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Impedance Characteristics of Yagi–Uda Antenna

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Abstract

Yagi-Uda Antenna is one of the most important and commercially used antennas for T.V. reception. Although the analysis of this antenna is reported in literature, the data of self impedance & mutual impedance is not fully available. But, this data is useful for the optimal design of the antenna. In view of this the above antenna with different characteristics and space distributions is considered and the analysis is carried out and such vital data is obtained and presented.

Introduction

Yagi-Uda antenna consists of a number of linear dipole elements. One element is energized directly by a feed transmission line while others act as parasitic radiators whose currents are induced by mutual coupling. A folded dipole is most commonly used as active element. The antenna is exclusively designed to operate as an endfire array which is accomplished by having the parasitic elements in the forward beam acting as directors while those in the rear act as reflectors. The Yagi-Uda antenna is widely use as a T.V. antenna. It is usually used at frequencies between about 30MHz and 300MHz, or a wavelength range of 10 metres to 1 cm [1]. The original design and operating principles of this radiator were first reported in papers [2, 3].

The active element of a Yagi-Uda antenna consists of a dipole whose length is λ/2 where λ is the operating wavelength. The parasitic elements consist of one reflector and few directors. The length of the reflector is greater than λ/2 and is located behind the active element. The length of each director is less than λ/2 and they are placed in front of the active element. The spacing between the each element is not identical and it can be considered as a non-linear array. The number of directors in the antenna depends on the gain requirements. [4]

Yagi-Uda array may be regarded as a structure supporting a traveling wave whose performance is determined by the current distribution in each element and the phase velocity of the traveling wave. The objective of the design is to make a "travelling wave" structure with currents in the elements all contributing to the far field in the forward direction. The contributions are designed to add up in phase in the forward direction, and to cancel in the reverse direction. The director elements are cut shorter than the driving element, which is itself a little shorter than a half wavelength at the design frequency. The reflector is cut to be about a half wavelength and it is longer than the driving element and spaced closer than are the directors. The directors present a capacitative impedance, acting like two lengths of open circuit transmission line each a little shorter than a quarter wavelength to a hypothetical generator at the centre formed from the "induced emf" set up by the impinging fields. For a closely spaced driving element and parasitic element, isolated from each other as far a electrical conduction currents are concerned, the currents are oppositely directed as can be seen in the discussion on folded dipoles with the folds cut off. As the spacing is increased, the currents remain oppositely directed until when the spacing is a half-wavelength, the contributions to the far field add up in phase in the "endfire direction". [4]

The major role of the reflector is played by the first element next to the one energized. Considerable improvements can be achieved if more directors are added the array. Usually most antennas have about 6 to 12 directors. The lengths and diameters of the directors and reflectors, as well as their respective spacings determine the optimum characteristics. Usually Yagi-Uda antenna arrays have low input impedance and relatively low bandwidth.

Salient features

- Yagi-Uda antenna consists of a driven element, a reflector and one or more directors.
- The active element is a folded dipole which has a length of $\lambda/2$ and it is at resonance. Length of director is less than $\lambda/2$ and length of reflector is greater than $\lambda/2$.
- Its radiation pattern is almost uni-directional.
- Reflector resonates at a lower frequency and director resonates at a higher frequency compared to that of a driven element.
- More directors can be added to increase the gain. In this case, directors can be of equal length or decreasing slightly away from the driven element.
- The mutual impedance of the antenna depends on the spacing and the length of the elements.
- Highest gain is obtained when the reflector is slightly greater than $\lambda/2$ in length and space at $\lambda/4$ from the driven element and when the length of director is about 10% less than $\lambda/2$ with an optimal spacing of $\lambda/3$.
- The reflector spacing and size have negligible effects on the forward gain and large effects on backward gain and input impedance.
- The maximum gain of a Yagi-Uda is limited to an amount given approximately by the gain of a dipole times the total number of elements.

Yagi-Uda antennas are quite common in practice because they are lightweight, simple to build, low cost and provide desirable characteristics for many applications. The performance of radiation, gain depends upon the self and mutual impedances of the array. The analysis of self impedances and mutual impedances and the formulae involved along with the results are discussed in the next sections. Section 2 explains the analysis of self impedance. Section 3 describes the analysis of mutual impedance. Section 4 depicts the calculations of self and mutual admittances. Section 5 shows the results of the impedances calculated for different characteristics of antenna.

Analysis of Self impedance

The radiation characteristics of an antenna in the presence of a lossy ground depend substantially on the infinite ground conductivity and in homogeneity [5, 6. 7]. The problem was conventionally simplified on the basis of a Hertzian dipole with specified current moment for very low frequency range [8]. But, for higher frequency a finite length antenna should be considered.

In the present analysis, the array is considered to be symmetric and the element half length is not greater than the limit of 5λ/8. The analysis is carried out by King and Wu's three term assumption for the current [9]. Thus, for a single centre-fed dipole in free space[10]

$$I(h) = \frac{jV}{60\Gamma_{dA}\cos\beta l}\left[\sin\beta(l-|h|)+A+B\right] \quad for \quad \beta l \neq \frac{\pi}{2} \tag{1}$$

where

$$A = X_X(\cos\beta h - \cos\beta l)$$

$$B = X_Y\left(\cos\frac{\beta h}{2} - \cos\frac{\beta l}{2}\right)$$

or

$$I(h) = \frac{-jV}{60\Gamma_{dA}}\left[\sin\beta|h| - 1 + C + D\right] \quad for \quad \beta l = \frac{\pi}{2} \tag{2}$$

where

$$C = X'_X \cos\beta h$$

$$D = X'_Y\left(\cos\frac{\beta h}{2} - \cos\frac{\pi}{4}\right)$$

In the above equations (1) and (2), V is the applied voltage, *h* is the distance along the dipole axis measured from the feeding point, *l* is the half-length of the dipole, *a* is the radius of dipole, $\beta=2\pi/\lambda$ and the other symbols are as defined below[10].

