IVF Almost Generalized Semi-Precontinuous Mappings

R. Jeyabalan

Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.
E-mail: jeyram84@gmail.com

Abstract

In this paper we introduce interval valued fuzzy almost generalized semi-precontinuous mappings. We investigate some of its properties. Also we provide the relation between interval valued fuzzy almost generalized semi-pre continuous mappings and other interval valued fuzzy continuous mappings.

Keywords: IVF-set, IVF-topological space, IVF-point, IVF-generalized semi-preclosed set, IVF-continuous mappings.

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1 INTRODUCTION

The concept of fuzzy subset was introduced and studied by L. A. Zadeh [13] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [12]. Tapas Kumar Mondal and S. K. Samantha [9] introduced the topology of interval valued fuzzy sets. Jeyabalan, R and Arjunan, K. [6] introduced interval valued fuzzy generalized semi-preclosed sets. In this paper, we introduce that IVF-almost generalized semi-precontinuous mappings and some properties are investigated.
2 PRELIMINARIES

Definition 2.1 [9] Let $X$ be a non empty set. A mapping $\overline{A} : X \rightarrow D[0,1]$ is called an interval valued fuzzy set (briefly IVFS) on $X$, where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and, for all $x \in X$, where $A^-(x)$ and $A^+(x)$ are fuzzy sets of $X$ such that $A^-(x) \leq A^+(x)$, for all $x \in X$. Thus $\overline{A}(x)$ is an interval (a closed subset of $[0,1]$) and not a number form the interval $[0,1]$ as in the case of fuzzy set.

Notation 2.2 $D^X$ denotes the set of all interval valued fuzzy subsets of a non empty set $X$.

Definition 2.3 [9] Let $X$ be a non empty set. Let $x_0 \in X$ and $\alpha \in D[0,1]$ be fixed such that $\alpha \neq [0,0]$. Then the interval valued fuzzy subset $p^\alpha_{x_0}$ is called an interval valued fuzzy point defined by,

$$p^\alpha_{x_0} = \begin{cases} \alpha & \text{if } x = x_0 \\ [0,0] & \text{if } x \neq x_0. \end{cases}$$

Definition 2.4 [9] Let $\overline{A}$ and $\overline{B}$ be any two IVFS of $X$, that is $\overline{A} = \{ x, [A^-(x), A^+(x)] : x \in X \}$, $\overline{B} = \{ x, [B^-(x), B^+(x)] : x \in X \}$. We define the following relations and operations:

(i) $\overline{A} \subseteq \overline{B}$ if and only if $A^-(x) \leq B^-(x)$ and $A^+(x) \leq B^+(x)$, for all $x \in X$.

(ii) $\overline{A} = \overline{B}$ if and only if $A^-(x) = B^-(x)$ and $A^+(x) = B^+(x)$, for all $x \in X$.

(iii) $(\overline{A})^c = \overline{1 - A} = \{ x, [1 - A^+(x), 1 - A^-(x)] : x \in X \}$.

(iv) $\overline{A} \cap \overline{B} = \{ x, [\min\{A(x), B(x)\}, \min\{A^+(x), B^+(x)\}] : x \in X \}$.

(v) $\overline{A} \cup \overline{B} = \{ x, [\max\{A(x), B(x)\}, \max\{A^+(x), B^+(x)\}] : x \in X \}$.

We denote by $\overline{0}_x$ and $\overline{1}_x$ for the interval valued fuzzy sets $\{ x, [0,0] \}$, for all $x \in X$ and $\{ x, [1,1] \}$, for all $x \in X$ respectively.

Definition 2.5 [9] Let $X$ be a set and $\mathcal{F}$ be a family of interval valued fuzzy sets (IVFSs) of $X$. The family $\mathcal{F}$ is called an interval valued fuzzy topology (IVFT) on $X$ if and only if $\mathcal{F}$ satisfies the following axioms:

(i) $\overline{0}_x, \overline{1}_x \in \mathcal{F}$,

(ii) If $\{ \overline{A}_i : i \in I \} \subseteq \mathcal{F}$, then $\bigcup_{i \in I} \overline{A}_i \in \mathcal{F}$,

(iii) If $\overline{A}_1, \overline{A}_2, \overline{A}_3, \ldots, \overline{A}_n \in \mathcal{F}$, then $\bigcap_{i = 1}^n \overline{A}_i \in \mathcal{F}$.

The pair $(X, \mathcal{F})$ is called an interval valued fuzzy topological space (IVFTS).
The members of \( \mathcal{S} \) are called interval valued fuzzy open sets \((IVFOS)\) in \( X \).

An interval valued fuzzy set \( \overline{A} \) in \( X \) is said to be interval valued fuzzy closed set \((IVFCS)\) in \( X \) if and only if \( (\overline{A})^c \) is an \( IVFOS \) in \( X \).

**Definition 2.6** [9] Let \((X, \mathcal{S})\) be an \( IVFTS \) and \( \overline{A} = \{ x, [A^-(x), A^+(x)] : x \in X \} \) be an \( IVFS \) in \( X \). Then the interval valued fuzzy interior and interval valued fuzzy closure of \( \overline{A} \) denoted by \( ivfint(\overline{A}) \) and \( ivfcl(\overline{A}) \) are defined by

\[
ivfint(\overline{A}) = \bigcup \{ \overline{G} : \overline{G} \text{ is an IVFOS in } X \text{ and } \overline{G} \subseteq \overline{A} \},
\]

\[
ivfcl(\overline{A}) = \bigcap \{ \overline{K} : \overline{K} \text{ is an IVFCS in } X \text{ and } \overline{A} \subseteq \overline{K} \}.
\]

For any \( IVFS \) \( \overline{A} \) in \((X, \mathcal{S})\), we have \( ivfcl(\overline{A}^c) = (ivfint(\overline{A}))^c \) and \( ivfint(\overline{A}^c) = (ivfcl(\overline{A}))^c \).

