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Cordiality of Transformation Graphs of Cycles

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Abstract

A graph is said to be cordial if it has a 0-1 labeling that satisfies certain properties. In this paper we show that transformation graphs of cycle are cordial. We also show that total graph of sunlet graph, comb and star are cordial.

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Keywords: Transformation Graph, Cordial Graph.

1. Introduction

All graphs G considered here are finite, undirected and simple. We refer to [1] for unexplained terminology and notations. In 2001, Wu and Meng [3] introduced some new graphical transformations which generalizes the concept of the total graph. As is the case with the total graph, these generalizations referred to as *transformation graphs* G^{xyz} have $V(G) \cup E(G)$ as the vertex set. The adjacency of two of its vertices is determined by adjacency and incidence nature of the corresponding elements in G.

Let α, β be two elements of $V(G) \cup E(G)$. Then associativity of α and β is taken as $+$ if they are adjacent or incident in G, otherwise $-$. Let xyz be a 3-permutation of the set $\{+, -\}$. The pair α and β is said to correspond to x or y or z of xyz if α and β are both in $V(G)$ or both are in $E(G)$, or one is in $V(G)$ and the other is in $E(G)$ respectively. Thus the *transformation graph* G^{xyz} of G is the graph whose vertex set is $V(G) \cup E(G)$. Two of its vertices α and β are adjacent if and only if their associativity in G is consistent with the corresponding element of xyz.

In particular the transformation graph G^{++-} of G is the graph with vertex set $V(G) \cup E(G)$ in which the vertices u and v are joined by an edge if one of the following holds

1. both $u, v \in V(G)$ and u and v are adjacent in G
2. both $u, v \in E(G)$ and u and v are adjacent in G
3. one is in $V(G)$ and the other is in $E(G)$ and they are not incident with each other in G.

The transformation graphs are investigated in [4], [5] and [6].

A *graph labeling* is an assignment of integers to the vertices or edges, or both, subject to certain conditions. A mapping $f : V(G) \to \{0, 1\}$ is called a binary labeling of the graph G. For each $v \in V(G)$, $f(v)$ is called the vertex label of the vertex v under f and for an edge uv the induced edge labeling $g : E(G) \to \{0, 1\}$ is given by $g(uv) = |f(u) - f(v)|$. Then f is called a *cordial labeling* of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1, and, the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1. A graph G is cordial if it admits a cordial labeling. This concept is introduced by Cahit [2]. For more cordial graphs we refer to [8], [9], [10], [12] and [11].

For convenience, the transformation graph G^{xyz} is partitioned into $G^{xyz} = S_x(G) \cup S_y(G) \cup S_z(G)$ where $S_x(G)$, $S_y(G)$ and $S_z(G)$ are the edge-induced subgraphs of G^{xyz}. The edge set of each of which is respectively determined by x, y and z of the permutation xyz. $S_x(G) \cong G$ when x is $+$ and $S_x(G) \cong \overline{G}$ when x is $-$. $S_y(G) \cong L(G)$ when y is $+$ and $S_y(G) \cong \overline{L(G)}$ when y is $-$. When z is $+$, $\alpha, \beta \in V(G^{xyz})$ are adjacent in $S_z(G)$ if they are incident with each other in G. When z is $-$, α, β are adjacent in $S_z(G)$ if they are not incident in G.

The following notations are used in relation to labeling of G^{xyz}:

Let V_0 and V_1 denote the set of vertices of G^{xyz} labeled 0 and 1.

E_0 and E_1 denote set of edges of G^{xyz} labeled 0 and 1 respectively.

$E_0(S_x)$ and $E_1(S_x)$ denote set of edges labeled 0 and 1 in $S_x(G)$. Similar meanings are associates with $E_0(S_y)$, $E_1(S_y)$, $E_0(S_z)$ and $E_1(S_z)$.

