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Robust Algorithm for Discrete Tomography with Gray Value Estimation

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Abstract

Discrete tomography (DT) named total variation regularized discrete algebraic reconstruction technique (TVR-DART) with automated gray value estimation. This algorithm is more robust and automated than the original DART algorithm, and is aimed at imaging of objects consisting of only a few different material compositions, each corresponding to a different gray value in the reconstruction.

INTRODUCTION

TOMOGRAPHY

Tomography is an important technique for noninvasive imaging with applications in medicine, industry, and science. It is applicable in scenarios where series of projection images of an object are available, acquired for a range of angles. A reconstruction of the object is subsequently computed from the projection images by a reconstruction algorithm. A range of reconstruction algorithms are available, which differ in reconstruction accuracy, requirements on the projection geometry, computational load, etc. Classical Filtered Back Projection (FBP) techniques are still commonly used.

Algebraic reconstruction methods, that are based on modelling the reconstruction problem as a large system of linear equations which is solved by iterative methods, are gradually becoming more common in tomography practice. Such algorithms can potentially yield more accurate reconstructions in some cases, at the expense of increased computation time. In many applications of tomography, it makes sense to exploit available prior knowledge of the unknown object. Incorporation of this knowledge in the reconstruction algorithm can potentially result in a reduction of the required number of projections, increased accuracy of the reconstruction, or an improved ability to deal with noisy projection data. The problem of reconstructing images, or more general signals, from a small number of weighted sums of their values has recently attracted considerable interest in the field of Compressed Sensing.

In particular, it was proved that if the image is sparse, it can be reconstructed accurately from a small number of measurements with very high probability, as long as the set of measurements satisfies certain randomization properties. In many images of objects that occur in practice, the image itself is not sparse, yet the boundary of the object is relatively small compared to the total number of pixels. In such cases, sparsity of the gradient image can be exploited by Total Variation Minimization. We consider a different type of prior knowledge, where it is assumed that the unknown object consists of a small number (i.e., 2-5) of different materials, each corresponding to a characteristic, approximately constant grey level in the reconstruction. Such prior knowledge is available in a wide range of tomography applications: when performing X-ray tomography of industrial objects, the compositions in these objects (e.g., aluminum, plastic, air) are often known in advance.

If a bone is scanned (in-vitro) in a micro-CT scanner, one can sometimes assume that the bone has a single constant density. As a third example we mention the reconstruction of homogeneous nanoparticles by electron tomography. The problem of reconstructing images containing a small set of grey levels from their projections has been studied in the fields of Discrete Tomography and Geometric Tomography. Geometric tomography deals with the reconstruction of geometric objects from data, its projections, or both . Images of such objects can be considered as binary images, where the first grey level (i.e., black) corresponds to the exterior of the object and the second grey level (white) corresponds to the interior. Much of the work on geometric tomography is concerned with rather specific objects, such as convex or starshaped objects.

According to the field of discrete tomography deals with the reconstruction of images from a small number of projections, where the set of pixel values is known to have only a few discrete values. The literature on discrete tomography contains some conflicting definitions of the field. Originally, the main focus was on the reconstruction of (typically binary) images for which the domain was a discrete set, inspired by applications in crystallography. The focus of the algorithm described in this paper is somewhat different from both geometric and discrete tomography. Firstly, our approach deals not only with binary images, but also with images that contain three or more grey levels. There is no fixed upper bound on the number of grey levels. Yet, the proposed techniques will only be effective if the number of grey levels is small (i.e., 5 or fewer). Compared to discrete tomography, which focuses on reconstruction from a small number of projections (i.e., 4 or fewer), our approach is more general. If tens or even hundreds of projection images are available, prior knowledge of the grey levels in the reconstruction can still be used effectively to improve the quality of the reconstruction, in particular when the projection data are noisy.

A variety of reconstruction algorithms have been proposed for discrete tomography problems. In a primal-dual subgradient algorithm is presented for reconstructing binary images from a small number of projections. This algorithm is applied to a suitable decomposition of the objective functional, yielding provable convergence to a binary solution. In [5], a similar reconstruction problem is modeled as a series of network flow problems in graphs, that are solved iteratively. Both consider reconstruction problems that may involve more than two grey levels, employing statistical models based on Gibbs priors for their solution.

For all these approaches, the required computation time becomes a major obstacles when dealing with image sizes used in practice. Recently, a new reconstruction algorithm for discrete tomography, called DART (Discrete Algebraic Reconstruction Technique) was proposed. DART alternates iteratively between “continuous” update steps, where the reconstruction is considered as an array of real-valued unknowns, and discretization steps, which incorporate the prior knowledge of the grey levels in the image. Application of this algorithm to experimental electron tomography data has already resulted in several important new insights in the properties of nanomaterials, as alternative techniques are not available at this scale.

However, a full description of the algorithmic details has been lacking thus far. Also, DART is a heuristic algorithm without guaranteed convergence properties which calls for a thorough experimental validation of algorithm properties. we provide a detailed presentation of the DART algorithm and validate this technique by extensive experiments based on simulated projection data, as well as real X-ray _CT data. We investigate its ability to reconstruct images from a small number of projections and from projections acquired along a small angular range, comparing DART with several alternative algorithms. We also present experimental results on the robustness of DART with respect to noise in the projection data and errors in the discrete grey levels used for reconstruction.

Mathematical notation is introduced to describe the tomographic reconstruction problem and the reconstruction problem for discrete tomography is stated formally. The Simultaneous Algebraic Reconstruction Technique (SART) algorithm for continuous tomography is briefly reviewed, as it is used as a subroutine in our implementation of DART. The DART algorithm is described.

We discuss how this algorithm can be implemented efficiently. The set of phantom images used in our simulation experiments and describes the experimental setup. Reports on extensive experiments, comparing DART with three alternative reconstruction algorithms, investigating its robustness with respect to noise and errors in the grey level assumptions, and desc

GRAY VALUE ESTIMATION

A **grayscale** or **greyscale** digital image is an image in which the value of each pixel is a single sample, that is, it carries only intensity information. Images of this sort, also known as black-and-white, are composed exclusively of shades of gray, varying from black at the weakest intensity to white at the strongest.

Grayscale images are distinct from one-bit bi-tonal black-and-white images, which in the context of computer imaging are images with only the two colors, black, and white (also called *bilevel* or *binary images*). Grayscale images have many shades of gray in between.

Grayscale images are often the result of measuring the intensity of light at each pixel in a single band of the electromagnetic spectrum (e.g.infrared, visible light, ultraviolet, etc.), and in such cases they are monochromatic proper when only a given frequency is captured. But also they can be synthesized from a full color image; see the section about converting to grayscale. ribing experimental convergence properties. Section VII concludes this paper.

EXISTING SYSTEM

Existing algorithms under noisy conditions from a small number of projection images and/or from a small angular range.

PROPOSED SYSTEM

The new algorithm requires less effort on parameter tuning compared with the original DART algorithm. With TVR-DART, we aim to provide the tomography society with a easy-to-use and robust algorithm for DT. Electron tomography data sets show that TVR-DART is capable of providing more accurate reconstruction

IMAGE RECONSTRUCTION

Iterative reconstruction refers to iterative algorithms used to reconstruct 2D and 3D images in certain imaging techniques. For example, in computed tomography an image must be reconstructed from projections of an object. Here, iterative reconstruction techniques are usually a better, but computationally more expensive alternative to the common filtered back projection (FBP) method, which directly calculates the image in a single reconstruction step. In recent research works, scientists have shown that extremely fast computations and massive parallelism is possible for iterative reconstruction, which makes iterative reconstruction practical for commercialization.

The reconstruction of an image from the acquired data is an inverse problem. Often, it is not possible to exactly solve the inverse problem directly. In this case, a direct algorithm has to approximate the solution, which might cause visible reconstruction artifacts in the image. Iterative algorithms approach the correct solution using multiple iteration steps, which allows to obtain a better reconstruction at the cost of a higher computation time.

In computed tomography, this approach was the one first used by Hounsfield. There are a large variety of algorithms, but each starts with an assumed image, computes projections from the image, compares the original projection data and updates the image based upon the difference between the calculated and the actual projections.

There are typically five components to iterative image reconstruction algorithms, e.g.