**Definition 2.7** An \( IVFS \) \( \overline{A} = \{ x, [A^-(x), A^+(x)] : x \in X \} \) in an \( IVFTS \) \((X, \mathcal{S})\) is said to be an

(i) IVF regular closed set \((IVFRCS)\) if \( \overline{A} = ivfcl(ivfint(\overline{A})) \);

(ii) IVF semi-closed set \((IVFSCS)\) if \( ivfint(ivfcl(\overline{A})) \subseteq \overline{A} \);

(iii) IVF preclosed set \((IVFPCS)\) if \( ivfcl(ivfint(\overline{A})) \subseteq \overline{A} \);

(iv) IVF \( \alpha \) closed set \((IVF\alpha CS)\) if \( ivfcl(ivfint(ivfcl(\overline{A}))) \subseteq \overline{A} \);

(v) IVF \( \beta \) closed set \((IVF\beta CS)\) if \( ivfint(ivfcl(ivfint(\overline{A}))) \subseteq \overline{A} \).

**Definition 2.8** An \( IVFS \) \( \overline{A} = \{ x, [A^-(x), A^+(x)] : x \in X \} \) in an \( IVFTS \) \((X, \mathcal{S})\) is said to be an

(i) interval valued fuzzy generalized closed set \((IVFGCS)\) if \( ivfcl(\overline{A}) \subseteq \overline{U} \), whenever \( \overline{A} \subseteq \overline{U} \) and \( \overline{U} \) in an \( IVFOS \);

(ii) interval valued fuzzy generalized regular closed set \((IVFGRCS)\) if \( ivfcl(\overline{A}) \subseteq \overline{U} \), whenever \( \overline{A} \subseteq \overline{U} \) and \( \overline{U} \) is an \( IVFROS \).

**Definition 2.9** An \( IVFS \) \( \overline{A} = \{ x, [A^-(x), A^+(x)] : x \in X \} \) in an \( IVFTS \) \((X, \mathcal{S})\) is said to be an

(i) interval valued fuzzy semi-preclosed set \((IVFSPCS)\) if there exist on
\textbf{Definition 2.10} Let $\overline{A}$ be an IVFS in IVFTS $(X, \mathcal{I})$. Then the interval valued fuzzy semi-preinterior of $\overline{A}$ (ivfspint(\overline{A})) and the interval valued fuzzy semi-preclosure of $\overline{A}$ (ivfspcl(\overline{A})) are defined by

\[
\text{ivfspint}(\overline{A}) = \bigcup\{G : G \text{ is an IVFPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},
\]

\[
\text{ivfspcl}(\overline{A}) = \bigcap\{K : K \text{ is an IVFSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.
\]

For any IVFS $\overline{A}$ in $(X, \mathcal{I})$, we have $\text{ivfspcl}(\overline{A}^c) = (\text{ivfspint}(\overline{A}))^c$ and $\text{ivfspint}(\overline{A}^c) = (\text{ivfspcl}(\overline{A}))^c$.

\textbf{Definition 2.11} [6] An IVFS $\overline{A}$ in IVFTS $(X, \mathcal{I})$ is said to be an interval valued fuzzy generalized semi-preclosed set (IVFGSPCS) if $\text{ivfspcl}(\overline{A}) \subseteq \overline{U}$, whenever $\overline{A} \subseteq \overline{U}$ and $\overline{U} \in \mathcal{I}$.

\textbf{Definition 2.12} [6] The complement $\overline{A}^c$ of an IVFGSPCS $\overline{A}$ in an IVFTS $(X, \mathcal{I})$ is called an interval valued fuzzy generalized semi-preopen set (IVFGSPOS) in $X$.

\textbf{Definition 2.13} Let $\overline{A}$ be an IVFS in IVFTS $(X, \mathcal{I})$. Then the interval valued fuzzy generalized semi-preinterior of $\overline{A}$ (ivfgspint(\overline{A})) and the interval valued fuzzy generalized semi-preclosure of $\overline{A}$ (ivfgspcl(\overline{A})) are defined by

\[
\text{ivfgspint}(\overline{A}) = \bigcup\{G : G \text{ is an IVFGSPOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},
\]

\[
\text{ivfgspcl}(\overline{A}) = \bigcap\{K : K \text{ is an IVFGSPCS in } X \text{ and } \overline{A} \subseteq \overline{K}\}.
\]

For any IVFS $\overline{A}$ in $(X, \mathcal{I})$, we have $\text{ivfgspcl}(\overline{A}^c) = (\text{ivfgspint}(\overline{A}))^c$ and $\text{ivfgspint}(\overline{A}^c) = (\text{ivfgspcl}(\overline{A}))^c$.

\textbf{Definition 2.14} An IVFTS $(X, \mathcal{I})$ is called an interval valued fuzzy semi-pre $1/2$ space (IVFSPT$_{1/2}$) if every IVFGSPCS is an IVFSPCS in $X$.