For P_n^{xyz} $xyz \in \{++-, +--, -++, -+-, --+, ---, +++\}$ we define a vertex labeling $f : V(P_n^{xyz}) \to \{0, 1\}$ and specify the induced edge labeling $g : E(P_n^{xyz}) \to \{0, 1\}$ then show that f is a cordial labeling.

2. Cordiality of Transformation Graphs of Cycle

Let $C_n : v_1 - v_2 - v_3 - \ldots - v_n (n \geq 3)$ be the cycle on n vertices and $e_i = v_i v_{i+1} (1 \leq i \leq n - 1)$ and $e_n = v_n v_1$ be the edges of C_n.

Theorem 2.1. For any positive integer $n \geq 3$

(a) C_n^{++-} is cordial

(b) C_n^{+++} is cordial

(c) C_n^{-++} is cordial when $n \equiv 0, 2, 3, 5, 6, 7 (mod\ 8)$

(d) C_n^{+--} is cordial when $n \equiv 0, 1, 2, 3, 5, 6 (mod\ 8)$

(e) C_n^{+-+} is cordial when $n \equiv 0, 2, 3, 5, 6, 7 (mod\ 8)$

(f) C_n^{---} is Cordial when $n \equiv 0, 1, 3, 4, 5, 7 (mod\ 8)$

(g) C_n^{-+-} is Cordial when $n \equiv 0, 1, 2, 3, 5, 6 (mod\ 8)$

(h) C_n^{--+} is Cordial when $n \equiv 0, 1 (mod\ 4)$.

Proof.

(a) For $n \geq 3$ the vertices of C_n^{-+-} are labeled as in table 1:

n	$f(v_i)$	$f(e_i)$	$\lvert V_0\rvert - \lvert V_1\rvert$
Even	$\begin{cases} 1, & 1 \leq i \leq \frac{n}{2} - 1 \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \frac{n}{2} + 1 \\ 0, & otherwise \end{cases}$	0
Odd	$\begin{cases} 1, & 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	

Table 1: Cordial labeling of C_n^{++-}.

$|E_0|$ and $|E_1|$ are calculated from the labeling of the vertices of $V(C_n^{++-})$, and are given in the table 2:

n	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_z)\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
Even	$\frac{n}{2} - 2$	$\frac{n}{2} - 2$	$n - 2 + \frac{n(n-2)}{4} + \frac{(n+2)(n-4)}{4}$	2	2	$n - 2 + \frac{n(n-2)}{4} + \frac{(n+2)(n-4)}{4}$	0
Odd			$\frac{n^2 - 4n + 7}{2}$			$\frac{n^2 - 7}{2}$	1

Table 2: $|E_0|$ and $|E_1|$ of C_n^{++-}.

Therefore C_n^{++-} is cordial.

(b) The vertices of C_n^{+++} are labeled as below:

$$f(v_i) = 1 \text{ for } 1 \leq i \leq n$$

$$f(e_i) = 1 \text{ for } 1 \leq i \leq n$$

Here $|V_0| - |V_1| = 0$
$|E_0|$ and $|E_1|$ are calculated from the labeling of the vertices of C_n^{+++}, and are:

$$|E_0| = |E_0(S_x)| + |E_0(S_y)| + |E_0(S_z)| = n + n + 0 = 2n$$

$$|E_1| = |E_1(S_x)| + |E_1(S_y)| + |E_1(S_z)| = 0 + 0 + 2n = 2n$$

Therefore $|E_0| - |E_1| = 0$.
Therefore C_n^{+++} is cordial.

(c) When $3 \leq n \leq 7$, the vertices of C_n^{-++} are labeled as in Table 3 which admits cordial labeling.

n	$f(v_1)f(v_2)...f(v_n)$	$f(e_1)f(e_2)...f(e_n)$
3	101	100
4	1010	1100
5	10101	11000
6	101010	110010
7	1010101	1100100

Table 3: Cordial labeling of C_n^{-++} for the case $n < 7$.

when $n \geq 8$, the vertex labeling of C_n^{-++} are given in Table 4 and $|E_0|$ and $|E_1|$ of C_n^{-++} are counted in Table 5.