For $\beta l \leq \pi/2$

$$\Gamma_{dA} = 2C1\overline{C_c}(\beta l) - 2\cos\beta l\overline{C_c}(2\beta l) - 2\cot\beta l C1C_s(\beta l) + EXP.C_s(2\beta l) \tag{3}$$

where
$$C1 = 1 + \cos\beta l$$
$$EXP = (\cot\beta l \cos\beta l - \sin\beta l)$$

For βl ≥ π/2

$$\Gamma_{dA} = \overline{C_c}(0.5\pi) + C2\overline{C_c}(E) - \cos 2\beta l\overline{C_c}(2E) + \sin 2\beta l\left[\overline{C_c}(\beta l) - \overline{C_c}(2\beta l)\right] - C2C_s(\beta l) + \cos 2\beta l C_s(2\beta l) + \sin 2\beta l\left[C_s(E) - C_s(2E)\right] \tag{4}$$

where $C2 = 1 + \cos 2\beta l$

When βl = π/2

$$\Gamma_{dA}\left(\beta l = \frac{\pi}{2}\right) = 2\overline{C_c}(0.5\pi) - C_s(\pi) \tag{5}$$

The complex functions X_X, X_Y, X'_X and X'_Y in equations (1) and (2) are expressed as

$$X_X = Z^{-1}(\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A) \tag{6}$$

$$X_Y = -jZ^{-1}\left[\Gamma_{dC}(\Gamma_{dE}\cos\beta l - \Gamma_U) + \Gamma_{dF}\Gamma_V\right] \tag{7}$$

$$X'_X = \frac{\Gamma_{dB}\Gamma_{dA}}{\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A} \tag{8}$$

$$X'_Y = \frac{j\Gamma_{dC}\Gamma_{dE}}{\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A} \tag{9}$$

where $Z = \Gamma_{dB}(\Gamma_{dE}\cos\beta l - \Gamma_U) + j\Gamma_{dF}\Gamma_A$ (10)

The other symbols in the above equations are as follows:

$$\Gamma_{dE} = \frac{2}{C3}\overline{C}_c(\beta l) - \frac{\cos\beta l}{C3}\overline{C}_c(2\beta l) - \frac{2\cos\beta l}{C3}\overline{C}(\beta l) + \frac{\cos\beta l}{C3}\overline{C}(2\beta l) - \frac{\sin\beta l}{C3}C_s(2\beta l) \tag{11}$$

$$\Gamma_{dF} = \frac{1}{C4}\begin{bmatrix} -2S_c(\beta l) + \cos\beta l S_c(2\beta l) + 2\cos\beta l S(\beta l) - \cos\beta l S(2\beta l) \\ +\sin\beta l S_s(2\beta l) \end{bmatrix} \tag{12}$$

$$\Gamma_{dC} = \frac{1}{C4}\begin{Bmatrix} -2\sin\beta l S_c(\beta l) + 2\cos\beta l S_s(\beta l) - \sin 2\beta l\left[S_c(\beta l) - S_c(2\beta l)\right] \\ +C2S_s(\beta l) - \cos 2\beta l S_s(2\beta l) \end{Bmatrix} \tag{13}$$

$$\Gamma_{dB} = \frac{1}{C4}\left[G_1 + jH_1\right] \tag{14}$$

$$\begin{aligned} & G_1 = 2\sinh^{-1}\left(\frac{l}{a}\right) - C(P) - C(Q) - \sinh^{-1}\left(\frac{2l}{a}\right)\cos\frac{\beta l}{2} \\ \text{with} \quad & +\frac{1}{2}\cos\frac{\beta l}{2}C(R) - \frac{1}{2}\sin\frac{\beta l}{2}S(R) + \cos\frac{\beta l}{2}\overline{C}(2\beta l) \\ & +\frac{1}{2}\cos\frac{\beta l}{2}C(T) + \frac{1}{2}\sin\frac{\beta l}{2}S(T) - 2\cos\frac{\beta l}{2}\overline{C}(\beta l) \end{aligned} \tag{15}$$

$$\begin{aligned} H_1 &= -S(P) - S(Q) + \frac{1}{2}\sin\frac{\beta l}{2}C(R) + \frac{1}{2}\cos\frac{\beta l}{2}S(R) \\ &+\frac{1}{2}\cos\frac{\beta l}{2}S(T) - \frac{1}{2}\sin\frac{\beta l}{2}C(T) + 2\cos\frac{\beta l}{2}S(\beta l) \\ &-\cos\frac{\beta l}{2}S(2\beta l) \end{aligned} \tag{16}$$

$$\Gamma_V = G_2 + jH_2 \tag{17}$$

with

$$G_2 = -\sin 2\beta l\left[\overline{C}_c(\beta l) - \overline{C}_c(2\beta l)\right] + \cos 2\beta l\left[C_s(\beta l) - C_s(2\beta l)\right] + C_s(\beta l) \tag{18}$$

$$H_2 = \sin 2\beta l\left[S_c(\beta l) - S_c(2\beta l)\right] - \cos 2\beta l\left[S_s(\beta l) - S_s(2\beta l)\right] - S_s(\beta l) \tag{19}$$

$$\begin{aligned} \Gamma_U &= \cos\beta l\left[\overline{C}_c(2\beta l) - \overline{C}_c(2\beta l)\right] + \sin khC_s(2\beta l) \\ &\quad + j\left\{\cos\beta l\left[S(2\beta l) - S_c(2\beta l)\right] - \sin\beta l S_s(2\beta l)\right\} \end{aligned} \tag{20}$$

$$\Gamma_A = G_3 + jH_3 \tag{21}$$

with

$$G_3 = \cos\frac{\beta l}{2}\left[\sinh^{-1}\left(\frac{l}{a}\right) - \frac{1}{2}C(R) - \frac{1}{2}C(T) - \overline{C}(2\beta l)\right] + \frac{1}{2}\sin\frac{\beta l}{2}\left[S(R) - S(T)\right] \tag{22}$$

$$H_3 = \cos\frac{\beta l}{2}\left[S(2\beta l) - \frac{1}{2}S(R) - \frac{1}{2}S(T)\right] + \frac{1}{2}\sin\frac{\beta l}{2}\left[C(T) - C(R)\right] \tag{23}$$

