Flow Past A Spherical Liquid Drop Embed in Infinite Medium

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Abstract

We discuss the problem of viscous fluid flow past liquid a sphere inside porous media extending to infinity and the pressure and velocity was calculated

THE BOUNDARY CONDITIONS OF THE PROBLEM :

The boundary conditions required the solution are

1- the continuity of pressures of different regions at the interface boundary

2- the continuity of normal velocities

3- \( -\beta (u_\theta - q_\theta) = \frac{\partial u_r}{\partial r} + \frac{1}{R_\theta} \left[ \frac{\partial u_r}{\partial \theta} - u_\theta \right] \)

Where \( \beta = \frac{\alpha}{\sqrt{k}} R_\theta \) is radius of curvature

PROBLEM DESCRIPTION AND FORMULATION :

Inside the sphere governed by stokes equations and outside the sphere governed by Darcy's law. the pressure and velocities of the flow inside the sphere

\[
p(r, \theta) = p_\infty + \gamma rUF\cos\theta
\]

\[
u_r = \gamma U \left[ F + D r^2 \right] \cos \theta
\]
\[ u_\theta = -U \sin \theta [F + 2D r^2] \]

the pressure and velocities of the flow outside the sphere

\[ p(r, \theta) = p_\infty + \gamma \frac{U F \cos \theta}{r^2} \]

\[ q_r = UK \cos \theta \left[ -1 + \frac{2L}{r^3} \right] \]

\[ q_\theta = UK \sin \theta \left[ 1 + \frac{2L}{r^3} \right] \]

Matching boundary conditions inside and outside the sphere (\(r=a\)):

\[ p_\infty + \gamma r U F \cos \theta = p_\infty + \gamma \frac{U F \cos \theta}{r^2} \]

\[ r F = \frac{L}{r^2} \]

At \(r=a\) we have

\[ a F = \frac{L}{a^2} \text{ or } a^3 F - L = 0 \]

\[ \gamma U [F + D r^2] \cos \theta = UK \gamma \cos \theta \left[ -1 + \frac{2L}{r^3} \right] \]

At \(r=a\) we have

\[ [F + D r^2] = K \left[ -1 + \frac{2L}{r^3} \right] \text{ or } a^3 F + a^5 D - 2KL - L = - K a^3 \]

From the condition

\[-\beta (u_\theta - q_\theta) = \frac{\partial u_r}{\partial r} + \frac{1}{r \theta} \left[ \frac{\partial u_r}{\partial \theta} - u_\theta \right] \]

We have

\[ \beta \left( U \sin \theta [F + 2D r^2] - UK \sin \theta \left[ 1 + \frac{2L}{r^3} \right] \right) \]

\[ = -U \sin \theta [F + 4D r] + \frac{1}{R \theta} \left[ - \gamma U [F + D r^2 \sin \theta] - U \sin \theta [F + 2D r^2] \right] \]

at \(r=a\)
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\[
\beta \left( U \sin \theta [F + 2D a^2] - UK \sin \theta \left[ 1 + \frac{2L}{a^3} \right] \right) \\
= -U \sin \theta [F + 4D a] + \frac{1}{R_\theta} \left[ - \gamma U [F + D a^2] \sin \theta \right] \\
- USin\theta[F + 2D a^2]
\]

or

\[
\beta a^3 F - (3a^4 + 2\beta a^5) D + \beta KL = -\beta a^3 K
\]

By solving the equations 3, 4 and 5 we get the following

\[
D = \frac{3\beta K^2}{aM} \\
L = \frac{-a^3 [3K + \beta aK]}{M} \\
F = \frac{-[3K + \beta aK]}{M}
\]

Where M is given by

\[
M = \beta a - 5\beta ak + 3 - 6k
\]

Conclusions:
In this problem of viscous fluid flow past liquid a sphere inside porous media extending to infinity and the Darsy's law was applied in the porous regions and stokes equations in free regions. the result was concluded the pressure and velocity

References:

