Characteristics of Radar Cross Section with Different Objects

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Abstract

Radar cross section is one of the key parameters in the detection of targets. It is used to describe the amount of scattered power from a target towards the radar. In stealth design of aircrafts and sea vessels, it is required to maintain low RCS. Lower the RCS, lower is the detectability and vice versa. In conventional aircrafts, bigger the aircraft, higher is the RCS value. RCS depends on several parameters like aspect angle, frequency, polarization, and dimensions of the target.

Most of the radar systems use RCS as a means of discrimination. Hence its prediction is critical. The measuring and identifying the scattering centres for a given target helps to reduce RCS. The exact method of prediction of RCS is very complex even for simple and conventional shaped objects. However, a few regular shaped objects are considered in the present work to estimate the variation of RCS. In order to generate data on RCS for stealth applications, its characteristics and dependency on several criteria are consolidated. The RCS of triangular plate, spherical, cylindrical, and ellipsoidal objects with aspect angle, frequency, and dimensions of the object are numerically computed. Its variation as a function of these parameters is presented.

Keywords: Radar cross section, Scattering, Stealth.

Introduction

When radar illuminates a target, RCS is a measure of scattered power by an incident electromagnetic wave. It is measured in a given direction. It doesn't depend on the
distance of the target from the source as it is normalized with respect to power density of the incident wave. In RCS measurement, it is not required to know the position of the receiver. The scattered echo is usually spherical in nature. RCS is used to obtain target characteristics, transmitter power, receiver sensitivity, position of transmitter and receiver distance. Sometimes it also represents an echo area. In fact, it is a function of frequency, aspect angle, shape of the target, position of the transmitter and receiver relative to the target geometry, material, and angular orientation of the target relative to the transmitter and the receiver and antenna polarization [1-5].

An EM wave incident on the target is normally scattered in all directions depending on the shape of the target and type of polarization of the wave. The scattered waves are divided into two parts: The first part belongs to the waves which have the polarization of the receiving antenna and the second set of scattered waves is characterized by different polarization which is no way related to the receiving antenna. These two sets of waves are said to be orthogonal to each other. The radar cross section is evaluated on the basis of intensity of back scattered energy whose polarization is same as that of receiving antenna.

An investigation of the scattering from rectangular flat plates has been carried out for analytical formulation suitable for the estimation of radar cross section. Simple physical optics theory provides an accurate means of predicting the near specular values of plate cross section, but fails to account for polarization dependence and detailed shape [6]. Radar cross section of flat plates painted with radar absorbing materials (RAM) in the range of 8-12GHz is reported in [7]. The obtained results demonstrate the RAM coating causes a radar cross section reduction of 94% of the radiation, when impinged at normal incidence.

The conceptual definition of RCS includes the fact that not all of the radiated energy falls on the target. A target’s RCS is most easily visualized as the product of three factors: projected cross section, reflectivity, directivity. Reflectivity is defined by the percent of intercepted (scattered) power reradiated by the target. Directivity represents ratio of power scattered back in the radar’s direction to the power that would have been backscattered had the backscattering been uniform in all directions [8]. RCS is not equal to the geometric area. For a sphere, RCS=$\pi r^2$, where $r$ is the radius of the sphere. The RCS of a sphere is independent of frequency if operating at sufficiently high frequencies where $\lambda<<$range, and $\lambda<<$ radius [9]. For a flat plate which is frequency dependant, RCS is equal to $4\pi a^2/\lambda^2$, where $a$ is area of the plate.

This paper presents sample set of simple objects radar cross section. Most of the expressions presented represent the radar cross section of the object when it is large compared to wavelength.

**Formulations**

The expressions for RCS for sphere, cylinder, ellipsoid, triangular plate are reported by Mahafza [10]. However, for the sake of completeness, they are reproduced here.