\textbf{Definition 2.15} [9] An IVFS $\overline{A}$ of a IVFTS of $(X, \mathcal{I})$ is said to be an interval valued fuzzy neighbourhood(IVFN) of an IVFP $p^\alpha_0$, if there exists an IVFOS $\overline{B}$ in $X$ such that $p^\alpha_0 \in \overline{B} \subseteq \overline{A}$. 
**Definition 2.16** Let \((X, \mathcal{I})\) and \((Y, \sigma)\) be IVFTSs. Then a map \(g : X \to Y\) is called an

(i) interval valued fuzzy continuous (IVF continuous mapping) if \(g^{-1}(\overline{B})\) is IVFOS in \(X\) for all \(\overline{B}\) in \(\sigma\).

(ii) interval valued fuzzy semi-continuous mapping (IVFS-continuous mapping) if \(g^{-1}(\overline{B})\) is IVFSOS in \(X\) for all \(\overline{B}\) in \(\sigma\).

(iii) interval valued fuzzy \(\alpha\)-continuous mapping (IVF\(\alpha\)-continuous mapping) if \(g^{-1}(\overline{B})\) is IVF\(\alpha\)OS in \(X\) for all \(\overline{B}\) in \(\sigma\).

(iv) interval valued fuzzy pre-continuous mapping (IVFP-continuous mapping) if \(g^{-1}(\overline{B})\) is IVFPSOS in \(X\) for all \(\overline{B}\) in \(\sigma\).

(v) interval valued fuzzy \(\beta\)-continuous mapping (IVF\(\beta\)-continuous mapping) if \(g^{-1}(\overline{B})\) is IVF\(\beta\)OS in \(X\) for all \(\overline{B}\) in \(\sigma\).

(vi) interval valued fuzzy semi-precontinuous mapping (IVFSP-continuous mapping) if \(g^{-1}(\overline{B})\) is IVFPSOS in \(X\) for all \(\overline{B}\) in \(\sigma\).

**Definition 2.17** An IVFS \(\overline{A}\) is said to be interval valued fuzzy dense (IVFD) in another IVFS \(\overline{B}\) in an IVFT \((X, \mathcal{I})\), if \(\text{ivfcl}(\overline{A}) = \overline{B}\).

**Definition 2.18** Let \((X, \mathcal{I})\) and \((Y, \sigma)\) be IVFTSs. Then a map \(g : X \to Y\) is called interval valued fuzzy generalized continuous (IVFG continuous mapping) if \(g^{-1}(\overline{B})\) is IVFGCS in \(X\) for all \(\overline{B}\) in \(\sigma^e\).

**Definition 2.19** A mapping \(g : (X, \mathcal{I}) \to (Y, \sigma)\) is called an interval valued fuzzy almost generalized semi-precontinuous (IVFaGSP continuous) mapping if \(g^{-1}(\overline{V})\) is an IVFaGSPCS in \(X\) for every \(\overline{V}\) in \(Y\).

**Example 2.20** Let \(X = \{a, b\}\), \(Y = \{u, v\}\) and
\[
\overline{K}_{a} = \{< a, [0.1,0.2]>, < b, [0.3,0.4] >\},
\overline{L}_{a} = \{< a, [0.3,0.4]>, < u, [0.4,0.6] >\}.
\]
Then \(\mathcal{I} = \{\overline{K}_{a}, \overline{L}_{a}\}\) and \(\sigma = \{\overline{u}, \overline{v}\}\) are IVFT on \(X\) and \(Y\) respectively. Define a mapping \(g : (X, \mathcal{I}) \to (Y, \sigma)\) by \(g(a) = u\) and \(g(b) = v\). Then \(g\) is an IVFaGSP continuous mapping.

### 3 MAIN RESULTS

**Theorem 3.1** Every IVF continuous mapping is an IVFaGSP continuous mapping.

**Proof.** Let \(g : (X, \mathcal{I}) \to (Y, \sigma)\) be an IVF continuous mapping. Let \(\overline{V}\) be an IVFRCS in \(Y\). Since every IVFRCS is an IVFCS, \(\overline{V}\) is an IVFCS in \(Y\). Then...
\(g^{-1}(\overline{V})\) is an IVFCS in \(X\), by hypothesis. Since every IVFCS is an IVFGSPCS, \(g^{-1}(\overline{V})\) is an IVFGSPCS in \(X\). Hence \(g\) is an IVFaGSP continuous mapping.

**Remark 3.2** The converse of the above theorem 3.1 need not be true from the following example: Let \(X = \{a, b\}, Y = \{u, v\}\) and
\[
\begin{align*}
\overline{K}_1 &= \{< a, [0.1,0.2] >, < b, [0.3,0.4] >\}, \\
\overline{L}_1 &= \{< u, [0.8,0.9] >, < v, [0.6,0.7] >\}.
\end{align*}
\]
Then \(\exists = \{0_X, \overline{K}_1, \overline{L}_1\}\) and \(\sigma = \{0_Y, \overline{M}_1, \overline{L}_1\}\) are IVFTs on \(X\) and \(Y\) respectively. Define a mapping \(g : (X, \exists) \to (Y, \sigma)\) by \(g(a) = u\) and \(g(b) = v\). Then \(g\) is an IVFaGSP continuous mapping but not an IVF continuous mapping. Since \(\overline{L}_1 = \{< u, [0.1,0.2] >, < v, [0.3,0.4] >\}\) is an IVFCS in \(Y\) but \(g^{-1}(\overline{L}_1) = \{< a, [0.1,0.2] >, < b, [0.3,0.4] >\}\) is not an IVFCS in \(X\), because
\[
\text{ivfc}(g^{-1}(\overline{L}_1)) = \overline{1}_X \neq g^{-1}(\overline{L}_1).
\]