n	$f(v_i)$	$f(e_i)$	$\|V_0\| \sim \|V_1\|$
$n = 8r$ $n = 8r+2$	1	$\begin{cases} 0 & 3 \leq i \equiv 0,3 \pmod 4 \leq 4r \\ 0 & 4r+2 \leq i \equiv 0 \pmod 2 \leq n \\ 1 & otherwise \end{cases}$	0
$n = 8r+3$ $n = 8r+5$ $n = 8r+7$	0	$\begin{cases} 0 & 3 \leq i \equiv 0,3 \pmod 4 \leq 4r+4 \\ 0 & 4r+6 \leq i \equiv 0 \pmod 2 \leq n-1 \\ 0 & i = n \\ 1 & otherwise \end{cases}$	
$n = 8r+6$	1	$\begin{cases} 0 & 3 \leq i \equiv 0,3 \pmod 4 \leq 4r+4 \\ 0 & 4r+6 \leq i \equiv 0 \pmod 2 \leq n \\ 1 & otherwise \end{cases}$	

Table 4: cordial labeling of C_n^{-++} for the case $n \geq 8$.

Therefore C_n^{-++} is cordial.

(d) For $n = 3, 5 and 6$, the vertices of C_n^{+--} are labeled as in Table 6 which admits cordial labeling.

For $n \geq 8$, the vertices of C_n^{+--} are labeled as in Table 7. In all the cases $|V_0| - |V_1| = 0$.

<table>
<tr><th>n</th><th>$|E_0(S_x)|$</th><th>$|E_0(S_y)|$</th><th>$|E_0(S_z)|$</th><th>$|E_1(S_x)|$</th><th>$|E_1(S_y)|$</th><th>$|E_1(S_z)|$</th><th>$|E_0| \sim |E_1|$</th></tr>
<tr><td>$n=8r$</td><td rowspan="2">$2\sum_{i=1}^{\frac{n-2}{2}} i$</td><td rowspan="2">$2r$</td><td rowspan="2">$n$</td><td rowspan="2">$2\sum_{i=1}^{\frac{n-4}{2}} i + \frac{n-4}{2}$</td><td rowspan="2">$(n-2r)$</td><td rowspan="2">$n$</td><td>0</td></tr>
<tr><td>$n=8r+2$</td><td>1</td></tr>
<tr><td>$n=8r+3$</td><td rowspan="3">$2\sum_{i=1}^{\frac{n-3}{2}} i + \frac{n-3}{2}$</td><td rowspan="3">$2r+3$</td><td rowspan="3">$n-1$</td><td rowspan="3">$2\sum_{i=1}^{\frac{n-3}{2}} i$</td><td rowspan="3">$n-2r-3$</td><td rowspan="3">$n+1$</td><td>1</td></tr>
<tr><td>$n=8r+5$</td><td>0</td></tr>
<tr><td>$n=8r+7$</td><td>1</td></tr>
<tr><td>$n=8r+6$</td><td>$2\sum_{i=1}^{\frac{n-2}{2}} i$</td><td>$2r+2$</td><td>$n$</td><td>$2\sum_{i=1}^{\frac{n-4}{2}} i + \frac{n-4}{2}$</td><td>$n-2r-2$</td><td>$n$</td><td>1</td></tr>
</table>

Table 5: $|E_0|$ and $|E_1|$ of C_n^{-++}.

n	$f(v_1)f(v_2)\ldots f(v_n)$	$f(e_1)f(e_2)\ldots f(e_n)$
3	110	001
5	11000	10101
6	110010	101010

Table 6: Cordial labeling of C_n^{+--} for the case $n < 8$.

$|E_0|$ and $|E_1|$ of C_n^{+--} for the case $n \geq 8$ are given Table 8.

From the Table 8 it is evident that C_n^{+--}.

(e) For $3 \leq n \leq 8$, the vertices of C_n^{+-+} are labeled as in Table 9 which admits cordial labeling.

For $n \geq 8$, f is defined as in (d) which admits cordial labeling.