**RCS of Sphere**

Radar designers and RCS engineers consider the perfectly conducting sphere to be the
simplest target to examine. Due to symmetry, waves scattered from a perfectly conducting sphere are co-polarized (have the same polarization) with the incident waves. This means that the cross-polarized backscattered waves are practically zero. Most formulas presented are Physical Optics (PO) approximation for the backscattered RCS measured by a far field radar in the direction \((\theta, \varphi)\). The direction of antenna receiving backscattered waves is shown in fig.1

\[ \sigma = \frac{1}{2\pi r^2} \sum_{n=1}^{\infty} (-1)^n (2n + 1) \left[ \left( \frac{kr j_n(kr) - n j_n(kr)}{kr H_n^{-1}(kr) - nH_n^1(kr)} \right) - \left( \frac{j_n(kr)}{H_n^1(kr)} \right) \right] \tag{1} \]

Here, \( r \) is radius of sphere, 
\( k = \frac{2\pi}{\lambda}, \) \( \lambda \) is wavelength 
\( j_n \) is spherical Bessel of the first kind of order \( n \),
\( H_n^1 \) is the hankel function of order \( n \), and is given by
\[ H_n^1(kr) = j_n(kr) + jy_n(kr) \tag{2} \]
\( y_n \) is the spherical Bessel function of the second kind of order \( n \).

**RCS of Cylinder**

![Figure 2: (a) Elliptical cylinder (b) circular cylinder.](image)
The geometry of elliptical and circular cylinders are shown in figure 2. The normal incident backscattered RCS for an elliptical cylinder due to a linearly polarized incident wave is given by equation 3.

\[
\sigma_{\theta n} = \frac{2\pi H^2 r_2^2 r_3^2}{\lambda(r_1^2\cos^2\varphi + r_2^2\sin^2\varphi)^{1.5}}
\]  
(3)

The RCS for an incident wave other than normal is given by eq. 4

\[
\sigma = \frac{\lambda r_1^2 r_3^2 \sin\theta}{8\pi (\cos\theta)^2 (r_1^2\cos^2\varphi + r_2^2\sin^2\varphi)}
\]  
(4)

For a circular cylinder of radius, equations (3-4) are reduced further and they appear in the form of

\[
\sigma_{\theta n} = \frac{2\pi l^2 r}{\lambda r_1 r_3 \sin\theta}
\]  
(5)

\[
\sigma = \frac{\lambda r^2 \sin\theta}{8\pi (\cos\theta)^2}
\]  
(6)

Here, H=length of the cylinder.
r=radius

When the incident electric field is perpendicular to the axis of a cylinder, cylinder exhibits resonance effect as a result of creeping waves. For a parallel incident wave, the effect of creeping waves is very small.

**RCS of Ellipsoid**

An ellipsoid centered at (0, 0, 0) shown in figure 3 is defined by Equation 7

\[
\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) + \left(\frac{z^2}{c^2}\right) = 1
\]  
(7)

Here, a = ellipsoid a-radius
b=ellipsoid b-radius  
c=ellipsoid c-radius

One widely accepted approximation for the ellipsoid backscattered RCS is given by

$$\sigma = \frac{\pi a^2 b^2 c^2}{(a^2(\sin \theta)^2 \cos \varphi)^2 + b^2(\sin \theta)^2 \sin \varphi)^2 + c^2(\cos \theta)^2)^2}$$

(8)

When $a = b$, ellipsoid becomes roll symmetric and RCS is independent of $\varphi$ and the above equation reduced to

$$\sigma = \frac{\pi b^4 c^2}{(a^2(\sin \theta)^2 + c^2(\cos \theta)^2)^2}$$

(9)

And for the case when $a = b = c$, $\sigma = \pi c^2$ it becomes backscattered RCS of sphere.

Here, $\varphi$=roll angle,  
$\theta$= angle between z axis and direction to receiving radar.

RCS of Triangular Plate

Consider the triangular flat plate defined by the isosceles triangle as oriented in Figure 4

The backscattered RCS can be approximated for small aspect angles (less than 30°) by

$$\sigma = \frac{4\pi A^2 \cos \theta \sigma_0}{\lambda^2}$$

(10)

$$\sigma_0 = \frac{[(\sin \alpha)^2 - (\sin \beta/2)^2]^2 + \sigma_{01}}{(\alpha^2 - (\beta/2)^2)}$$

(11)

Figure 4: Triangular plate.
\[ \sigma_{01} = 0.25(\sin \varphi)^2 \left( \frac{2a}{b} \cos \varphi \sin \beta - \sin \varphi \sin 2\alpha \right) \]  
\[ \alpha = ka \sin \theta \cos \varphi, \beta = ka \sin \theta \sin \varphi \]  