**Theorem 3.3** Every IVFG continuous mapping is an IVFaGSP continuous mapping.

**Proof.** Let \(g : (X, \exists) \to (Y, \sigma)\) be an IVF continuous mapping. Let \(\overline{V}\) be an IVFRCS in \(Y\). Since every IVFRCS is an IVFCS, \(\overline{V}\) is an IVFCS in \(Y\). Then \(g^{-1}(\overline{V})\) is an IVFGCS in \(X\). Since every IVFGCS is an IVFGSPCS, \(g^{-1}(\overline{V})\) is an IVFGSPCS in \(X\). Hence \(g\) is an IVFaGSP continuous mapping.

**Remark 3.4** The converse of the above theorem 3.3 need not be true from the following example: Let \(X = \{a, b\}, Y = \{u, v\}\) and
\[
\begin{align*}
\overline{K}_1 &= \{< a, [0.4,0.5] >, < b, [0.6,0.7] >\}, \\
\overline{L}_1 &= \{< a, [0.6,0.7] >, < b, [0.7,0.8] >\}, \\
\overline{M}_1 &= \{< u, [0.4,0.8] >, < v, [0.7,0.9] >\}.
\end{align*}
\]
Then \(\exists = \{0_X, \overline{K}_1, \overline{L}_1, \overline{M}_1\}\) and \(\sigma = \{0_Y, \overline{M}_1, \overline{L}_1\}\) are IVFTs on \(X\) and \(Y\) respectively. Define a mapping \(g : (X, \exists) \to (Y, \sigma)\) by \(g(a) = u\) and \(g(b) = v\). Then \(g\) is an IVFaGSP continuous mapping but not an IVFG continuous mapping, since \(\overline{M}_1\) is IVFCS in \(Y\) but \(g^{-1}(\overline{M}_1)\) is not an IVFGCS in \(X\), because \(g^{-1}(\overline{M}_1) \subseteq \overline{K}_1\) and \(g^{-1}(\overline{M}_1) \subseteq \overline{L}_1\) but
\[
\text{ivfc}(g^{-1}(\overline{M}_1)) = \overline{1}_X \subset \overline{K}_1\text{ and } \text{ivfc}(g^{-1}(\overline{M}_1)) = \overline{1}_Y \subset \overline{L}_1.
\]

**Theorem 3.5** Every IVFS continuous mapping is an IVFaGSP continuous mapping.

**Proof.** Let \(g : (X, \exists) \to (Y, \sigma)\) be an IVF continuous mapping. Let \(\overline{V}\) be an IVFRCS in \(Y\). Since every IVFRCS is an IVFCS, \(\overline{V}\) is an IVFCS in \(Y\). Then
$g^{-1}(\overline{V})$ is an IVFSCS in $X$. Since every IVFSCS is an IVFGSPCS, $g^{-1}(\overline{V})$ is an IVFGSPCS in $X$. Hence $g$ is an IVFaGSP continuous mapping.

**Remark 3.6** The converse of the above theorem 3.5 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$\overline{K}_0 = \{<a, [0.1, 0.2]>, <b, [0.3, 0.4]>\},$$

$$\overline{L}_0 = \{<u, [0.3, 0.4]>, <v, [0.4, 0.6]>\}.$$

Then $\mathcal{A} = \left\{0_x, \overline{K}_0, \overline{L}_0 \right\}$ and $\sigma = \left\{0_y, \overline{L}_y, \overline{L}_y \right\}$ are IVFTs on $X$ and $Y$ respectively. Define a mapping $g : (X, \mathcal{A}) \to (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then $g$ is an IVFaGSP continuous mapping but not an IVFS continuous mapping. Since $\overline{L}_y = \{<u, [0.6, 0.7]>, <v, [0.4, 0.6]>\}$ is an IVFCS in $Y$ but $g^{-1}(\overline{L}_y) = \{<a, [0.6, 0.7]>, <b, [0.4, 0.6]>\}$ is not an IVFSCS in $X$, because $ivfint(ivfcl(g^{-1}(\overline{L}_y))) = ivfint(\overline{L}_0) = \overline{L}_0 \notin g^{-1}(\overline{L}_y)$.

**Theorem 3.7** Every IVFP continuous mapping is an IVFaGSP continuous mapping.

**Proof.** Let $g : (X, \mathcal{A}) \to (Y, \sigma)$ be an IVF continuous mapping. Let $\overline{V}$ be an IVFRCS in $Y$. Since every IVFRCS is an IVFCS, $\overline{V}$ is an IVFCS in $Y$. Then $g^{-1}(\overline{V})$ is an IVFPCS in $X$. Since every IVFPCS is an IVFGSPCS, $g^{-1}(\overline{V})$ is an IVFGSPCS in $X$. Hence $g$ is an IVFaGSP continuous mapping.