(f) For $n = 3, 4, 5, 7$, the vertices of C_n^{---} are labeled as in Table 10 which admits cordial labeling.

For $n \geq 8$, the vertices of C_n^{---} are labeled as in Table 11 and $|E_0|$ and $|E_1|$ of C_n^{---} for the case $n \geq 8$ are given in Table 12.

Therefore C_n^{---} is cordial.

(g) For $3 \leq n \leq 8$, the vertices of C_n^{-+-} are labeled as in Table 13 which admits cordial labeling.

For $n \geq 8$, the vertices of C_n^{-+-} labeled as in Table 14 and $|E_0|$ and $|E_1|$ of C_n^{-+-} for the case $n \geq 8$ are given table 15:

Therefore C_n^{-+-} is cordial.

n	$f(e_i)$	$f(v_i)$	$\lvert V_0\rvert \sim \lvert V_1\rvert$
$n=8r$ $n=8r+2$	0	$\begin{cases} 0 & 3\le i\equiv 0,3 \pmod 4 \le 4r \\ 0 & 4r+2\le i\equiv 0,2 \pmod 4 \le n \\ 1 & otherwise \end{cases}$	0
$n=8r+1$, $n=8r+3$, $n=8r+5$	1	$\begin{cases} 0 & 3\le i\equiv 0,3 \pmod 4 \le 4r+4 \\ 0 & 4r+6\le i\equiv 0,2 \pmod 4 \le n-1 \\ 0 & i=n \\ 1 & otherwise \end{cases}$	
$n=8r+6$	0		

Table 7: Cordial labeling of C_n^{+--} for the case $n \geq 8$.

n	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_z)\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
$n=8r$	$2r$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$(n-2)r+\left(\frac{n}{2}-2\right)(n-2r)$	$n-2r$	$2\sum_{i=1}^{\frac{n-4}{2}} i+\frac{n-4}{2}$	$(n-2)r+\left(\frac{n}{2}-2\right)(n-2r)$	0
$n=8r+2$							1
$n=8r+3$	$2r+3$	$2\sum_{i=1}^{\frac{n-3}{2}} i+\frac{n-3}{2}$	$\frac{n^2-2n-3}{2}$	$n-2r-3$	$2\sum_{i=1}^{\frac{n-3}{2}} i$	$\frac{n^2-2n+3}{2}$	1
$n=8r+5$							0
$n=8r+7$							1
$n=8r+6$	$2r+2$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$\frac{n(n-2)}{2}$	$n-2r-2$	$2\sum_{i=1}^{\frac{n-4}{2}} i+\frac{n-4}{2}$	$\frac{n(n-2)}{2}$	1

Table 8: $|E_0|$ and $|E_1|$ of C_n^{+--} for the case $n \geq 8$

(h) Vertices of $V(C_n^{--+})$ are as in Table 16 and $|E_0|$ and $|E_1|$ of C_n^{--+} for the case $n \geq 8$ are given table 17:

■

3. Cordiality of G^{+++} when G isomorphic to Sunlet, Comb and Star Graphs

We recall the following definitions:

Definition 3.1. Corona $G_1 \odot G_2$ of G_1 and G_2 is the graph obtained by taking one copy of G_1(which has n_1 vertices) and n_1 copies of G_2 and then joining i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. So the order $G_1 \odot G_2$ is $n_1 + n_1 n_2$ where n_2 is the order of G_2 and its size is $m_1 + n_1 n_2 + n_1 m_2$ where m_i is the size of G_i, $\forall\, i = 1, 2$.

n	$f(v_1)f(v_2)\cdots f(v_n)$	$f(e_1)f(e_2)\cdots f(e_n)$
3	001	011
4	1101	1000
5	11001	01010
6	110010	010101
7	1100101	0101010

Table 9: Cordial labeling ofC_n^{+-+} for the case $3 \le n \le 8$.

n	$f(v_1)f(v_2)\cdots f(v_n)$	$f(e_1)f(e_2)\cdots f(e_n)$
3	010	110
4	0101	1100
5	01010	11001
7	0101010	1100101

Table 10: Cordial labeling ofC_n^{---} for the case $n < 8$.

Definition 3.2. $P_n \odot K_1$ $(n \ge 3)$ is called *comb graph* of order $2n$.
Definition 3.3. $C_n \odot K_1$ $(n \ge 3)$ is called *sunlet graph* of order $2n$.

Theorem 3.4. For any positive integer $n \ge 3$ total graph of sunlet graph S_n is cordial.

Proof. Let $v_i (1 \le i \le n)$ be the consecutive vertices on the cycle C_n, v_i' be the pendant vertices adjacent to v_i respectively. Let $e_i = v_i v_{i+1}$ be the edges on the cycle and e_i' be the pendant edges incident to v_i respectively.

Define a binary labeling $f : V(S_n^{+++}) \to \{0, 1\}$ as in Table 18 and $|E_0|$ and $|E_1|$ of S_n^{+++} for the case $n \ge 6$ are given Table 19:

Therefore S_n^{+++} is cordial. ■

<table>
<tr><th>n</th><th>$f(v_i)$</th><th>$f(e_i)$</th><th>$|V_0| \sim |V_1|$</th></tr>
<tr><td>$n \equiv 0,4 \pmod 8$</td><td rowspan="3">$\begin{cases} 0 & i \text{ is odd} \\ 1 & i \text{ is even} \end{cases}$</td><td rowspan="2">$\begin{cases} 1 & i \equiv 1,2 \pmod 4 \\ 0 & \text{otherwise} \end{cases}$</td><td rowspan="4">0</td></tr>
<tr><td>$n \equiv 1, 5 \pmod 8$</td></tr>
<tr><td>$n \equiv 7 \pmod 8$</td><td>$\begin{cases} 1 & i \equiv 1,2 \pmod 4 \le n-5 \\ 0 & i \equiv 0,3 \pmod 4 \le n-3 \\ 1 & i = n-2, n \\ 0 & i = n-1 \end{cases}$</td></tr>
<tr><td>$n \equiv 3 \pmod 8$</td><td>$\begin{cases} 0 & i > 1 \text{ and is odd} \\ 1 & i = 1 \\ 1 & i \text{ is even} \end{cases}$</td><td>$\begin{cases} 1 & i \equiv 3,4 \pmod 4 \\ 0 & \text{otherwise} \end{cases}$</td></tr>
</table>

Table 11: cordial labeling of C_n^{---} for the case $n \ge 8$.

n	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_z)\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
$n \equiv 0, 4 \pmod 8$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$8\sum_{i=1}^{\frac{n-4}{4}} i$	$\frac{n(n-2)}{2}$	$2\sum_{i=1}^{\frac{n-4}{2}} i + \frac{n-4}{2}$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$\frac{n(n-2)}{2}$	0
$n \equiv 1, 5 \pmod 8$	$2\sum_{i=1}^{\frac{n-3}{2}} i + \frac{n-3}{2}$	$2\sum_{i=1}^{\frac{n-3}{2}} i - 1$	$\frac{(n-1)^2}{2}$	$2\sum_{i=1}^{\frac{n-3}{2}} i$	$2\sum_{i=1}^{\frac{n-3}{2}} i + \frac{n-1}{2}$	$\frac{n^2-2n-1}{2}$	1
$n \equiv 7 \pmod 8$		$2\sum_{i=1}^{\frac{n-3}{2}} i$			$2\sum_{i=1}^{\frac{n-3}{2}} i + \frac{n-3}{2}$		1
$n \equiv 3 \pmod 8$							1

Table 12: $|E_0|$ and $|E_1|$ of C_n^{---} for the case $n \geq 8$

n	$f(v_1)f(v_2)\cdots f(v_n)$	$f(e_1)f(e_2)\cdots f(e_n)$
3	101	100
4	11100	0001
5	10101	00011
6	10101	110101
7	1010101	1101000

Table 13: Cordial labeling ofC_n^{-+-} for the case $3 \leq n \leq 8$.

Theorem 3.5. Total graph of Comb graph is cordial.

Proof. Let $G = P_n \odot K_1 (n \geq 3)$ be a comb on $2n$ vertices. Let $v_i (1 \leq i \leq n)$ be the consecutive vertices of degree 2, $v_i' (1 \leq i \leq n)$ be the pendant vertices adjacent to v_i respectively. Let $e_i = v_i v_{i+1} (1 \leq i \leq n-1)$ be the edges and $e_i' (1 \leq i \leq n)$ be the pendant edges incident to v_i respectively. Define a binary labeling $f : V(G) \to \{0, 1\}$ as in Table 20 and $|E_0|$ and $|E_1|$ of total graph of comb for the case $n \geq 6$ are as shown in Table 21.

■

n	$f(v_i)$	$f(e_i)$	$\lvert V_0\rvert \sim \lvert V_1\rvert$
$n=8r$, $8r+2$	$\begin{cases} 1 & i \text{ is odd} \\ 0 & i \text{ is even} \end{cases}$	$\begin{cases} 0 & i \equiv 3, 0 \pmod 4 \leq 4r \\ 0 & 4r+2 \leq i \equiv 2, 4 \pmod 4 \leq n \\ 1 & \textit{otherwise} \end{cases}$	0
$n=8r+6$			
$n=8r+1$, $8r+3$, $8r+5$		$\begin{cases} 0 & i \equiv 3, 0 \pmod 4 \leq 4r+4 \\ 0 & 4r+6 \leq i \equiv 2, 4 \pmod 4 \leq n \\ 1 & \textit{otherwise} \end{cases}$	

Table 14: cordial labeling of C_n^{-+-} for the case $n \geq 8$.

n	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_z)\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
$n \equiv 0 \pmod 8$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$2r$	$\frac{n(n-2)}{2}$	$2\sum_{i=1}^{\frac{n-4}{2}} i + \frac{n-4}{2}$	$n-2r$	$\frac{n(n-2)}{2}$	0
$n \equiv 2 \pmod 8$							1
$n \equiv 1 \pmod 8$	$2\sum_{i=1}^{\frac{n-3}{2}} i + \frac{n-3}{2}$	$2r+1$	$\frac{(n-1)^2}{2}$	$2\sum_{i=1}^{\frac{n-3}{2}} i$	$n-1-2r$	$\frac{n^2-2n-1}{2}$	1
$n \equiv 3 \pmod 8$							0
$n \equiv 5 \pmod 8$							1
$n \equiv 6 \pmod 8$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$2r+2$	$\frac{n(n-2)}{2}$	$2\sum_{i=1}^{\frac{n-4}{2}} i + \frac{n-4}{2}$	$n-2-2r$	$\frac{n(n-2)}{2}$	1

Table 15: $|E_0|$ and $|E_1|$ of C_n^{-+-} for the case $n \geq 8$

n	$f(v_i)$	$f(e_i)$	$\lvert V_0\rvert - \lvert V_1\rvert$
$n \equiv 0 \pmod 4$	$\begin{cases} 0 & i \equiv 0,3 \pmod 4 \\ 1 & otherwise \end{cases}$		0
$n \equiv 1 \pmod 4$	$\begin{cases} 0 & i \equiv 0,3 \pmod 4 \\ 1 & otherwise \end{cases}$	$\begin{cases} 0 & 6 \leq i \equiv 0,3 \pmod 4 \leq n-2 \\ 0 & i = 2,3,n \\ 1 & otherwise \end{cases}$	

Table 16: Cordial labeling of C_n^{--+}.

Theorem 3.6. Total graph of Star $K_{1,n}$ where$n \equiv 0, 1, 3, 4, 5, 6 (mod\ \ 8)$ is cordial.

Proof. Let v_0 be the central vertex and $v_i (1 \leq i \leq n)$ be the pendant vertices of $K_{1,n}$. Let $e_i = v_0 v_i (1 \leq i \leq n)$ be the pendant edges.