For a set of values for a and l all the functions shown above can be calculated. Thus from equations (1) and (2) the self impedance of the dipole is computed as

$$Z_s = \frac{-j60\Gamma_{dA}\cos\beta l}{\sin\beta l + X_x C3 + X_Y C4} \qquad \text{for} \quad \beta l \neq \frac{\pi}{2} \tag{24}$$

$$Z_s = \frac{j60\Gamma_{dA}}{-1 + X'_X - (1-\sqrt{2}/2)X'_Y} \qquad \text{for} \quad \beta l = \frac{\pi}{2} \tag{25}$$

In the equations from (1) through (25) the various symbols used are defined as follows. The generalized sine and cosine integrals are defined as:

$$C_s(\beta l) = C_s(\beta a, \beta l) \qquad C_s(2\beta l) = C_s(\beta a, 2\beta l) \qquad C_s(0.5\pi) = C_s(\beta a, \pi/2)$$

$$C_s(E) = C_s(\beta a, \beta l - \pi/2) \qquad C_s(2E) = C_s(\beta a, 2\beta l - \pi/2) \qquad C_s(\pi) = C_s(\beta a, \pi)$$

$$C_s(E) = C_s(\beta a, \beta l - \pi/2) \qquad C_s(2E) = C_s(\beta a, 2\beta l - \pi/2) \qquad C_s(P) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \frac{\beta l}{2}\right)$$

$$C_s(Q) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \frac{3\beta l}{2}\right) \qquad C_s(R) = C_s\left(\beta a\frac{\sqrt{3}}{2}, 3\beta l\right) \qquad C_s(T) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \beta l\right)$$

The expressions hold good for other integrals shown in equations (1) through (25). These generalized integrals are given in the reference [10]. The other symbols used are defined as follows:

$$C1 = (1 + \cos\beta l) \qquad C2 = (1 + \cos 2\beta l) \qquad C3 = (1 - \cos\beta l) \qquad C4 = (1 - \cos\frac{\beta l}{2})$$

Analysis of Mutual Impedance

The open circuit mutual impedance between two parallel dipoles of half-lengths l_1 and l_2 spaced a distance *s* apart as shown in Fig. 1 is calculated by

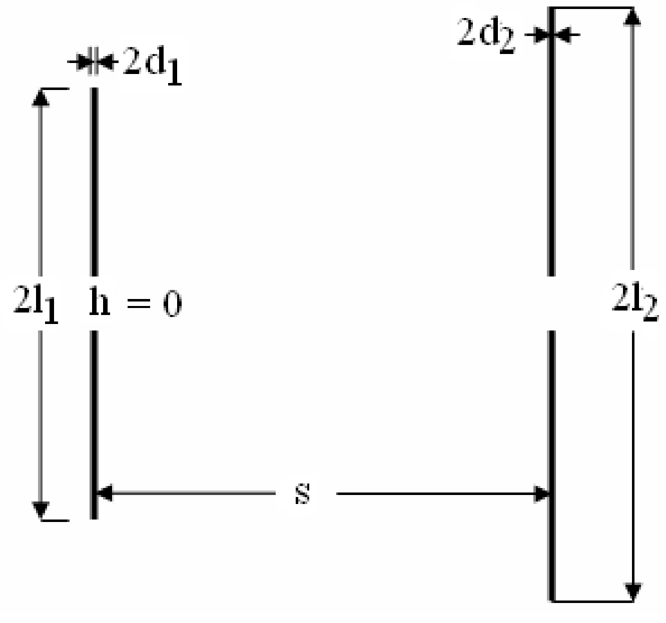

Figure 1: Two non-identical element array.

$$Z_m = \frac{-1}{I_1(0)I_2(0)}\int_{-l2}^{l1} E_{s1}(h)I_2(h)dh \tag{26}$$

where $I_1(0)$ and $I_2(0)$ are the input currents of No.1 and No.2 antennas respectively. The *h* component of electric field $E_{s1}(h)$ produced at No.2 antenna by the current on No. 1 antenna is given by [9]

$$E_{h1} = \frac{-j30}{\beta}\left(\beta^2 + \frac{\partial^2}{\partial h^2}\right)\int_{-l1}^{l1} I_1(h')K(h,h')dh' \tag{27}$$

where $K(h,h') = \dfrac{e^{-j\beta R}}{R}$ (28)

$$R = \left[(h-h')^2 + s^2\right]^{1/2} \tag{29}$$

Substituting equations (35) into (34) and using (1) and (2) for I(h') and I(h) the expression for mutual impedance can be obtained as

for $\beta l_1 \neq \pi/2$ and $\beta l_2 \neq \pi/2$,

$$Z_m = \frac{-j30\{\beta M_a + 2(\cos\beta l_1)M_b - [1 + X_{X1}\sin\beta l_1 + \tfrac{1}{2}X_{Y1}\sin(\beta l_1/2)]M_c\}}{Y_aY_b} \tag{30}$$

where

$$Y_a = \sin\beta l_1 + X_{X1}(1-\cos\beta l_1) + X_{Y1}\left(1-\cos\frac{\beta l_1}{2}\right) \tag{31}$$

$$Y_b = \sin\beta l_2 + X_{X2}(1-\cos\beta l_2) + X_{Y2}\left(1-\cos\frac{\beta l_2}{2}\right) \tag{32}$$

$$M_a = \int_{-l2}^{l2}[\sin\beta(l_2-|h|) + X_{X2}C5 + X_{Y2}C6] \times$$

$$\int_0^{l1}\left(X_{X1}\cos\beta l_1 - \tfrac{3}{4}X_{Y1}\cos\frac{\beta h'}{2} + X_{Y1}\cos\frac{\beta l_1}{2}\right)KK\ dh'dh \tag{33}$$

$$M_b = \int_{-l2}^{l2}K(h,0)[\sin\beta(l_2-|h|) + X_{X2}C5 + X_{Y2}C6]dh \tag{34}$$

$$M_c = \int_{-l2}^{l2}KK[\sin\beta(l_2-|h|) + X_{X2}C5 + X_{Y2}C6]dh \tag{35}$$

for $\beta l_1 = \pi/2$ and $\beta l_2 \neq \pi/2$,

$$Z_m = \frac{j30\{\beta M_d + 2M_b + [X'_{X1} - \tfrac{1}{2}X'_{Y1}\sin(\pi/4)]M_c\}}{Y_cY_b} \tag{36}$$

where

$$Y_c = -1 + X'_{X1} - X'_{Y1}\left(1 - \cos\frac{\pi}{4}\right) \tag{37}$$

$$M_d = \int_{-l2}^{l2}\left[\sin\beta(l_2 - |h|) + X_{X2}C5 + X_{Y2}C6\right] \quad \times$$

$$\int_0^{l1}\left(-1 + \tfrac{3}{4}X'_{X1}\cos\frac{kh'}{2} + X'_{Y1}\cos\frac{\pi}{4}\right)KK\,dh'\,dh \tag{38}$$

for $\beta l_1 = \beta l_2 = \pi/2$,

$$Z_m = \frac{j30\left\{\beta M_e + 2M_f + \left[X'_{X1} - \tfrac{1}{2}X'_{Y1}\sin(\pi/4)\right]M_g\right\}}{Y_c Y_d} \tag{39}$$

where

$$Y_d = -1 + X'_{X2} - X'_{Y2}\left(1 - \cos\frac{\pi}{4}\right) \tag{40}$$

(which will be identical to M_c if $d_2=d_1$)

$$M_e = \int_{-l2}^{l2}\left[\sin\beta|h| - 1 + X'_{X2}\cos\beta h - X'_{Y2}C7\right]$$

$$\times\int_0^{l1}\left(-1 + \tfrac{3}{4}X'_{X1}\cos\frac{kh'}{2} + X'_{Y1}\cos\frac{\pi}{4}\right)KK\,dh'\,dh \tag{41}$$

$$M_f = \int_{-l2}^{l2} K(h,0)\left[\sin\beta|h| - 1 + X'_{X2}\cos\beta h - X'_{Y2}C7\right]dh \tag{42}$$

$$M_g = \int_{-l2}^{l2} KK\left[\sin\beta|h| - 1 + X'_{X2}\cos\beta h - X'_{Y2}C7\right]dh \tag{43}$$

In the above equations the symbols used are defined as:

$$C5 = \cos\beta h - \cos\beta l_2 \qquad C6 = \cos\frac{\beta h}{2} - \cos\frac{\beta l_2}{2} \qquad C7 = \cos\frac{\beta h}{2} - \cos\frac{\pi}{4}$$

$$KK = K(h,h') + K(h,-h')$$

Since X_X, X_Y, X'_X and X'_Y are given in equations (6) to (9), the integrals *Ma* through *Mg* can all be computed numerically once the values for d_1, d_2, l_1, l_2 and h are specified. Hence the mutual impedance in (30), (36) or (39) can also be computed. Also, when $X_{X1} = X_{X2} = X_{Y1} = X_{Y2} = 0$, equation (30) reduces to familiar expression for the case of simple sinusoidal current distribution. In such case the mutual impedance is calculated as [11]

$$Z_m = -30\int_0^{l_2}\left(\frac{-je^{-j\beta r_1}}{r_1} - \frac{je^{-j\beta r_2}}{r_2} + \frac{2j\cos kh_1 e^{-j\beta r_0}}{r_0}\right)\sin\beta(l_2 - h)\,dh \tag{44}$$

$$\begin{aligned} & r_0 = \sqrt{s^2 + h^2} \\ \text{where } & r_1 = \sqrt{s^2 + (l_1 - h)^2} \qquad (45) \\ & r_2 = \sqrt{s^2 + (l_1 + h)^2} \end{aligned}$$

Results

In order to validate the impedances, different sets of Yagi_Uda antennas with five elements where the directors have fixed length and ten elements with tapering wavelength are considered as follows. The numerical integrations of the generalized sine and cosine integrals and the impedances were computed using MATLAB and are listed in the tables [11-34] shown below. The radius of all elements is assumed as 0.001 m. For each configuration the impedances were computed for different frequencies at 200MHz, 250MHz and 300MHz as listed.

Impedances (Ohms)

Table 11: Frequency = 200 MHz.

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.4 λ;									
Z_{11}	131.1 – 589.80i	Z_{21}	118.3 – 28.10i	Z_{31}	67.8 – 54.20i	Z_{41}	18.9 - 71.80i	Z_{51}	-24.1 – 61.90i
Z_{12}	118.3 – 28.10i	Z_{22}	1638.7 – 1252.5i	Z_{32}	180.5 + 51.20i	Z_{42}	134.2 - 44.10i	Z_{52}	70.6 – 91.80i
Z_{13}	67.8 – 54.20i	Z_{23}	180.5 + 51.20i	Z_{33}	590.5 + 828.70i	Z_{43}	184.0 + 53.7i	Z_{53}	138.3 – 42.6i
Z_{14}	18.9 – 71.80i	Z_{24}	134.2 - 44.10i	Z_{34}	184.0 + 53.7i	Z_{44}	590.5 + 828.7i	Z_{54}	184.0 + 53.7i
Z_{15}	-24.1 - 61.90i	Z_{25}	70.6 – 91.80i	Z_{35}	138.3 - 42.6i	Z_{45}	184.0 + 53.7i	Z_{55}	590.5 + 828.7i

Table 12: Frequency = 200 MHz.

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.3 λ;									
Z_{11}	131.1 – 589.8i	Z_{21}	118.3 – 28.10i	Z_{31}	49.5 – 33.4i	Z_{41}	13.3 – 50.0i	Z_{51}	-18.4 – 44.2i
Z_{12}	118.3 – 28.10i	Z_{22}	1638.7 – 1252.5i	Z_{32}	137.3 + 66.7i	Z_{42}	102.5 – 24.5i	Z_{52}	54.5 – 66.2i
Z_{13}	49.5 – 33.4i	Z_{23}	137.3 + 66.7i	Z_{33}	142.9 + 254.4i	Z_{43}	110.2 + 27.9i	Z_{53}	83.8 – 26.8i
Z_{14}	13.3 – 50.0i	Z_{24}	102.5 – 24.5i	Z_{34}	110.2 + 27.9i	Z_{44}	142.9 + 254.4i	Z_{54}	110.2 + 27.9i
Z_{15}	-18.4 – 44.2i	Z_{25}	54.5 – 66.2i	Z_{35}	83.8 – 26.8i	Z_{45}	110.2 + 27.9i	Z_{55}	142.9 + 254.4i

Table 13: Frequency = 200 MHz

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.2 λ;									
Z_{11}	131.1 – 589.8i	Z_{21}	118.3 – 28.10i	Z_{31}	26.2 – 14.8i	Z_{41}	6.9 – 25.4i	Z_{51}	-10.0 – 22.9i
Z_{12}	118.3 – 28.10i	Z_{22}	1638.7 – 1252.5i	Z_{32}	74.5 + 52.9i	Z_{42}	55.7 – 8.5i	Z_{52}	29.8 – 34.2i
Z_{13}	26.2 – 14.8i	Z_{23}	74.5 + 52.9i	Z_{33}	41.5 – 157.9i	Z_{43}	33.3 – 6.i	Z_{53}	25.5 – 11.9i
Z_{14}	6.9 – 25.4i	Z_{24}	55.7 – 8.5i	Z_{34}	33.3 – 6.0i	Z_{44}	41.5 – 157.9i	Z_{54}	33.3 – 6.0i
Z_{15}	-10.0 – 22.9i	Z_{25}	29.8 – 34.2i	Z_{35}	25.5 – 11.9i	Z_{45}	33.3 – 6.0i	Z_{55}	41.5 – 157.9i

Table 14: Frequency = 200 MHz

Lreflector= 0.6 λ; Lactive = 0.5 λ; Other directors = 0.45 λ to 0.1 λ with tapering interval of 0.5 λ ;									
Z_{11}	131.1 – 589.8i	Z_{21}	118.3 – 28.10i	Z_{31}	72.5 – 62.6i	Z_{41}	18.9 – 71.8i	Z_{51}	-21.8 – 54.0i
Z_{12}	118.3 – 28.10i	Z_{22}	1638.7 – 1252.5i	Z_{32}	187.2 + 36.4i	Z_{42}	134.2 – 44.1i	Z_{52}	64.3 – 80.7i
Z_{13}	72.5 – 62.6i	Z_{23}	187.2 + 36.4i	Z_{33}	1724.1 + 911.7i	Z_{43}	188.8 + 53.1i	Z_{53}	128.6 – 37.1i
Z_{14}	18.9 – 71.8i	Z_{24}	134.2 – 44.1i	Z_{34}	188.8 + 53.1i	Z_{44}	590.5 + 828.7i	Z_{54}	167.5 + 53.6i
Z_{15}	-21.8 – 54.0i	Z_{25}	64.3 – 80.7i	Z_{35}	128.6 – 37.1i	Z_{45}	167.5 + 53.6i	Z_{55}	268.4 + 503.5i
Z_{16}	-38.3 – 23.9i	Z_{26}	5.3 – 74.2i	Z_{36}	59.2 – 70.1i	Z_{46}	107.2 – 29.9i	Z_{56}	129.7 + 39.6i
Z_{17}	-32.9 + 1.6i	Z_{27}	25.8 – 44.3i	Z_{37}	6.7 – 60.8i	Z_{47}	46.5 – 53.6i	Z_{57}	76.9 – 22.9i
Z_{18}	17.4 + 13.0i	Z_{28}	-29.1 – 14.1i	Z_{38}	-17.3 – 33.5i	Z_{48}	5.8 – 42.1i	Z_{58}	30.5 – 35.0i
Z_{19}	-4.1 + 11.6i	Z_{29}	-17.6 + 2.4i	Z_{39}	-17.7 – 9.9i	Z_{49}	-9.5 – 20.3i	Z_{59}	3.7 – 23.5i
Z_{110}	1.3 + 5.2i	Z_{210}	-5.7 + 5.0i	Z_{310}	-8.5 +0.7i	Z_{410}	-7.8 – 4.8i	Z_{510}	-3.8 – 8.8i
Z_{61}	-38.3 – 23.9i	Z_{71}	-32.9 + 1.6i	Z_{81}	-17.4 + 13.0i	Z_{91}	-4.1 + 11.6i	Z_{101}	1.3 + 5.2i
Z_{62}	5.3 – 74.2i	Z_{72}	-25.8 – 44.3i	Z_{82}	-29.1 – 14.1i	Z_{92}	-17.6 + 2.4i	Z_{102}	-5.7 + 5.0i
Z_{63}	59.2 – 70.1i	Z_{73}	6.7 – 60.8i	Z_{83}	-17.3 – 33.5i	Z_{93}	-17.7 - .9.9i	Z_{103}	-8.5 + 0.7i
Z_{64}	107.2 – 29.9i	Z_{74}	46.5 – 53.6i	Z_{84}	5.8 – 42.1i	Z_{94}	-9.5 – 20.3i	Z_{104}	-7.8 -4.8i
Z_{65}	129.7 + 39.6i	Z_{75}	76.9 – 22.9i	Z_{85}	30.5 – 35.0i	Z_{95}	3.7 – 23.5i	Z_{105}	-3.8 – 8.8i
Z_{66}	142.9 + 254.4i	Z_{76}	86.1 + 18.9i	Z_{86}	46.2 – 16.0i	Z_{96}	15.9 – 18.6i	Z_{106}	1.6 – 9.5i
Z_{67}	86.1 + 18.9i	Z_{77}	82.1 + 35.6i	Z_{87}	47.3 + 1.0i	Z_{97}	21.9 – 9.5i	Z_{107}	5.9 – 7.1i
Z_{68}	46.2 – 16.0i	Z_{78}	47.3 + 1.0i	Z_{88}	41.5 – 157.9i	Z_{98}	20.1 – 7.5i	Z_{108}	7.2 – 4.1i
Z_{69}	15.9 – 18.6i	Z_{79}	21.9 – 9.5i	Z_{89}	20.1 – 7.5i	Z_{99}	20.3 – 370.1i	Z_{109}	5.7 – 6.4i
Z_{610}	1.6 – 9.5i	Z_{710}	5.9 – 7.1i	Z_{810}	7.2 – 4.1i	Z_{910}	5.7 – 6.4i	Z_{1010}	8.2 – 655.5i

Table 21: Frequency = 250 MHz

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.4 λ;									
Z_{11}	125.9 – 557.1i	Z_{21}	118.3 – 28.10i	Z_{31}	67.8 – 54.20i	Z_{41}	18.9 - 71.80i	Z_{51}	-24.1 – 61.90i
Z_{12}	118.3 – 28.10i	Z_{22}	1500.9 – 1170.3i	Z_{32}	180.5 + 51.20i	Z_{42}	134.2 - 44.10i	Z_{52}	70.6 – 91.80i
Z_{13}	67.8 – 54.20i	Z_{23}	180.5 + 51.20i	Z_{33}	588.0 + 783.9i	Z_{43}	184.0 + 53.7i	Z_{53}	138.3 – 42.6i
Z_{14}	18.9 – 71.80i	Z_{24}	134.2 - 44.10i	Z_{34}	184.0 + 53.7i	Z_{44}	588.0 + 783.9i	Z_{54}	184.0 + 53.7i
Z_{15}	-24.1 - 61.90i	Z_{25}	70.6 – 91.80i	Z_{35}	138.3 - 42.6i	Z_{45}	184.0 + 53.7i	Z_{55}	588.0 + 783.9i

Table 22: Frequency = 250 MHz

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.3 λ;									
Z_{11}	125.9 – 557.1i	Z_{21}	118.3 – 28.10i	Z_{31}	49.5 – 33.4i	Z_{41}	13.3 – 50.0i	Z_{51}	-18.4 – 44.2i
Z_{12}	118.3 – 28.10i	Z_{22}	1500.9 – 1170.3i	Z_{32}	137.3 + 66.7i	Z_{42}	102.5 – 24.5i	Z_{52}	54.5 – 66.2i
Z_{13}	49.5 – 33.4i	Z_{23}	137.3 + 66.7i	Z_{33}	143.3 + 245.1i	Z_{43}	110.2 + 27.9i	Z_{53}	83.8 – 26.8i
Z_{14}	13.3 – 50.0i	Z_{24}	102.5 – 24.5i	Z_{34}	110.2 + 27.9i	Z_{44}	143.3 + 245.1i	Z_{54}	110.2 + 27.9i
Z_{15}	-18.4 – 44.2i	Z_{25}	54.5 – 66.2i	Z_{35}	83.8 – 26.8i	Z_{45}	110.2 + 27.9i	Z_{55}	143.3 + 245.1i

Table 23: Frequency = 250 MHz

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.2 λ;									
Z_{11}	125.9 – 557.1i	Z_{21}	118.3 – 28.10i	Z_{31}	26.2 – 14.8i	Z_{41}	6.9 – 25.4i	Z_{51}	-10.0 – 22.9i
Z_{12}	118.3 – 28.10i	Z_{22}	1500.9 – 1170.3i	Z_{32}	74.5 + 52.9i	Z_{42}	55.7 – 8.5i	Z_{52}	29.8 – 34.2i
Z_{13}	26.2 – 14.8i	Z_{23}	74.5 + 52.9i	Z_{33}	41.6 – 149.3i	Z_{43}	33.3 – 6.i	Z_{53}	25.5 – 11.9i
Z_{14}	6.9 – 25.4i	Z_{24}	55.7 – 8.5i	Z_{34}	33.3 – 6.0i	Z_{44}	41.6 – 149.3i	Z_{54}	33.3 – 6.0i
Z_{15}	-10.0 – 22.9i	Z_{25}	29.8 – 34.2i	Z_{35}	25.5 – 11.9i	Z_{45}	33.3 – 6.0i	Z_{55}	41.6 – 149.3i

Table 24: Frequency = 250 MHz.

