

# Computational Complexity of a Dorsal Hand Vein Pattern Recognition System Based on a Statistical Approach: Quadratic Inference Function

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## Abstract

Different methods are explored to extract and represent dorsal hand vein pattern. To evaluate the efficiency of the methods, their computational complexities are calculated. In this paper, generalized method of moment via QIF is being explored and evaluated. To further reduce the computational cost, LU factorization is used. FAR, FRR and matching time are also computed. The experimental results show that generalized method of moment via LU factorization provides better results than pixel by pixel method and therefore proves to be an efficient algorithm for dorsal hand vein feature extraction.

**Key words:** Dorsal hand vein pattern, quadratic inference function, LU factorization, computational complexity.

## Introduction

In this ubiquitous society, anyone can easily gain access to information anywhere and at anytime. Traditional personal verification methods such as passwords, personal identification numbers (PINS), magnetic swipe cards, keys and smart cards offer very limited security and are unreliable [1,2]. The recent increase in crimes has pushed researchers to scrutinise for better security provisions. Consequently, biometrics which involves the analysis of human biological, physical and behavioral characteristics have been developed to ensure more reliable security. Compared to traditional methods, biometric features are harder for intruders to copy and forge [3, 6]. The most popular biometric features that are used are fingerprints, hand geometry, iris scans, faces, as well as handwritten signatures. Recently dorsal hand vein pattern biometric is attracting the attention of researchers and is gaining momentum. Anatomically, aside from surgical intervention, the shape of vascular patterns in the

back of the hand is distinct from each other [1, 2]. It is a randotypic trait, which is formed during the early phases of embryonic development and hence unique to everyone [3]. Veins are found beneath the skin and cannot be seen with naked eyes. Its uniqueness, stability and immunity to forgery are attracting researchers. These feature makes it a more reliable biometric for personal identification [4]. Furthermore, the state of skin, temperature and humidity has little effect on the vein image, unlike fingerprint and facial feature acquirement [5]. The hand vein biometrics principle is non- invasive in nature where dorsal hand vein pattern are used to verify the identity of individuals [7]. Vein pattern is also stable, that is, the shape of the vein remains unchanged even when human being grows.

Extensive researches are carried out on vein patterns and researchers are striving hard to find methods and techniques to develop dorsal hand vein security system. Any biometric system consists of four main steps namely the preprocessing, feature extraction, processing and matching phase. Feature extraction is a crucial step in biometric system and its capability directly influence the performance of the system. Different methods such as Principle Component Analysis (PCA) [6], modified Principle component analysis with Lanczos and Cholesky decomposition [7] have been explored to extract and represent dorsal hand vein features. However, to improve the accuracy of the dorsal hand vein verification system, more methods are being scrutinized.

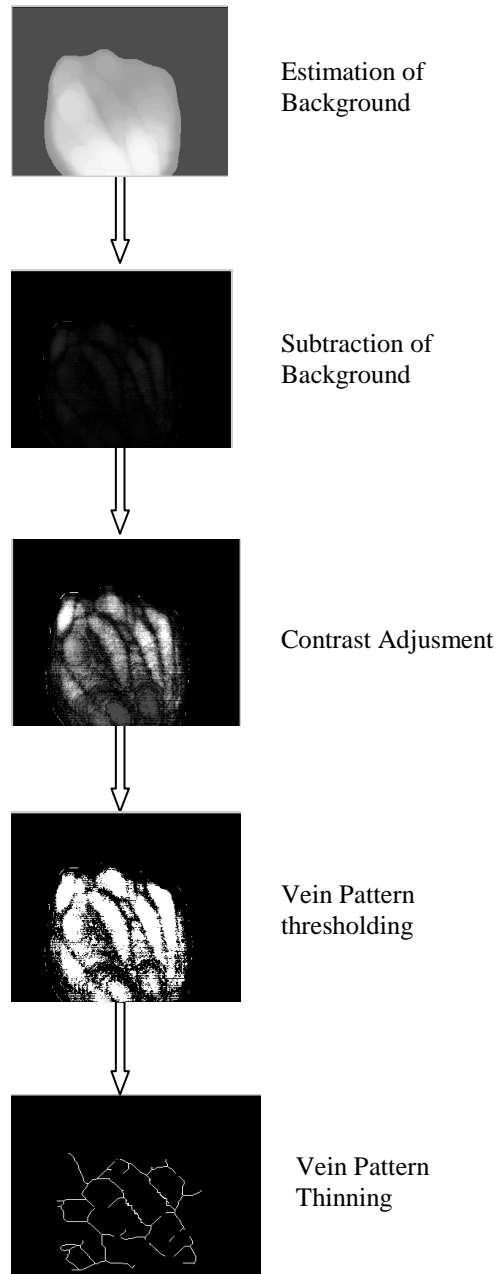
With the methods previously developed, it was found that the size of matrices obtained for the feature extraction is very large and consequently new ways have to be obtained to reduce these matrices dimension. In this work, the method proposed aims at reducing the dimension of the training set by building an adaptive estimating equation or a quadratic inference function [8,9,10] that combines the covariance matrix and the vectors in the training set. Furthermore the computational complexity of the method has been calculated and analysed.