Define a binary labeling $f : V(G) \rightarrow \{0, 1\}$ as in Table 22 and $|E_0|$ and $|E_1|$ of total graph of Star for the case $n \geq 6$ are as shown in Table 23. ■

4. Conclusion

We did not get the cordial labeling of G^{xyz} for some specific values of n when G is isomorphic to path, cycle and star. One of the future work on this paper is to find some more standard graphs G for which the transformation graphs are cordial.

n	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_z)\rvert$	$\lvert E_0\rvert-\lvert E_1\rvert$
$n\equiv 0\,(\mathrm{mod}\,4)$	$8\sum_{i=1}^{\frac{n-4}{4}} i$	$8\sum_{i=1}^{\frac{n-4}{4}} i$	$n+\frac{n}{2}$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$2\sum_{i=1}^{\frac{n-2}{2}} i$	$\frac{n}{2}$	0
$n\equiv 1\,(\mathrm{mod}\,4)$	$(n-4)+2\sum_{i=1}^{\frac{n-5}{2}} i$	$n-2r+4\sum_{i=1}^{r}(2i-1)$	$\frac{n-1}{2}$	$\frac{n-1}{2}+2\sum_{i=1}^{\frac{n-3}{2}} i$	$\frac{n-3}{2}+2\sum_{i=1}^{\frac{n-3}{2}} i-1$	$\frac{n-1}{2}+2$	0

Table 17: $|E_0|$ and $|E_1|$ of C_n^{--+}

$(n\geq 6)$	$f(v_i)$	$f(v_i')$	$f(e_i)$	$f(e_i')$	$\lvert V_0\rvert-\lvert V_1\rvert$
$n=4r$	0 $(1\leq i\leq n)$	1 $(1\leq i\leq n)$	$\begin{cases}1, & 1\leq i\leq r\\ 0, & otherwise\end{cases}$	$\begin{cases}0, & 1\leq i\leq r\\ 1, & otherwise\end{cases}$	0
$n=4r+1$	0 $(1\leq i\leq n)$	1 $(1\leq i\leq n)$	$\begin{cases}1, & 1\leq i\leq r\\ 0, & otherwise\end{cases}$	$\begin{cases}0, & 1\leq i\leq r\\ 1, & otherwise\end{cases}$	0
$n=4r+3$	0 $(1\leq i\leq n)$	1 $(1\leq i\leq n)$	$\begin{cases}1, & 1\leq i\leq r+1\\ 0, & otherwise\end{cases}$	$\begin{cases}0, & 1\leq i\leq r+1\\ 1, & otherwise\end{cases}$	

Table 18: Cordial labeling of S_n^{+++} for the case $n\geq 6$.

$(n\geq 6)$	$\lvert E_0(S_1)\rvert$	$\lvert E_0(S_2)\rvert$	$\lvert E_0(S_3)\rvert$	$\lvert E_1(S_1)\rvert$	$\lvert E_1(S_2)\rvert$	$\lvert E_1(S_3)\rvert$	$\lvert E_0\rvert-\lvert E_1\rvert$
n=4r, 4r+1	n	n	3n-2r	n	2n	n+2r	0, 1
$n=4r+3$	n	n	3n-2r-2	n	2n	n+2r+2	-1