**Remark 3.8** The converse of the above theorem 3.7 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$\overline{K}_0 = \{<a, [0.3, 0.4]>, <b, [0.4, 0.7]>\},$$

$$\overline{L}_0 = \{<u, [0.1, 0.2]>, <v, [0.3, 0.4]>\}.$$

Then $\mathcal{A} = \left\{0_x, \overline{K}_0, \overline{L}_0 \right\}$ and $\sigma = \left\{0_y, \overline{L}_y, \overline{L}_y \right\}$ are IVFTs on $X$ and $Y$ respectively. Define a mapping $g : (X, \mathcal{A}) \to (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then $g$ is an IVFaGSP continuous mapping but not an IVFP continuous mapping. Since $\overline{L}_y = \{<u, [0.8, 0.9]>, <v, [0.6, 0.7]>\}$ is an IVFCS in $Y$ and $g^{-1}(\overline{L}_y) = \{<a, [0.8, 0.9]>, <b, [0.6, 0.7]>\}$ is not an IVFPCS in $X$, because $ivfcl(ivfint(g^{-1}(\overline{L}_y))) = ivfcl(\overline{K}_0) = \overline{K}_0 \notin g^{-1}(\overline{L}_y)$.

**Theorem 3.9** Every IVFSP continuous mapping is an IVFaGSP continuous mapping.

**Proof.** Let $g : (X, \mathcal{A}) \to (Y, \sigma)$ be an IVFSP continuous mapping. Let $\overline{V}$ be an IVFRCS in $Y$. Since every IVFRCS is an IVFSPCS, $\overline{V}$ is an IVFSPCS in $Y$. Since every...
Then $g^{-1}(\overline{V})$ is an IVFSPCS in X. Since every IVFSPCS is an IVFGSPCS, $g^{-1}(\overline{V})$ is an IVFGSPCS in X. Hence g is an IVFaGSP continuous mapping.

**Remark 3.10** The converse of the above theorem 3.9 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$\overline{K}_a = \{a, [0.3, 0.4], b, [0.1, 0.2]\},$$

$$\overline{L}_a = \{u, [0.1, 0.2], v, [0.3, 0.4]\}.$$

Then $\mathcal{A} = \{0_x, \overline{K}_a, \overline{L}_a\}$ and $\sigma = \{0_y, \overline{L}_a, \overline{L}_a\}$ are IVFTs on X and Y respectively. Define a mapping $g : (X, \mathcal{A}) \rightarrow (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an IVFaGSP continuous mapping but not an IVFSP continuous mapping. Since $\overline{L}_a = \{u, [0.8, 0.9], v, [0.6, 0.7]\}$ is an IVFCS in Y but $g^{-1}(\overline{L}_a) = \{a, [0.8, 0.9], b, [0.6, 0.7]\}$ is not an IVFSPCS in X, because there exist no IVFPDS $\overline{B}$ in X such that $\text{ivfint}(g^{-1}(\overline{L}_a)) \subseteq \overline{B} \subseteq g^{-1}(\overline{L}_a)$.

**Theorem 3.11** Every IVFβ continuous mapping is an IVFαGSP continuous mapping.

**Proof.** Let $g : (X, \mathcal{A}) \rightarrow (Y, \sigma)$ be an IVFSP continuous mapping. Let $\overline{V}$ be an IVFRCS in Y. Since every IVFRCS is an IVFβCS, $\overline{V}$ is an IVFβCS in Y. Then $g^{-1}(\overline{V})$ is an IVFβCS in X. Since every IVFβCS is an IVFGSPCS, $g^{-1}(\overline{V})$ is an IVFGSPCS in X. Hence g is an IVFaGSP continuous mapping.

**Remark 3.12** The converse of the above theorem 3.11 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and

$$\overline{K}_a = \{a, [0.5, 0.7], b, [0.3, 0.4]\},$$

$$\overline{L}_a = \{u, [0.3, 0.4], v, [0.4, 0.6]\}.$$

Then $\mathcal{A} = \{0_x, \overline{K}_a, \overline{L}_a\}$ and $\sigma = \{0_y, \overline{L}_a, \overline{L}_a\}$ are IVFTs on X and Y respectively. Define a mapping $g : (X, \mathcal{A}) \rightarrow (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an IVFaGSP continuous mapping but not an IVFβ continuous mapping. Since $\overline{L}_a = \{u, [0.6, 0.7], v, [0.4, 0.6]\}$ is an IVFCS in Y but $g^{-1}(\overline{L}_a) = \{a, [0.6, 0.7], b, [0.4, 0.6]\}$ is not an IVFβCS in X, because $\text{ivfint}(\text{ivfcl}(\text{ivfint}(g^{-1}(\overline{L}_a)))) = \text{ivfint}(\text{ivfcl}(\overline{K}_a)) = \text{ivfint}(\overline{L}_a) = \overline{L}_a \not\subseteq g^{-1}(\overline{L}_a)$.

**Theorem 3.13** Every IVFα continuous mapping is an IVFαGSP continuous mapping.

**Proof.** Let $g : (X, \mathcal{A}) \rightarrow (Y, \sigma)$ be an IVFSP continuous mapping. Let $\overline{V}$ be an
IVFRCS in Y. Since every IVFRCS is an IVFαCS, \( \overline{V} \) is an IVFαCS in Y. Then \( g^{-1}(\overline{V}) \) is an IVFαCS in X. Since every IVFαCS is an IVFGSPCS, \( g^{-1}(\overline{V}) \) is an IVFGSPCS in X. Hence \( g \) is an IVFaGSP continuous mapping.