The organization of the paper is as follows: in section 2 we describe the pre-processing phases applied on the dorsal hand vein pattern and a Cartesian-based block matrix representation of the dorsal hand vein pattern, section 3 explains generalized method of moment, in section 4 we compute and analyze the computational complexity of the generalized method of moments and in next section, we perform the vein pattern matching, experimental results are presented in section 6 and finally section 7 concludes the paper.

## **A Cartesian-based block matrix representation of the dorsal hand vein pattern**

In this section, we present a novel approach of representing dorsal hand vein pattern. This approach involves the construction of a block matrix training set to represent the dorsal hand vein features. Firstly, it is important to obtain the vein pattern in the image captured. This procedure requires image acquisition, hand segmentation, vein pattern segmentation, noise filtering and thinning of the vein pattern. After these preprocessing techniques, we obtain an image consisting of a background represented

in black and a thinned vein pattern in white. The following figure shows the resulting image after applying all the preprocessing steps.



**Figure 1:** Biometric Procedure.

The resulting image is represented as a binary matrix of size  $320 \times 240$  where the black color is coded as 0 and the white color is coded as 1. We provide an alternative

matrix representation that converts these binary codes into a two-dimensional cartesian coordinate system where the black color takes value 0 for both the  $x$  and  $y$  co-ordinates and the white color is indexed by its  $i^{th}$  and  $j^{th}$  position in the binary matrix. We illustrate this concept through the following examples: Assume a  $3 \times 3$  sub-matrix from the  $320 \times 240$  binary image matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

We convert this matrix in cartesian system as follows:

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 \end{pmatrix} \quad (2)$$

Ultimately, the size of the two-dimensional cartesian coordinate based image matrix will be of size  $320 \times 240 \times 2$ .

Assuming assume  $I$  images for the training set  $X$ , i.e.,

$$X = [X_1, X_2, \dots, X_i, \dots, X_I] \quad (3)$$

$$X_i = \begin{bmatrix} X_{i,1,1} & X_{i,1,2} & \dots & X_{i,1,240 \times 2} \\ X_{i,2,1} & X_{i,2,2} & \dots & X_{i,2,240 \times 2} \\ \vdots & \vdots & X_{i,j,k} & \vdots \\ X_{i,320,1} & \dots & \dots & X_{i,320,240 \times 2} \end{bmatrix} \quad (4)$$

where

$$X_{i,j,k} = (x_{ij}, y_{ik}) \quad (5)$$

where  $i$  is the index for the  $i^{th}$  image,  $j$  is the corresponding index for the  $x$  co-ordinate of the  $i^{th}$  image,  $k$  is the corresponding index for the  $k$  co-ordinate of the  $i^{th}$  image where  $i = 1, \dots, I$ ,  $j = 1, \dots, 320$  and  $k = 1, \dots, 240 \times 2$ . Thus, the training block matrix  $X$  is of dimension  $320 \times 2 \times I \times 240$ . As it can be noted, the size of this matrix is very large especially when  $I$  is large. Thus, it becomes difficult to work with the original matrix  $X$ . To overcome this problem, we combine the cartesian-based co-ordinate matrices  $X_i$  via a quadratic inference function that is independent of the number of images  $I$ .

### Generalized method of moments (GMM)

Generalized method of moments developed by Hansen [11] is a powerful statistical tool that yields reliable and robust parameter estimates especially in regression analysis. It has been used in a variety of research applications particularly in the field

of econometrics [12], in the analysis of longitudinal data and in solving generalized estimating equations [9], [10]. So far, it has not been applied in the field of biometric security. In this paper, we introduce GMM to detect vein pattern. Conceptually, it is made up of moment functions and an empirical covariance matrix of these moments. Both are combined to form a quadratic inference function (QIF) of the form

$$Q = g^T C^{-1} g \quad (6)$$

where  $g$  is a matrix comprising of moments  $\phi_i$  and  $C$  is the covariance matrix given by

$$C = \frac{1}{I} \sum_{i=1}^I \phi_i \phi_i^T \quad (7)$$

In our context, we define  $\phi_i$  as a  $320 \times 240$  matrix where the  $(j, k)^{th}$  element of  $\phi_i$  are given by