(12)

For incidence in the plane \( \varphi = 0 \)

\[ \sigma = \frac{4\pi A^2}{\lambda^2} (\cos \theta)^2 \left[ \left( \frac{\sin \alpha}{a} \right)^4 + \left( \frac{\sin 2\alpha - 2\alpha}{4a} \right)^4 \right], \]  
\[ A = ab/2 \]  

(13)

For incidence in the plane \( \varphi = \frac{\pi}{2} \)

\[ \sigma = \frac{4\pi A^2}{\lambda^2} (\cos \theta)^2 \left[ \left( \frac{\sin \beta/2}{\beta/2} \right)^4 \right] \]  

(14)

**Results**

Computations are carried out using equations (1-2) and the variation of RCS of a sphere is presented in figs (5-6). In fig.5, its variation is normalized with the circumference of the sphere. On the other hand, in fig.6, RCS is represented in decibels.

Computations are repeated for the evaluation of RCS using the equations (3-14) for the following cases.

For an elliptic cylinder, \( r_1=0.125 \text{m}, r_2=0.05 \text{m}, h=1 \text{m} \) and \( r_1=0.125 \text{m}, r_2=0.05 \text{m}, h=5 \text{m} \).

For circular cylinder, \( r=0.125 \text{m}, h=1 \text{m} \) and \( r=0.125 \text{m}, h=5 \text{m} \).

Ellipsoid with roll angle=0\(^\circ\) and 90\(^\circ\).

Triangular plate with \( a=0.2 \text{m}, b=0.75 \); \( a=1 \text{m}, b=1 \text{m} \) and \( a=2 \text{m}, b=2 \text{m} \).

The variation of RCS with frequency and aspect angle are also obtained. The results are presented in figs (7-20).

![Figure 5: Variation of RCS with circumference of sphere.](image-url)
Figure 6: Variation of RCS in dB with circumference of sphere.

Figure 7: Variation of RCS with aspect angle of circular cylinder\((r=0.125\,\text{m}, \, h=1\,\text{m})\).

Figure 8: Variation of RCS with aspect angle of circular cylinder\((r=0.125\,\text{m}, \, h=5\,\text{m})\).
Figure 9: Variation of RCS with aspect angle of elliptic cylinder ($r_1=0.125\text{m}$, $r_2=0.05$, h=1m).

Figure 10: Variation of RCS with aspect angle of elliptic cylinder ($r_1=0.125\text{m}$, $r_2=0.05$, h=5m).

Figure 11: Variation of RCS with frequency of cylinder ($r=0.125$, h=1m).
Figure 12: Variation of RCS with frequency of cylinder(r=0.125, h=5m).

Figure 13: Variation of RCS with aspect angle of ellipsoid at phi=0.

Figure 14: Variation of RCS with aspect angle of ellipsoid at phi=90.
Figure 15: Variation of RCS with aspect angle of triangular plate (a=0.2m, b=0.75m).

Figure 16: Variation of RCS with aspect angle of triangular plate (a=1m, b=1m).

Figure 17: Variation of RCS with aspect angle of triangular plate (a=2m, b=2m).
Figure 18: Variation of RCS with frequency of triangular plate (theta=30°).

Figure 19: Variation of RCS with frequency of triangular plate (theta=60°).

Figure 20: Variation of RCS with frequency of triangular plate (theta=90°).
Conclusion
It is evident from the results on the variation of RCS of sphere, RCS is high for small spheres and low and fluctuating with increase in circumference of the sphere. The RCS has a peak value at an aspect angle of $90^\circ$ for circular and elliptical cylinders. The variation is found to be independent of the dimensions of the cylinder. However, for large cylinders, the peak value of RCS is also large. The variation of RCS with frequency is found to be linear and exhibit higher values for large cylinders.

RCS is also found to have similar variation for ellipsoids and it has peak values at an aspect angle of $90^\circ$. Interestingly, for triangular plates of different sizes, RCS is highly fluctuating with aspect angle as well as frequency. However, RCS is uniform with frequency at an aspect angle of $90^\circ$.

References