technology Research Project of Colleges and Universities in Hebei Province， No. ZC2016009) , Hebei University, Natural Science Foundation of Shandong Province (No. BS2015SF006)

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