$$\phi_{ij} = x_{ij} - \psi_j \quad (8)$$

and

$$\phi_{ik} = y_{ij} - \psi_k \quad (9)$$

where

$$\psi_j = \frac{1}{I} \sum_{i=1}^I x_{ij} \quad (10)$$

and

$$\psi_k = \frac{1}{I} \sum_{i=1}^I y_{ik} \quad (11)$$

where  $x_{ij}$  and  $y_{ik}$  are the corresponding  $x$  and  $y$  coordinates from equation (5). Thus, the dimension of  $g$  is  $320 \times 240 \times 2$  and  $C$  is of  $320 \times 320$ . Ultimately, the GMM based moment objective function is of size  $2 \times 240$  by  $2 \times 240$  following equation (6). Note the dimension of the training set has been reduced considerably from  $320 \times 240 \times 2$  to  $2 \times 240 \times 2 \times 240$  and equation (6) does not depend on the number of images  $I$ . Thus, even if  $I$  is very large, this will not affect the GMM based QIF.

To generate the space of vein pattern, we use

$$Qv_i = \mu_i v_i \quad (12)$$

For each eigenvector, a family of eigenvein has to be generated. However, many eigenveins are being generated. In order to determine how many eigenveins are required, the following formulae are being used. We have accounted for 90% and 95% of the variation in the training set.

$$\frac{\sum_{i=1}^{2N'} \mu_i}{480} > 0.9 \quad (13)$$

$$\frac{\sum_{i=1}^{2N'} \mu_i}{480} > 0.95 \quad (14)$$

We have already obtained  $2N'$  eigenveins. For each element in the training set, the weight is calculated. This weight will demonstrate the contribution of each eigenvein to respective training element. If the weight is bigger, then the eigenvein has shown the real vein. If the value is less, there is no big contribution with the real vein for that particular eigenvalue. The following operation shows how each element in the training set is projected onto the vein space:

$$\omega_k = \frac{1}{320 \times 240} \sum_{i=1}^{320} \sum_{j=1}^{240} (Qv_k)^T (X_{ij}^T - \phi_j^T) \quad (15)$$

where  $1 < k < 2N'$

Each element in the training set has a weight to determine their contribution to the vein space.

### Computational complexity of QIF

Assuming  $m = 240$  and  $n = 320$ , the QIF algorithm can be summarized in three main steps:

1. Firstly, we require the computation of the matrix  $Q = g^T C^{-1} g$ , where  $g^T$  is of dimension  $2m \times n$ ,  $C^{-1}$  is of dimension  $n \times n$  and  $g$  is of dimension  $n \times 2m$ . It consists of 3 matrix multiplications which requires  $O(mn)$  number of operations and 1 inverse calculation which require  $O(n!)$  number of operations. Thus, the computational complexity is given by  $O(mn) + O(n!)$

2. Next, we need to compute the eigenvalues using the following equation

$$(Q - \mu_i I)v_i = 0, \quad (16)$$

where  $I$  is the identity matrix of size  $2m \times 2m$ . To obtain  $\mu_i$ , we form a characteristic polynomial and evaluate the determinant of  $|Q - \mu_i I| = 0$ . The determinant is evaluated using the Laplacian method and this requires  $O(m!)$  number of operations.

3. The third step involves the calculation of the Eigenveins using the equation

$$(Q - \mu_i I)v_i = 0. \quad (17)$$

We note that this system of equations is homogeneous and assuming a unique solution set, we may solve equation (4.16) using the Gaussian elimination via forward or backward substitution. This requires  $\frac{2m(2m+1)}{2}$  divisions,  $\frac{16m^3 + 12m^2 - 10m}{6}$  multiplications and  $\frac{16m^3 + 12m^2 - 10m}{6}$  subtractions which finally yield  $O(m^3)$  operations.

Therefore, the QIF algorithm requires an overall complexity cost of  $O(mn) + O(n!) + O(m^3) + O(m!)$  operations, which is very huge.

### Improvement of QIF by using LU factorization

We can further improve the complexity by splitting the quadratic inference matrix  $Q$  by an LU factorization or Cholesky decomposition. In order to decrease this computational cost, we propose to change the way of calculating  $(Q - \mu_i I)v_i = 0$  and to calculate the determinant of the characteristic matrix (16). That is, we re-write equation (16) as the follows:

$$LUv_i = 0 \quad (18)$$

where  $(Q - \mu_i I)$  is decomposed into a lower triangular matrix  $L$  and an upper triangular matrix  $U$  such that  $(Q - \mu_i I) = LU$ . Thus, the determinant or the characteristic polynomial of  $(Q - \mu_i I) = 0$  is simply the product of the determinants of  $L$  and  $U$ . This operation requires  $O(m^3)$  operations. Thus, our computations are reduced by a margin of  $|O(m!)|$  operations. Overall, by using the LU factorization, the computational complexity is  $O(mn) + O(n!) + O(m^3)$ .