1. An object model that expresses the unknown continuous-space function that is to be reconstructed in terms of a finite series with unknown coefficients that must be estimated from the data.
2. A system model that relates the unknown object to the "ideal" measurements that would be recorded in the absence of measurement noise. Often this is a linear model of the form, where represents the noise.
3. A statistical model that describes how the noisy measurements vary around their ideal values. Often Gaussian noise or Poisson statistics are assumed. Because Poisson statistics are closer to reality, it is more widely used.
4. A cost function that is to be minimized to estimate the image coefficient vector. Often this cost function includes some form of regularization. Sometimes the regularization is based on Markov random fields.
5. An algorithm, usually iterative, for minimizing the cost function, including some initial estimate of the image and some stopping criterion for terminating the iterations.

Advantages

The advantages of the iterative approach include improved insensitivity to noise and capability of reconstructing an optimal image in the case of incomplete data. The method has been applied in emission tomography modalities like SPECT and PET, where there is significant attenuation along ray paths and noise statistics are relatively poor.

Statistical, likelihood-based approaches: Statistical, likelihood-based iterative expectation-maximization algorithms [4] are now the preferred method of reconstruction. Such algorithms compute estimates of the likely distribution of annihilation events that led to the measured data, based on statistical principle, often providing better noise profiles and resistance to the streak artifacts common with FBP. Since the density of radioactive tracer is a function in a function space, therefore of extremely high-dimensions, methods which regularize the maximum-likelihood solution turning it towards penalized or maximum a-posteriori methods can have significant advantages for low counts. Examples such as Ulf Grenander's Sieve estimator or Bayes penalty methods or via I.J. Good's roughness method may yield superior performance to expectation-maximization-based methods which involve a Poisson likelihood function only.

As another example, it is considered superior when one does not have a large set of projections available, when the projections are not distributed uniformly in angle, or when the projections are sparse or missing at certain orientations. These scenarios may occur in intraoperative CT, in cardiac CT, or when metal artifacts require the exclusion of some portions of the projection data.

In Magnetic Resonance Imaging it can be used to reconstruct images from data acquired with multiple receive coils and with sampling patterns different from the conventional Cartesian grid and allows the use of improved regularization techniques (e.g. total variation) or an extended modeling of physical processes to improve the reconstruction. For example, with iterative algorithms it is possible to reconstruct images from data acquired in a very short time as required for Real-time MRI.

In Cryo Electron Tomography, where the limited number of projections are acquired due to the hardware limitations and to avoid the biological specimen damage, it can be used along with compressive sensing techniques or regularization functions (e.g. Huber function) to improve the reconstruction for better interpretation

Image Reconstruction Techniques

Image reconstruction in CT is a mathematical process that generates tomographic images from X-ray projection data acquired at many different angles around the patient. Image reconstruction has fundamental impacts on image quality and therefore on radiation dose. For a given radiation dose it is desirable to reconstruct images with the lowest possible noise without sacrificing image accuracy and spatial resolution. Reconstructions that improve image quality can be translated into a reduction of radiation dose because images of the same quality can be reconstructed at lower dose.

Two major categories of reconstruction methods exist, analytical reconstruction and Iterative Reconstruction (IR). Let’s focus on the analytical reconstruction methods at first. There are many types of analytical reconstruction methods. The most commonly used analytical reconstruction methods on commercial CT scanners are all in the form of Filtered Back Projection (FBP), which uses a 1D filter on the projection data before back projecting (2D or 3D) the data onto the image space. The popularity of FBP-type of method is mainly because of its computational efficiency and numerical stability. Various FBP-type of analytical reconstruction methods were developed for different generations of CT data-acquisition geometries, from 2D parallel- and fan-beam CT in the 1970s and 1980s to helical and multi-slice CT with narrow detector coverage in late 1990s and early 2000s, and to multi-slice CT with a wide detector coverage (up to 320 detector rows and 16 cm width). 3D weighted FBP methods are generally adopted on scanners with more than 16 detector rows. For a general introduction of the fundamental principles of CT image reconstruction, please refer to Chapter 3 in Kak and Slaney’s book. An introduction to reconstruction methods in helical and multi-slice CT can be found in Hsieh’s book. A review of analytical CT image reconstruction methods used on clinical CT scanners.