**Remark 3.14** The converse of the above theorem 3.13 need not be true from the following example: Let \( X = [a,b], Y = \{u,v\} \) and
\[
\begin{align*}
\overline{K}_x &= \{<a,[0.3,0.4]>,<b,[0.4,0.6]>, \overline{L}_x = \{<u,[0.1,0.2]>,<v,[0.3,0.4]>. \\
\end{align*}
\]

Then \( \mathcal{Z} = \{0_x,K_x,\overline{K}_x\} \) and \( \sigma = \{0_y,L_y,\overline{L}_y\} \) are IVFT on X and Y respectively. Define a mapping \( g : (X,\mathcal{Z}) \rightarrow (Y,\sigma) \) by \( g(a) = u \) and \( g(b) = v \). Then \( g \) is an IVFaGSP continuous mapping but not an IVFα continuous mapping. Since \( \overline{L}_x = \{<u,[0.8,0.9]>,<v,[0.6,0.7]>\} \) is an IVFCS in Y but \( g^{-1}(\overline{L}_x) = \{<a,[0.8,0.9]>,<b,[0.6,0.7]>\} \) is not an IVFαCS in X, because
\[
ivfcl(ivfint(ivfcl(g^{-1}(\overline{L}_x)))) = ivfcl(ivfcl(\overline{L}_x)) = ivfcl(\overline{L}_x) = \overline{I}_x \neq g^{-1}(\overline{L}_x).
\]

**Theorem 3.15** Let \( g : (X,\mathcal{Z}) \rightarrow (Y,\sigma) \) be a mapping where \( g^{-1}(\overline{V}) \) is an IVFRCS in X, for every IVFCS \( \overline{V} \) in Y. Then \( g \) is an IVFaGSP continuous mapping.

**Proof.** Let \( \overline{A} \) be an IVFRCS in Y. Since every IVFRCS is an IVFCS, \( \overline{V} \) is an IVFCS in Y. Then \( g^{-1}(\overline{V}) \) is an IVFRCS in X. Since every IVFRCS is an IVFGSPCS, \( g^{-1}(\overline{V}) \) is an IVFGSPCS in X. Hence \( g \) is an IVFaGSP continuous mapping. Then \( g^{-1}(\overline{V}) \) is an IVFRCS in X. Since every IVFRCS is an IVFGSPCS, \( g^{-1}(\overline{V}) \) is an IVFGSPCS in X. Hence \( g \) is an IVFaGSP continuous mapping.

**Remark 3.16** The converse of the above theorem 3.15 need not be true from the following example: Let \( X = [a,b], Y = \{u,v\} \) and
\[
\begin{align*}
\overline{K}_x &= \{<a,[0.3,0.4]>,<b,[0.4,0.7]>, \overline{L}_x = \{<u,[0.1,0.2]>,<v,[0.3,0.4]>. \\
\end{align*}
\]

Then \( \mathcal{Z} = \{0_x,K_x,\overline{K}_x\} \) and \( \sigma = \{0_y,L_y,\overline{L}_y\} \) are IVFT on X and Y respectively. Define a mapping \( g : (X,\mathcal{Z}) \rightarrow (Y,\sigma) \) by \( g(a) = u \) and \( g(b) = v \). Then \( g \) is an IVFGSP continuous mapping but not a mapping as defined in theorem 3.15, since \( \overline{L}_x = \{<u,[0.8,0.9]>,<v,[0.6,0.7]>\} \) is an IVFCS in Y and \( g^{-1}(\overline{L}_x) = \{<a,[0.8,0.9]>,<b,[0.6,0.7]>\} \) is not an IVFRCS in X, because
\[
ivfcl(ivfint(ivfcl(g^{-1}(\overline{L}_x)))) = ivfcl(\overline{K}_x) = \overline{I}_x \neq g^{-1}(\overline{L}_x).
\]
Theorem 3.17 Every IVFGSP continuous mapping is an IVFaGSP - continuous mapping.

Proof. Let \( g : X \rightarrow Y \) be an IVFGSP -continuous mapping. Let \( \tilde{A} \) be an IVFRCS in \( Y \). Then \( \tilde{A} \) is an IVFCS in \( Y \). By hypothesis \( g^{-1}(\tilde{A}) \) is an IVFGSPS in \( X \). Hence \( g \) is an IVFaGSP continuous mapping.

Remark 3.18 The converse of the above theorem 3.17 need not be true from the following example: Let \( X = [a,b] \), \( Y = [u,v] \) and

\[
\begin{align*}
K_1 &= \{<a,[0.4,0.5]>, <b,[0.6,0.7]>\}, \\
L_1 &= \{<a,[0.6,0.7]>, <b,[0.7,0.8]>\}, \\
M_1 &= \{<u,[0.4,0.8]>, <v,[0.7,0.9]>\}, \\
N_1 &= \{<u,[0.3,0.5]>, <v,[0.5,0.7]>\},
\end{align*}
\]

Then \( \mathcal{S} = \{0_\mathcal{K}, K_1, L_1, M_1 \} \) and \( \sigma = \{0_\mathcal{Y}, M_1, N_1, L_1 \} \) are IVFT on \( X \) and \( Y \) respectively. Define a mapping \( g : (X, \mathcal{S}) \rightarrow (Y, \sigma) \) by \( g(a) = u \) and \( g(b) = v \). Then \( g \) is an IVFaGSP continuous mapping but not an IVFGSP continuous mapping, since \( M_1^- = \{<u,[0.2,0.6]>, <v,[0.1,0.3]>\} \) is IVFCS in \( Y \) but \( g^{-1}(M_1^-) \) is not an IVFGSPCS in \( Y \), \( g^{-1}(M_1^-) = \{<a,[0.2,0.6]>, <b,[0.1,0.3]>\} \subseteq \overline{K}_1 \), but ivfscr\( (g^{-1}(M_1^-)) = \overline{L}_1 \subset \overline{K}_1 \).

