

Empirical Mode Decomposition: Theory & Applications

Sonam Maheshwari¹ and Ankur Kumar²

^{1,2} *M. Tech (ECE), Kurukshetra University, Haryana*

Abstract

Empirical Mode Decomposition (EMD), introduced by Huang et al, in 1998 is a new and effective tool to analyze non-linear and non-stationary signals. With this method, a complicated and multiscale signal can be adaptively decomposed into a sum of finite number of zero mean oscillating components called as Intrinsic Mode Functions (IMF) whose instantaneous frequency computed by the analytic signal method (process known as Hilbert Huang Transform) give a physically meaningful characterization of the signal. The EMD is based on the sequential extraction of energy associated with various intrinsic time scales of the signal starting from finer temporal scales (high frequency modes) to coarser ones (low frequency modes). This paper reviews the method of applying EMD to a signal and its various applications.

Keywords— Empirical mode decomposition, Intrinsic mode function, Hilbert-Huang transform, Signal denoising, Adaptive, Biomedical signal analysis

Introduction

Signal analysis for extracting useful information embedded in it is an important area of signal processing and has been an area of research for decades. Many algorithms have been so far reported in the literature for analyzing the signal. The simpler and well known method for signal analysis is Fourier transform which decompose the signal into a weighted sum of sinusoids for efficient extraction of information [1]. Though FT can effectively capture signal frequency content, it cannot provide both time and frequency localization of the signal simultaneously. Time-frequency analysis methods, such as Short Time Fourier Transform (STFT) and Wavelets thus are considered more appropriate to handle non-stationary signals. However, STFT suffers from its inability to precisely localize the signal in time, while simultaneously maintaining adequate frequency resolution. Moreover, STFT requires piece-wise stationary of the data, and also assumes that the stationary scales coincide with the sliding window length used for the decomposition, something that is practically

impossible to guarantee. The wavelet transform is another method for the time-frequency analysis of signals [2], [3]. Like the Fourier transform, which decomposes signals into a weighted sum of sinusoids, the wavelet transform decomposes signals into a weighted sum of wavelets. Unlike the Fourier representation of a signal, which retains only frequency information, the wavelet representation contains both time and frequency information; this makes it more suited to the analysis of non-stationary signals. The empirical mode decomposition (EMD) is a relatively new method proposed by Huang et al [4] in 1998 for decomposing non linear and non stationary signals into a series of Intrinsic Mode Functions (IMFs). IMF captures the repeating behaviour of the signal at some particular time scale. Like the Fourier or wavelet transforms, the empirical mode decomposition reduces a time signal into a set of basis signals; unlike the Fourier or wavelet transforms, however, the basis functions are derived from the data itself. Consequently the results preserve the full non stationary of the signal under consideration. When Hilbert transform is applied to the IMFs instantaneous frequency and amplitude of the signal can be determined. This process is called Hilbert-Huang transform (HHT). The biggest advantage of the method is that it is totally adaptive and data driven, without the need for a-priori basis function selection for signal decomposition. The purpose of this paper is to investigate its capabilities and limitations in real world signal analysis.

Empirical mode decomposition

- A. Principle: Empirical Mode Decomposition (EMD) [4] is a technique which decomposes multiscale non-linear, non-stationary signal into number of AM-FM zero mean signals, known as Intrinsic Mode Functions (IMF), in an adaptive, fully data-driven, way. Principle of EMD is derived from the simple assumption that any signal consists of different IMFs, each of them representing an embedded characteristic oscillation on a separated time scale.
- B. Algorithm: Empirical mode decomposition (EMD) adaptively decomposes a multiscale signal $x(t)$ into a number L , of the so called, Intrinsic Mode Functions (IMFs), $h_{(i)}(t)$, $1 \leq i \leq L$,

$$x(t) = \sum_{i=1}^L h_{(i)}(t) + r(t) \quad (1)$$

where $r(t)$ is a remainder which is a non zero-mean slowly varying function with only few extrema. The basic concept of EMD is to identify proper time scales that reveal physical characteristics of the signal, and then decompose the signal into modes intrinsic to the function. These modes are referred to as Intrinsic Mode Functions (IMF). IMFs are signals satisfying the following conditions:

- 1) In the whole dataset, the number of extrema and the number of zero crossings must either be equal or differ at most by one,
- 2) At any point, the mean value of the envelope defined by local maxima and the envelope defined by the local minima is zero.

Steps are as follows-

1. Extract all the local maxima and minima of $x(t)$.
2. Form the upper and lower envelope $e_u(t)$ and $e_l(t)$ by cubic spline interpolation of the extrema point developed in step (1).
3. Calculate the mean function of the upper and lower envelop, $m_1(t)$ as $m_1(t) = e_u(t) + e_l(t) / 2$.
4. Let $d_1(t) = x(t) - m_1(t)$. If $d_1(t)$ is a zero-mean function, then the iteration stops and $d_1(t)$ is accepted as first IMF, ie, $h_1(t) = d_1(t)$.
5. If not, use $d_1(t)$ as the new data and repeat steps 1-4 until ending up with an IMF.