Users of clinical CT scanners usually have very limited control over the inner workings of the reconstruction method and are confined principally to adjusting various parameters that potentially affect image quality. The reconstruction kernel, also referred to as “filter” or “algorithm” by some CT vendors, is one of the most important parameters that affect the image quality. Generally speaking, there is a tradeoff between spatial resolution and noise for each kernel. A smoother kernel generates images with lower noise but with reduced spatial resolution. A sharper kernel generates images with higher spatial resolution, but increases the image noise.

The selection of reconstruction kernel should be based on specific clinical applications. For example, smooth kernels are usually used in brain exams or liver tumor assessment to reduce image noise and enhance low contrast detectability, whereas sharper kernels are usually used in exams to assess bony structures due to the clinical requirement of better spatial resolution.

Another important reconstruction parameter is slice thickness, which controls the spatial resolution in the longitudinal direction, influencing the tradeoffs among resolution, noise, and radiation dose. It is the responsibility of CT users to select the most appropriate reconstruction kernel and slice thickness for each clinical application so that the radiation dose can be minimized consistent with the image quality needed for the examination.

In addition to the conventional reconstruction kernels applied during image reconstruction, many noise reduction techniques, operating on image or projection data, are also available on commercial scanners or as third-party products. Many of these methods involve non-linear de-noising filters, some of which have been combined into the reconstruction kernels for the users’ convenience. In some applications these methods perform quite well to reduce image noise while maintaining high-contrast resolution. If applied too aggressively, however, they tend to change the noise texture and sacrifice the low-contrast detectability in the image. Therefore, careful evaluation of these filters should be performed for each diagnostic task before they are deployed into wide-scale clinical usage.

Scanning techniques and image reconstructions in ECG-gated cardiac CT have a unique impact on image quality and radiation dose. Half-scan (or short-scan) reconstruction is typically used to obtain better temporal resolution. For the widely employed retrospective ECG-gated helical scan mode, the helical pitch is very low (~0.2 to 0.3) in order to avoid anatomical discontinuities between contiguous heart cycles. A significant dose reduction technique in helical cardiac scanning is ECG tube-current pulsing, which involves modulating the tube current down to 4% to 20% of the full tube current for phases that are of minimal interest. Prospective ECG-triggered sequential (or step-and-shoot) scans are a more dose-efficient scanning mode for cardiac CT, especially for single-phase studies. An overview of scanning and reconstruction techniques in cardiac CT.

Different from analytical reconstruction methods, IR reconstructs images by iteratively optimizing an objective function, which typically consists of a data fidelity term and an edge-preserving regularization term. The optimization process in IR involves iterations of forward projection and backprojection between image space and projection space. With the advances in computing technology, IR has become a very popular choice in routine CT practice because it has many advantages compared with conventional FBP techniques. Important physical factors including focal spot and detector geometry, photon statistics, X-ray beam spectrum, and scattering can be more accurately incorporated into IR, yielding lower image noise and higher spatial resolution compared with FBP. In addition, IR can reduce image artifacts such as beam hardening, windmill, and metal artifacts.

Due to the intrinsic difference in data handling between FBP and iterative reconstruction, images from IR may have a different appearance (e.g., noise texture) from those using FBP reconstruction. More importantly, the spatial resolution in a local region of IR-reconstructed images is highly dependent on the contrast and noise of the surrounding structures due to the non-linear regularization term and other factors during the optimization process . Measurements on different commercial IR methods have demonstrated this contrast- and noise-dependency of spatial resolution . Because of this dependency, the amount of potential radiation dose reduction allowable by IR is dependent on the diagnostic task since the contrast of the subject and the noise of the exam vary substantially in clinical exams . For low-contrast detection tasks, several phantom and human observer studies on multiple commercial IR methods demonstrated that only marginal or a small amount of radiation dose reduction can be allowed. Careful clinical evaluation and reconstruction parameter optimization are required before IR can be used in routine practice. Task-based image quality evaluation using model observers have been actively investigated so that image quality and dose reduction can be quantified objectively in an efficient manner.

METHOD

(1) Algebraic Reconstruction Algorithms

An entirely different approach for tomographic imaging consists of assuming that the cross section consists of an array of unknowns, and then setting up algebraic equations for the unknowns in terms of the measured projection data. Although conceptually this approach is much simpler than the transform-based methods, for medical applications it lacks the accuracy and the speed of implementation. However, there are situations where it is not possible to measure a large number of projections, or the projections are not uniformly distributed over 180 or 360”) both these conditions being necessary requirements for the transform based techniques to produce results with the accuracy desired in medical imaging.