Theorem 3.19 Let \( p^a_0 \) be an IVFP in \( X \). A mapping \( g : X \rightarrow Y \) is an IVFaGSP continuous mapping, then for every IVFO \( \overline{A} \) in \( Y \) with \( g(p^a_0) \in \overline{A} \), there exists an IVFOS \( \overline{B} \) in \( X \) with \( p^a_0 \in \overline{B} \) such that \( g^{-1}(\overline{A}) \) is IVFD in \( \overline{B} \).

Proof. Let \( \overline{A} \) be an IVFROS in \( Y \). Then \( \overline{A} \) is an IVFOS in \( Y \). Let \( g(p^a_0) \in \overline{A} \), then there exists an IVFOS \( \overline{B} \) in \( X \) such that \( p^a_0 \in \overline{B} \) and ivfcl\( (g^{-1}(\overline{A})) = \overline{B} \). Since \( \overline{B} \) is an IVFOS, ivfcl\( (g^{-1}(\overline{A})) \) is also an IVFOS in \( X \). Therefore ivfint(ivfcl\( (g^{-1}(\overline{A})) = ivfcl\( (g^{-1}(\overline{A})) \).

Now \( g^{-1}(\overline{A}) \subseteq ivfcl\( (g^{-1}(\overline{A})) \subseteq ivfcl\( (ivfint(ivfcl\( (g^{-1}(\overline{A}))\))) \). This implies \( g^{-1}(\overline{A}) \) is an IVFROS in \( X \) and hence an IVFGSPOS in \( X \). Thus \( g \) is an IVFaGSP continuous mappings. W

Theorem 3.20 Let \( f : X \rightarrow Y \) be a mapping where \( X \) is an IVFSPT\(_{1/2} \) space. Then the following are equivalent:

(i) \( g \) is an IVFaGSP continuous mapping.
(ii) \( \text{ivfspcl}(g^{-1}(A)) \subseteq g^{-1}(\text{ivfcl}(A)) \) for every IVFSPOS \( A \) in \( Y \),
(iii) \( \text{ivfspcl}(g^{-1}(A)) \subseteq g^{-1}(\text{ivfcl}(A)) \) for every IVFSOS \( A \) in \( Y \),
(iv) \( g^{-1}(A) \subseteq \text{ivfspint}(g^{-1}(\text{ivfint}(\text{ivfcl}(A)))) \) for every IVFPOS \( A \) in \( Y \).

**Proof.** (i) \( \Leftrightarrow \) (ii) Let \( \bar{A} \) be an IVFSPOS in \( Y \). Then by definition 2.9, there exists an IVFPOS \( \bar{B} \) such that \( \bar{B} \subseteq \bar{A} \subseteq \text{ivfcl}(\bar{B}) \) and \( \bar{B} \subseteq \text{ivfint}(\text{ivfcl}(\bar{B})) \). Now \( \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \supseteq \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{B}))) \supseteq \text{ivfcl}(\bar{B}) \supseteq \bar{A} \). Hence \( \bar{A} \subseteq \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \). Therefore \( \text{ivfcl}(\bar{A}) \subseteq \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \). But \( \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \subseteq \text{ivfcl}(\bar{A}) \). Hence \( \text{ivfcl}(\text{ivfint}(\text{ivfcl}(\bar{A}))) = \text{ivfcl}(\bar{A}) \). This implies \( \text{ivfcl}(\bar{A}) \) is an IVFRCS in \( (X, \mathfrak{A}) \). By hypothesis \( g^{-1}(\text{ivfcl}(\bar{A})) \) is an IVFGSCPS in \( X \) and hence \( g^{-1}(\text{ivfcl}(\bar{A})) \) is an IVFPS in \( X \), since \( X \) is an IVFSP\(_{1/2} \) space. This implies \( \text{ivfspcl}(g^{-1}(\text{ivfcl}(\bar{A}))) = g^{-1}(\text{ivfcl}(\bar{A})) \). Now \( \text{ivfspcl}(g^{-1}(\bar{A})) \subseteq \text{ivfspcl}(g^{-1}(\text{ivfcl}(\bar{A}))) = g^{-1}(\text{ivfcl}(\bar{A})) \). Thus \( \text{ivfspcl}(g^{-1}(\bar{A})) \subseteq g^{-1}(\text{ivfcl}(\bar{A})) \).

(ii) \( \Leftrightarrow \) (iii) Since every IVFSOS is an IVFSPOS, proof is similar in (i) \( \Rightarrow \) (ii).