Lreflector= 0.6 λ; Lactive = 0.5 λ; Other directors = 0.45 λ to 0.1 λ with tapering interval of 0.5 λ ;									
Z_{11}	125.9 – 557.1i	Z_{21}	118.3 – 28.10i	Z_{31}	72.5 – 62.6i	Z_{41}	18.9 – 71.8i	Z_{51}	-21.8 – 54.0i
Z_{12}	118.3 – 28.10i	Z_{22}	1500.9 – 1170.3i	Z_{32}	187.2 + 36.4i	Z_{42}	134.2 – 44.1i	Z_{52}	64.3 – 80.7i
Z_{13}	72.5 – 62.6i	Z_{23}	187.2 + 36.4i	Z_{33}	164.9 + 802.1i	Z_{43}	188.8 + 53.1i	Z_{53}	128.6 – 37.1i
Z_{14}	18.9 – 71.8i	Z_{24}	134.2 – 44.1i	Z_{34}	188.8 + 53.1i	Z_{44}	588.0 + 783.9i	Z_{54}	167.5 + 53.6i
Z_{15}	-21.8 – 54.0i	Z_{25}	64.3 – 80.7i	Z_{35}	128.6 – 37.1i	Z_{45}	167.5 + 53.6i	Z_{55}	268.6 + 482.1i
Z_{16}	-38.3 – 23.9i	Z_{26}	5.3 – 74.2i	Z_{36}	59.2 – 70.1i	Z_{46}	107.2 – 29.9i	Z_{56}	129.7 + 39.6i
Z_{17}	-32.9 + 1.6i	Z_{27}	25.8 – 44.3i	Z_{37}	6.7 – 60.8i	Z_{47}	46.5 – 53.6i	Z_{57}	76.9 – 22.9i
Z_{18}	17.4 + 13.0i	Z_{28}	-29.1 – 14.1i	Z_{38}	-17.3 – 33.5i	Z_{48}	5.8 – 42.1i	Z_{58}	30.5 – 35.0i
Z_{19}	-4.1 + 11.6i	Z_{29}	-17.6 + 2.4i	Z_{39}	-17.7 – 9.9i	Z_{49}	-9.5 – 20.3i	Z_{59}	3.7 – 23.5i
Z_{110}	1.3 + 5.2i	Z_{210}	-5.7 + 5.0i	Z_{310}	-8.5 +0.7i	Z_{410}	-7.8 – 4.8i	Z_{510}	-3.8 – 8.8i
Z_{61}	-38.3 – 23.9i	Z_{71}	-32.9 + 1.6i	Z_{81}	-17.4 + 13.0i	Z_{91}	-4.1 + 11.6i	Z_{101}	1.3 + 5.2i
Z_{62}	5.3 – 74.2i	Z_{72}	-25.8 – 44.3i	Z_{82}	-29.1 – 14.1i	Z_{92}	-17.6 + 2.4i	Z_{102}	-5.7 + 5.0i
Z_{63}	59.2 – 70.1i	Z_{73}	6.7 – 60.8i	Z_{83}	-17.3 – 33.5i	Z_{93}	-17.7 - .9.9i	Z_{103}	-8.5 + 0.7i
Z_{64}	107.2 – 29.9i	Z_{74}	46.5 – 53.6i	Z_{84}	5.8 – 42.1i	Z_{94}	-9.5 – 20.3i	Z_{104}	-7.8 - 4.8i
Z_{65}	129.7 + 39.6i	Z_{75}	76.9 – 22.9i	Z_{85}	30.5 – 35.0i	Z_{95}	3.7 – 23.5i	Z_{105}	-3.8 – 8.8i
Z_{66}	143.3 + 245.1i	Z_{76}	86.1 + 18.9i	Z_{86}	46.2 – 16.0i	Z_{96}	15.9 – 18.6i	Z_{106}	1.6 – 9.5i
Z_{67}	86.1 + 18.9i	Z_{77}	82.5 + 35.2i	Z_{87}	47.3 + 1.0i	Z_{97}	21.9 – 9.5i	Z_{107}	5.9 – 7.1i
Z_{68}	46.2 – 16.0i	Z_{78}	47.3 + 1.0i	Z_{88}	41.6 – 149.3i	Z_{98}	20.1 – 7.5i	Z_{108}	7.2 – 4.1i
Z_{69}	15.9 – 18.6i	Z_{79}	21.9 – 9.5i	Z_{89}	20.1 – 7.5i	Z_{99}	20.3 – 350.8i	Z_{109}	5.7 – 6.4i
Z_{610}	1.6 – 9.5i	Z_{710}	5.9 – 7.1i	Z_{810}	7.2 – 4.1i	Z_{910}	5.7 – 6.4i	Z_{1010}	8.2 – 619.1i

Table 31: Frequency = 300 MHz.

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.4 λ;									
Z_{11}	121.6 – 530.6i	Z_{21}	118.3 – 28.10i	Z_{31}	67.8 – 54.20i	Z_{41}	18.9 - 71.80i	Z_{51}	-24.1 – 61.90i
Z_{12}	118.3 – 28.10i	Z_{22}	1392.9 – 1104.9i	Z_{32}	180.5 + 51.20i	Z_{42}	134.2 - 44.10i	Z_{52}	70.6 – 91.80i
Z_{13}	67.8 – 54.20i	Z_{23}	180.5 + 51.20i	Z_{33}	585.6 + 746.9i	Z_{43}	184.0 + 53.7i	Z_{53}	138.3 – 42.6i
Z_{14}	18.9 – 71.80i	Z_{24}	134.2 - 44.10i	Z_{34}	184.0 + 53.7i	Z_{44}	585.6 + 746.9i	Z_{54}	184.0 + 53.7i
Z_{15}	-24.1 - 61.90i	Z_{25}	70.6 – 91.80i	Z_{35}	138.3 - 42.6i	Z_{45}	184.0 + 53.7i	Z_{55}	585.6 + 746.9i

Table 32: Frequency = 300 MHz.

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.3 λ;									
Z_{11}	121.6 – 530.6i	Z_{21}	118.3 – 28.10i	Z_{31}	49.5 – 33.4i	Z_{41}	13.3 – 50.0i	Z_{51}	-18.4 – 44.2i
Z_{12}	118.3 – 28.10i	Z_{22}	1392.9 – 1104.9i	Z_{32}	137.3 + 66.7i	Z_{42}	102.5 – 24.5i	Z_{52}	54.5 - 0.0662i
Z_{13}	49.5 – 33.4i	Z_{23}	137.3 + 66.7i	Z_{33}	143.6 + 237.5i	Z_{43}	110.2 + 27.9i	Z_{53}	83.8 – 26.8i
Z_{14}	13.3 – 50.0i	Z_{24}	102.5 – 24.5i	Z_{34}	110.2 + 27.9i	Z_{44}	143.6 + 237.5i	Z_{54}	110.2 + 27.9i
Z_{15}	-18.4 – 44.2i	Z_{25}	54.5 – 66.2i	Z_{35}	83.8 – 26.8i	Z_{45}	110.2 + 27.9i	Z_{55}	143.6 + 237.5i

Table 33: Frequency = 300 MHz.