An example of such a situation is earth resources imaging using cross-borehole measurements. Problems of this type are sometimes more amenable to solution by algebraic techniques. Algebraic techniques are also useful when the energy propagation paths between the source and receiver positions are subject to ray bending on account of refraction, or when the energy propagation undergoes attenuation along ray paths as in emission CT. Unfortunately, many imaging problems where refraction is encountered also suffer from diffraction effects. As will be obvious from the discussion to follow, in algebraic methods it is essential to know ray paths that connect the corresponding transmitter and receiver positions. When refraction and diffraction effects are substantial (medium in homogeneities exceed 10% of the average background value and the correlation length of these in homogeneities is comparable to a wavelength), it becomes impossible to predict these ray paths. If algebraic techniques are applied under these conditions, we often obtain meaningless results.

If the refraction and diffraction effects are small (medium in homogeneities are less than 2 to 3% of the average background value and the correlation width of these in homogeneities is much greater than a wavelength), in some cases it is possible to combine algebraic techniques with digital ray tracing techniques and devise iterative procedures in which we first construct an image ignoring refraction, then trace rays connecting the corresponding transmitter and receiver locations through this distribution, and finally use these rays to construct a more accurate set of algebraic equations. Experimental verification of this iterative procedure for weakly refracting objects has been obtained. Space limitations prevent us from discussing here the combined ray tracing and algebraic reconstruction algorithms. Our aim in this section is to merely introduce the reader to the algebraic approach for image reconstruction. First we will show how we may construct a set of linear equations whose unknowns are elements of the object cross section. The Kaczmarz method for solving these equations will then be presented. This will be followed by the various approximations that are used in this method to speed up its computer implementation.

Algebraic reconstruction technique

The **algebraic reconstruction technique (ART)** is a class of iterative algorithms used in computed tomography. These reconstruct an image from a series of angular projections (a sinogram). Gordon, Bender and Herman first showed its use in image reconstruction; whereas the method is known as Kaczmarz method in numerical linear algebra.

ART can be considered as an iterative solver of a system of linear equations The values of the pixels are considered as variables collected in a vector and the image process is described by a matrix .The measured angular projections are collected in a vector . Given a real or complex matrix and a real or complex vector respectively, the method computes an approximation of the solution of the linear systems of equations as in the following formula, where is the *i*-th row of the matrix is the *i*-th component of the vector, and is a relaxation parameter. The above formulae gives a simple iteration routine. An advantage of ART over other reconstruction methods (such as filtered backprojection) is that it is relatively easy to incorporate prior knowledge into the reconstruction process. ART falls into the category of Iterative reconstruction techniques.

TVR-DART FORMULATION

The key assumption in TVR-DART is that the image we seek consists of n grayscale levels that are separated by a narrow, but smooth, transition layer. Thus, we introduce a (parametrized) segmentation operator $T : X \times \Theta \to X$ that acts as a kind of segmentation map. It is here given as

$$T(f, \theta)(x) = nX-1\ \Sigma\ i=1\ (\rho i - \rho i-1) uki\ (f(x) - \tau i) \qquad \text{for } x \in \Omega \text{ and } \theta \in \Theta.$$

The parameter space $\Theta := (R \times R \times R)$ n defines the transition characteristics of the n layers (the background $\rho 0$ is often set to 0). Concretely, $\theta = (\theta 1, \ldots, \theta n) \in \Theta$ with $\theta i := (\rho i, \tau i, Ki)$ where ρi is the gray-scale level of the i:th level, τi is the mid-point gray-scale value, $ki := Ki/(\rho i - \rho i-1)$ is the sharpness of the smooth gray-scale transition, and $u: R \to [0, 1]$ is the logistic function that models the transition itself

$$uk(s) := 1/\ 1 + e-2ks \text{ for } s \in R.$$

The TVR-DART algorithm for solving (1) is now defined as a method that yields a minimizer to

$$\min\ f \in X, \theta \in \Theta\ [\ L\ [A \circ T](f, \theta), g) + \lambda[S \circ T](f, \theta)\ ]$$