A stopping criteria is applied to the number of shifting iterations so that IMF component can retain amplitude and frequency modulation. Once the first IMF $h_1(t)$ is obtained, remaining IMF's are obtained by applying shifting process to the residual signal. Residual signal $r_1(t)$ can be defined as

$$r_1(t) = x(t) - h_1(t) \tag{2}$$

Residual signal now contains information about the lower frequency components. Shifting process will be continued until the final residue is a constant, a monotonic function or a function with only one maxima and minima from which no IMF can be obtained. At the end of decomposition process noisy signal $x(t)$ can be represented as a sum of IMFs plus a residue signal.

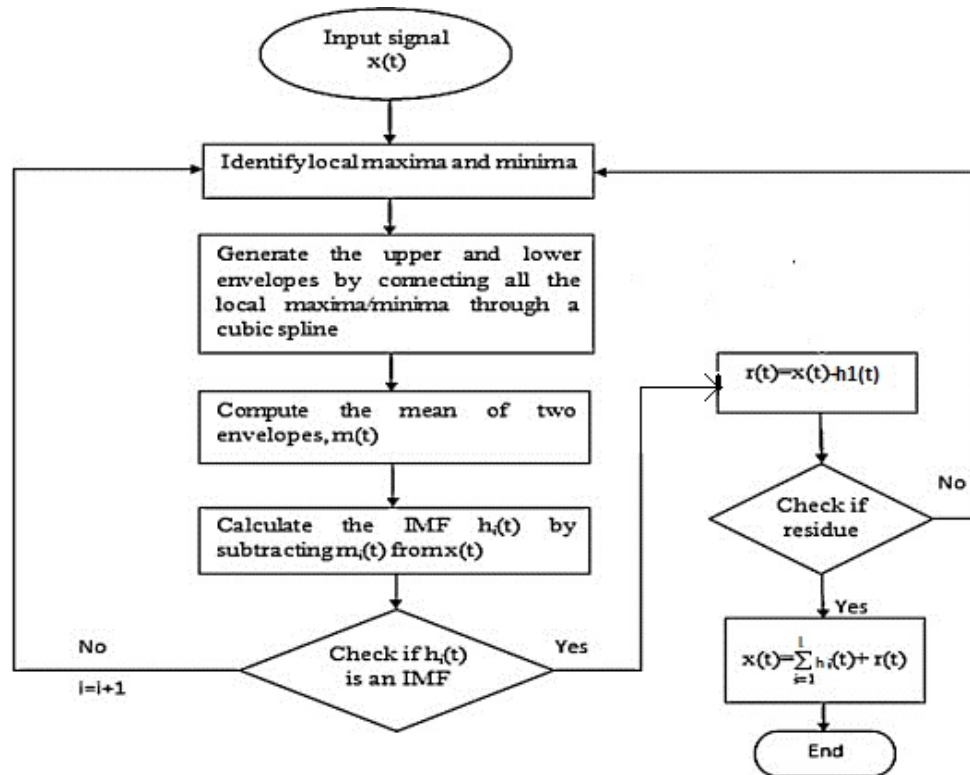


Figure1. depicts the flowchart of EMD process

EMD procedure can be applied to decompose the time series into a set of IMFs and a residue. By applying the Hilbert transform to each IMF signal can be further analyzed to calculate the instantaneous frequency and amplitude of each IMF. The whole process is called Hilbert Huang Transform.