Table 19: $|E_0|$ and $|E_1|$ of S_n^{+++}

for the case $n\geq 6$

$(n \geq 6)$	$f(v_i)$	$f(v_i')$	$f(e_i)$	$f(e_i')$	$\|V_0\|-\|V_1\|$
$n=4r$	0 $(1 \leq i \leq n)$	1 $(1 \leq i \leq n)$	$\begin{cases} 0 & r-1 \leq i \leq n-2 \\ 1 & otherwise \end{cases}$	$\begin{cases} 1 & r-1 \leq i \leq n-1 \\ 0 & otherwise \end{cases}$	0
$n=4r+1$	0 $(1 \leq i \leq n)$	1 $(1 \leq i \leq n)$	$\begin{cases} 1 & 2 \leq i \leq r \\ 1 & i=n-1 \\ 0 & otherwise \end{cases}$	$\begin{cases} 0 & 1 \leq i \leq r \\ 0 & i=n \\ 1 & otherwise \end{cases}$	0
$n=4r+2$	0 $(1 \leq i \leq n)$	1 $(1 \leq i \leq n)$	$\begin{cases} 1 & 1 \leq i \leq r-1 \\ 1 & i=n-1 \\ 0 & otherwise \end{cases}$	$\begin{cases} 0 & 1 \leq i \leq r-1 \\ 0 & i=n \\ 1 & otherwise \end{cases}$	0
$n=4r+3$	0 $(1 \leq i \leq n)$	1 $(1 \leq i \leq n)$	$\begin{cases} 1 & 1 \leq i \leq r-1 \\ 1 & i=n-1 \\ 0 & otherwise \end{cases}$	$\begin{cases} 0 & 1 \leq i \leq r-1 \\ 0 & i=n \\ 1 & otherwise \end{cases}$	0

Table 20: Cordial labeling of total graph of Comb for the case $n \geq 6$.

$(n \geq 6)$	$\|E_0(S_x)\|$	$\|E_0(S_y)\|$	$\|E_0(S_z)\|$	$\|E_1(S_x)\|$	$\|E_1(S_y)\|$	$\|E_1(S_z)\|$	$\|E_0\| \sim \|E_1\|$
$n=4r$	n-1	n-2	3n-2r	n	2n-2	n+2r-2	1
$n=4r+1$	n-1	n-1	3n-2r-2	n	2n-3	n+2r	0
$n=4r+2$	n-1	n-2	3n-2r-2	n	2n-2	n+2r	1
$n=4r+3$	n-1	n-2	3n-2r-2	n	2n-2	n+2r	0

Table 21: $|E_0|$ and $|E_1|$ of total graph of comb for the case $n \geq 6$.

$(n \geq 6)$	$f(v_i)$	$f(e_i)$	$\|V_0\| \sim \|V_1\|$
$n=8r$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+1$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+2$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+3$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+4$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+5$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 1 \leq i \leq n \\ 0, & otherwise \end{cases}$	1
$n=8r+6$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases}$	$\begin{cases} 1, & 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - r - 1 \quad and \quad \left\lceil \frac{n}{2} \right\rceil + r + 2 \leq i \leq n \\ 0, & otherwise \end{cases}$	1

Table 22: Cordial labeling of total graph of Star for the case $n \geq 6$.

$(n \geq 6)$	$\lvert E_0(S_x)\rvert$	$\lvert E_0(S_y)\rvert$	$\lvert E_0(S_z)\rvert$	$\lvert E_1(S_x)\rvert$	$\lvert E_1(S_y)\rvert$	$\lvert E_1(S_3)\rvert$	$\lvert E_0\rvert \sim \lvert E_1\rvert$
$n = 8r$	$\left\lfloor \frac{n}{2} \right\rfloor$	$\binom{\lfloor \frac{n}{2} \rfloor}{2} \times \binom{\lceil \frac{n}{2} \rceil}{2}$	$\left\lceil \frac{n}{2} \right\rceil + 2n - 2r$	$\left\lceil \frac{n}{2} \right\rceil$	$\left\lfloor \frac{n}{2} \right\rfloor \times \left\lceil \frac{n}{2} \right\rceil$	$\left\lceil \frac{n}{2} \right\rceil + 2r$	0
$n = 8r + 1$							1
$n = 8r + 2$							1
$n = 8r + 3$							0
$n = 8r + 4$							0
$n = 8r + 5$							1
$n = 8r + 6$			$\frac{3n}{2} - 2r - 2$			$\frac{n}{2} + 2r + 2$	1

Table 23: $|E_0|$ and $|E_1|$ of total graph of Star for the case $n \geq 6$.

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