In the above, $L: Y \times Y \to R+$ is an appropriate data-fit term and $S : X \to R+$ is the regularization. The variant considered in [17] uses a data-fit term $L(\cdot, g) = k \cdot - gk\ 2\ 2$ and a regularizing functional $TV\varepsilon = [H\varepsilon \circ \nabla]$, where $H\varepsilon$ is the Huber norm and ∇ is the spatial gradient operator. The Huber norm is a smooth surrogate functional for the L 1 -norm, and $H\varepsilon \circ \nabla$ is thus a smoothed version of TV. Hence, (6) becomes

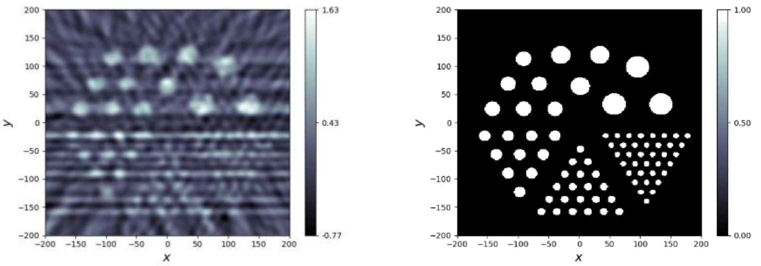
$$\min f \in X, \theta \in \Theta\ [\ [A \circ T](f, \theta) - g\ 2\ 2 + \lambda[H\varepsilon \circ \nabla \circ T](f, \theta)\ ]$$

Gradient based methods can be used to solve (7) since its objective functional is smooth. In [17] one such solution method was presented, based on an alternating optimization over f and θ. We take a similar approach here, and to this end define the two operator $T\ \theta : X \to X$, defined by $T\ \theta(f) = T(f, \theta)$, and $T\ f : \Theta \to X$, defined $T\ f\ (\theta) = T(f, \theta)$, where θ and f are seen as fix parameters, respectively. In the current implementation we view the sharpness parameter as fixed, but optimize over gray-scale value and mid-point.

REQUIREMENTS

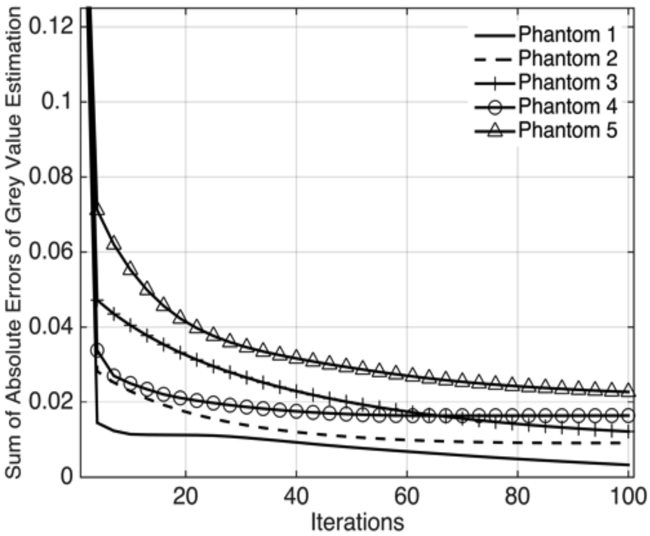
The complete algorithm is implemented in Matlab using the open source ASTRA Tomography Toolbox

RESULTS AND DISCUSSION



Phantom x Filtered Back projection RME: 1:96, and SSIM: 0:111.,

FBP reconstructions from the data with Poisson noise



Using TVR-DART under the same noise level and number of projection images as in Reconstruction. Convergence of the objective function through iterations. Convergence of gray value estimation as sum of absolute errors of the estimated gray values through iterations.

CONCLUSIONS

The algorithm is aimed at tomographic reconstruction of objects consisting of a few different material compositions, each approximately corresponding to a constant gray value in the reconstruction. By defining a soft segmentation function within the objective function of the reconstruction algorithm, TVR-DART smoothly steers the solution toward discrete gray values while minimizing the total variation of the boundaries of the discrete solution. Since it is very difficult to know the exact gray values in most practical applications, the gray values and thresholds of the segmentation function are automatically estimated in an alternating manner with the reconstruction assuming the total number of gray values is known.

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