To recognize an image means to check whether the image exists in the database. When a person wants to get access to the system, the picture of the vein, known as the test image is captured. The coordinates of the test image are obtained and represented as the training set. The weight of the new image is calculated and projected on the vein space [7, 8]. The vein space contains all the vein images. Thus, we have to check whether the input image exist in that space. The Euclidean distance between the projected image and those stored is being calculated. First of all, our system checks whether the test image is a vein by testing it with an arbitrary value. Then the Euclidean distance is computed to check whether the test image exist in the database. If it is vein image, then it is accepted. The results were recorded and analyzed.

### Vein pattern matching

To test the accuracy and efficiency of our proposed method, we need to perform vein pattern matching. Vein pattern matching involves recognizing an image, which means checking whether the image exist in the database. When a person wants to get access to the system, the picture of the vein, known as the test image is captured. The

coordinates of the test image are obtained and represented as the training set. The weight of the new image is calculated and projected on the vein space [6],[7],[8]. The vein space contains all the vein images. Thus, we have to check whether the input image exist in that space. The Euclidean distance between the projected image and those stored is being calculated. First of all, our system checks whether the test image is a vein by testing it with an arbitrary value. Then the Euclidean distance is computed to check whether the test image exist in the database. If it is vein image, then it is accepted. The results were recorded and analyzed.

### Experimental results

The hand dorsal vein biometric was tested using pixel by pixel method, generalized method of moment discussed in this paper and its improved version which uses the LU factorisation. It is to be noted that pixel by pixel method test each individual pixel by counting the number of overlapped pixel in the test image and that of the template found in the database.

In order to test the efficiency and accuracy of the method proposed, false acceptance rate (FAR) and false rejection rate(FRR) are computed. False Acceptance Rate refers to the total number of unauthorized persons getting access to the system over the total number of people attempting to use the system. False Rejection Rate refers to the total number of authorized persons not getting access to the system over the total number of people attempting to get access to the system. The table below shows the FAR and FRR for 20,40,60,80 and 100 images tested.

**Table 1:** FAR and FRR using pixel by pixel method.

Number of images	FAR(%)	FRR(%)
20	0.1000	0.1500
40	0.0250	0.0750
60	0.0340	0.0670
80	0.0375	0.0250
100	0.0400	0.0300

**Table 2:** FAR and FRR using Generalized Method of Moment.

Number of images	FAR(%)	FRR(%)
20	0.0500	0.0600
40	0.0250	0.0500
60	0.0340	0.0340
80	0.0250	0.0125
100	0.0200	0.0300

**Table 2:** FAR and FRR using Generalized Method of Moment with LU factorization.

Number of images	FAR(%)	FRR(%)
20	0.0500	0.0600
40	0.0250	0.0500
60	0.0340	0.0340
80	0.0250	0.0125
100	0.0200	0.0310

According to the results obtained, the FAR and FRR is less when using generalized method of moment compared to pixel by pixel method. LU factorization does not affect the FAR and FRR of generalized method of moment.

In order to test the efficiency of our proposed method, we have computed the matching time of the method illustrated in the table below:

**Table 2:** Comparison of matching time.

Number of images	Matching time using pixel by pixel(in second)	Matching time using Generalized method of moment (in second)	Matching time using Generalized method of moment integrating LU factorization(in second)
20	275	131	105
40	580	300	225
60	843	452	342
80	1130	576	489
100	1400	703	535

From the results obtained, it is noticed that the matching time of the proposed method Generalized Method of Moment is less compared to the pixel by pixel method. It is also noted that the matching time of the Generalized method of moment with LU factorization consumes less time compared to the other two methods.

## Conclusion

Different methods are devised to extract and represent the dorsal hand vein pattern features. In this paper, we have applied the generalized method of moments and the generalized method of moments with LU factorization to extract dorsal hand vein pattern. We have also provided their computational complexity to determine their efficiencies. Generalized method of moment through improvement by the LU factorization proves to be more efficient than pixel by pixel and generalized method of moment.

The FRR and FAR were computed and are found to be less when using our proposed methods. It also reduces the dimension of the matrices which consequently has an impact on matching time. The matching time is improved in our proposed method and this is desired in all biometric security system. We have compared our method with pixel by pixel based method with generalized method of moments and LU factorization.

### Acknowledgments

We express our deepest thanks to Prof. Ahmed M. Badawi, from University of Tennessee, Knoxville, for providing us with a dataset of 100 images of hand dorsal vein pattern.

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