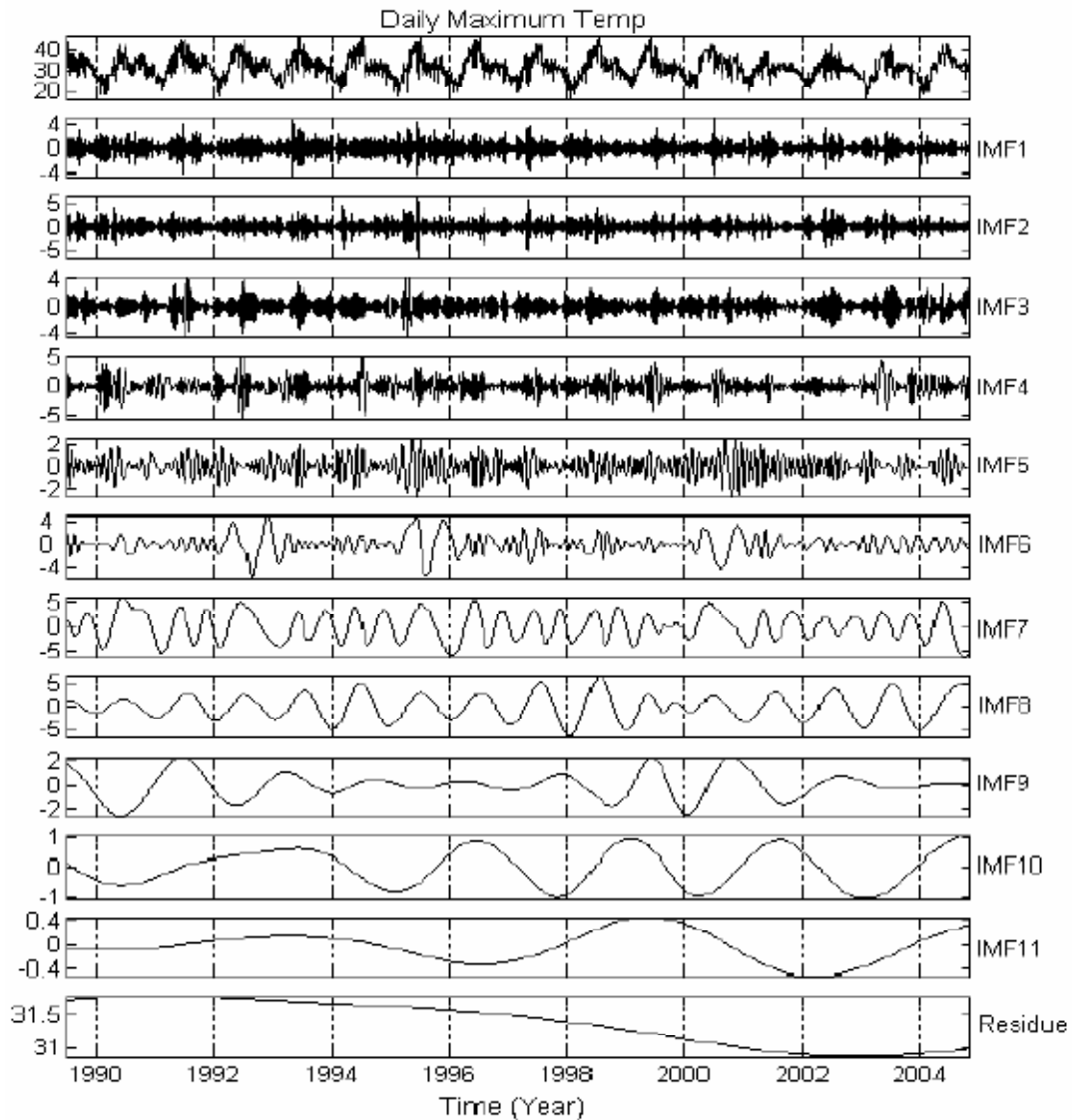


Figure 2. depicts the decomposition of a real signal into IMFs and a residual signal using EMD. It can be observed that higher order IMFs contain lower frequency components than lower order IMFs

- C. **Significance:** EMD is a method of breaking down a signal without leaving the time domain. This process is useful for analyzing natural real signals which are most often non-linear and non-stationary. EMD filters out functions which

form a complete and nearly orthogonal basis for the original signal. These functions known as IMFs ensure completeness and thus are sufficient to describe the signal. Essence of EMD is the fact that the function into which a signal is decomposed are all in the time domain and of the same length as the original signal allows for varying frequency in time to be preserved. This is generally hidden in Fourier and Wavelet transform. Generally speaking, the overall result of decomposition is to successively remove the highest frequencies from a signal. In this context, Rilling et al (2004) described the EMD to behave as a dyadic filter bank just as those involved in wavelet representation. This can be interpreted as having several filters of overlapping frequency content. Thus EMD can be used to extract useful information inherent to the signal and has received more and more attention in signal denoising area.

- D. Applications: The EMD algorithm decomposes a function into a function-tailored, fine to coarse multi resolution of IMFs. The procedure is extremely attractive, both for its effectiveness in a wide range of applications and for its simplicity of implementation. Owing to its impressive performance EMD has been used to address several problems in the field of engineering, mathematics and sciences. A plethora of documented literature is available on use of EMD in signal denoising. In [8], Boudraa and Cexus have demonstrated the use of EMD in denoising signals. EMD proves to be a novel technique for analyzing biomedical signals [9]. K. Khaldi and others demonstrated the application of EMD in denoising voiced signals [11], [12]. Zhang et al proposed a new technique for analyzing crude oil prices based on EMD [9]. Zhu integrated EMD with Genetic Algorithm to predict and model the carbon price [13].

Conclusion and Discussions-

EMD is an appealing new approach for adaptively decomposing non linear and non stationary signals. It decomposes a signal based on the time scale characteristics of the signal itself. So, it effectively overcomes the limitations of Fourier transform uncertainty principle and more fits with non-linear and non-stationary data. Moreover, it can get the frequency distribution at any time and provide higher frequency resolution on time-frequency domain. Seen in this light EMD approach is superior to wavelet representation and thus paves the way for a new dimension in signal analysis. In spite of its practical achievements, the EMD technique is essentially an algorithmic approach which lacks a well established theoretical proof and a direct, systematic optimization of the method. The wealth of literature on EMD, cries out the lack of a rigorous mathematical base for the definition of EMD. Several issues relating to theoretical analysis and understanding, performance enhancement, optimization and so on need to be addressed.

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