(iii) \( \Leftrightarrow \) (i) Let \( \bar{A} \) be an IVFRCS in \( Y \). Then \( \bar{A} = \text{ivfcl}(\text{ivfint}(\bar{A})) \). Therefore \( \bar{A} \) is an IVFSOS in \( Y \). By hypothesis, \( \text{ivfspcl}(g^{-1}(\bar{A})) \subseteq g^{-1}(\text{ivfcl}(\bar{A})) = g^{-1}(\bar{A}) \subseteq \text{ivfspcl}(g^{-1}(\bar{A})) \).

Hence \( g^{-1}(\bar{A}) \) is an IVFPS in \( X \) and hence is an IVFGSCPS in \( X \). Thus \( g \) is an IVF\(_{a}\)GSP continuous mapping.

(i) \( \Leftrightarrow \) (iv) Let \( \bar{A} \) be an IVFPOS in \( Y \). Then \( \bar{A} \subseteq \text{ivfint}(\text{ivfcl}(\bar{A})) \). Since \( \text{ivfint}(\text{ivfcl}(\bar{A})) \) is an IVFROS in \( Y \), by hypothesis, \( g^{-1}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \) is an IVFGSPOS in \( X \). Since \( X \) is an IVFSP\(_{1/2} \) space, \( g^{-1}(\text{ivfint}(\text{ivfcl}(\bar{A}))) \) is an IVFPOS in \( X \).

Therefore \( g^{-1}(\bar{A}) \subseteq g^{-1}(\text{ivfint}(\text{ivfcl}(\bar{A}))) = \text{ivfspint}(g^{-1}(\text{ivfint}(\text{ivfcl}(\bar{A})))) \).

(iv) \( \Leftrightarrow \) (i) Let \( \bar{A} \) be an IVFROS in \( Y \). Then \( \bar{A} \) is an IVFPOS in \( X \). By hypothesis, \( g^{-1}(\bar{A}) \subseteq \text{ivfspint}(g^{-1}(\text{ivfint}(\text{ivfcl}(\bar{A})))) = \text{ivfspint}(g^{-1}(\bar{A})) \subseteq g^{-1}(\bar{A}) \).

This implies \( g^{-1}(\bar{A}) \) is an IVFPS in \( X \) and hence is an IVFGSPOS in \( X \). Therefore \( g \) is an IVF\(_{a}\)GSP continuous mapping.

**Theorem 3.21** Let \( g : X \to Y \) be a mapping. If \( g \) is an IVFGSP continuous mapping, then \( \text{ivfgspcl}(g^{-1}(\bar{A})) \subseteq g^{-1}(\text{ivfcl}(\text{bar}A)) \) for every IVFSPOS \( \bar{A} \) in \( Y \).

**Proof.** Let \( \bar{A} \) be an IVFSPOS in \( Y \). Then \( \text{ivfcl}(\bar{A}) \) is an IVFRCS in \( Y \). By hypothesis \( g^{-1}(\text{ivfcl}(\bar{A})) \) is an IVFGSCPS in \( X \). Then
\[ \text{ivfgspcl}(g^{-1}(\text{ivfcl}(\bar{A}))) = g^{-1}(\text{ivfcl}(\bar{A})) \]. Now
\( \text{ivfspcl}(g^{-1}(\overline{A})) \subseteq \text{ivfgspcl}(g^{-1}(\text{ivfcl}(\overline{A}))) = g^{-1}(\text{ivfcl}(\overline{A})). \)

**Theorem 3.22** Let \( g : X \to Y \) be a mapping where \( X \) is an IVFSPT\(_{1/2} \) space. If \( g \) is an IVFaGSP continuous mapping, then 
\( \text{ivfint}(\text{ivfcl}(\text{ivfint}(g^{-1}(\overline{B})))) \subseteq g^{-1}(\text{ivfspcl}(\overline{B})) \) for every \( \overline{B} \in \text{IVFRC}(Y) \).

**Proof.** Let \( \overline{B} \subseteq Y \) be an IVFRCS. By hypothesis, \( g^{-1}(\overline{B}) \) is an IVFGSPCS in \( X \). Since \( X \) is an IVFSPT\(_{1/2} \) space, \( g^{-1}(\overline{B}) \) is an IVFSPCS in \( X \). Therefore 
\( \text{ivfspcl}(g^{-1}(\overline{B})) = g^{-1}(\overline{B}). \) Now 
\( \text{ivfint}(\text{ivfcl}(\text{ivfint}(g^{-1}(\overline{B})))) \subseteq g^{-1}(\overline{B}) \)
\( \cup \)
\( \text{ivfint}(\text{ivfcl}(\text{ivfint}(g^{-1}(\overline{B})))) \subseteq \text{ivfspcl}(g^{-1}(\overline{B})) = g^{-1}(\overline{B}) = g^{-1}(\text{ivfspcl}(\overline{B})). \)
Hence 
\( \text{ivfint}(\text{ivfcl}(\text{ivfint}(g^{-1}(\overline{B})))) \subseteq \text{ivfspcl}(g^{-1}(\overline{B})). \)

**Theorem 3.23** Let \( g : X \to Y \) be a mapping where \( X \) is an IVFSPT\(_{1/2} \) space. If \( g \) is an IVFaGSP continuous mapping, then 
\( g^{-1}(\text{ivfspint}(\overline{B})) \subseteq \text{ivfcl}(\text{ivfint}(\text{ivfcl}(g^{-1}(\overline{B})))) \) for every \( \overline{B} \in \text{IVFRO}(Y) \).

**Proof.** This theorem can be easily proved by taking complement in theorem 3.22.

**REFERENCES**