Lreflector= 0.6 λ ; Lactive = 0.5 λ; Other 3 directors = 0.2 λ;									
Z_{11}	121.6 – 530.6i	Z_{21}	118.3 – 28.10i	Z_{31}	26.2 – 14.8i	Z_{41}	6.9 – 25.4i	Z_{51}	-10.0 – 22.9i
Z_{12}	118.3 – 28.10i	Z_{22}	1392.9 – 1104.9i	Z_{32}	74.5 + 52.9i	Z_{42}	55.7 – 8.5i	Z_{52}	29.8 – 34.2i
Z_{13}	26.2 – 14.8i	Z_{23}	74.5 + 52.9i	Z_{33}	41.7 – 142.3i	Z_{43}	33.3 – 6.i	Z_{53}	25.5 – 11.9i
Z_{14}	6.9 – 25.4i	Z_{24}	55.7 – 8.5i	Z_{34}	33.3 – 6.0i	Z_{44}	41.7 – 142.3i	Z_{54}	33.3 – 6.0i
Z_{15}	-10.0 – 22.9i	Z_{25}	29.8 – 34.2i	Z_{35}	25.5 – 11.9i	Z_{45}	33.3 – 6.0i	Z_{55}	41.7 – 142.3i

Table 34: Frequency = 300 MHz.

Lreflector= 0.6 λ; Lactive = 0.5 λ; Other directors = 0.45 λ to 0.1 λ with tapering interval of 0.5 λ ;									
Z_{11}	121.6 – 530.6i	Z_{21}	118.3 – 28.10i	Z_{31}	72.5 – 62.6i	Z_{41}	18.9 – 71.8i	Z_{51}	-21.8 – 54.0i
Z_{12}	118.3 – 28.10i	Z_{22}	1392.9 – 1104.9i	Z_{32}	187.2 + 36.4i	Z_{42}	134.2 – 44.1i	Z_{52}	64.3 – 80.7i
Z_{13}	72.5 – 62.6i	Z_{23}	187.2 + 36.4i	Z_{33}	1613.1 + 714.3i	Z_{43}	188.8 + 53.1i	Z_{53}	128.6 – 37.1i
Z_{14}	18.9 – 71.8i	Z_{24}	134.2 – 44.1i	Z_{34}	188.8 + 53.1i	Z_{44}	585.6 + 746.9	Z_{54}	167.5 + 53.6i
Z_{15}	-21.8 – 54.0i	Z_{25}	64.3 – 80.7i	Z_{35}	128.6 – 37.1i	Z_{45}	167.5 + 53.6i	Z_{55}	268.8 + 464.5i
Z_{16}	-38.3 – 23.9i	Z_{26}	5.3 – 74.2i	Z_{36}	59.2 – 70.1i	Z_{46}	107.2 – 29.9i	Z_{56}	129.7 + 39.6i
Z_{17}	-32.9 + 1.6i	Z_{27}	25.8 – 44.3i	Z_{37}	6.7 – 60.8i	Z_{47}	46.5 – 53.6i	Z_{57}	76.9 – 22.9i
Z_{18}	17.4 + 13.0i	Z_{28}	-29.1 – 14.1i	Z_{38}	-17.3 – 33.5i	Z_{48}	5.8 – 42.1i	Z_{58}	30.5 – 35.0i
Z_{19}	-4.1 + 11.6i	Z_{29}	-17.6 + 2.4i	Z_{39}	-17.7 – 9.9i	Z_{49}	-9.5 – 20.3i	Z_{59}	3.7 – 23.5i
Z_{110}	1.3 + 5.2i	Z_{210}	-5.7 + 5.0i	Z_{310}	-8.5 +0.7i	Z_{410}	-7.8 – 4.8i	Z_{510}	-3.8 – 8.8i
Z_{61}	-38.3 – 23.9i	Z_{71}	-32.9 + 1.6i	Z_{81}	-17.4 + 13.0i	Z_{91}	-4.1 + 11.6i	Z_{101}	1.3 + 5.2i
Z_{62}	5.3 – 74.2i	Z_{72}	-25.8 – 44.3i	Z_{82}	-29.1 – 14.1i	Z_{92}	-17.6 + 2.4i	Z_{102}	-5.7 + 5.0i
Z_{63}	59.2 – 70.1i	Z_{73}	6.7 – 60.8i	Z_{83}	-17.3 – 33.5i	Z_{93}	-17.7 - .9.9i	Z_{103}	-8.5 + 0.7i
Z_{64}	107.2 – 29.9i	Z_{74}	46.5 – 53.6i	Z_{84}	5.8 – 42.1i	Z_{94}	-9.5 – 20.3i	Z_{104}	-7.8 -4.8i
Z_{65}	129.7 + 39.6i	Z_{75}	76.9 – 22.9i	Z_{85}	30.5 – 35.0i	Z_{95}	3.7 – 23.5i	Z_{105}	-3.8 – 8.8i
Z_{66}	143.6 +.237.5i	Z_{76}	86.1 + 18.9i	Z_{86}	46.2 – 16.0i	Z_{96}	15.9 – 18.6i	Z_{106}	1.6 – 9.5i
Z_{67}	86.1 + 18.9i	Z_{77}	82.8 + 34.8i	Z_{87}	47.3 + 1.0i	Z_{97}	21.9 – 9.5i	Z_{107}	5.9 – 7.1i
Z_{68}	46.2 – 16.0i	Z_{78}	47.3 + 1.0i	Z_{88}	41.7 – 142.3i	Z_{98}	20.1 – 7.5i	Z_{108}	7.2 – 4.1i
Z_{69}	15.9 – 18.6i	Z_{79}	21.9 – 9.5i	Z_{89}	20.1 – 7.5i	Z_{99}	20.3 – 335.1i	Z_{109}	5.7 – 6.4i
Z_{610}	1.6 – 9.5i	Z_{710}	5.9 – 7.1i	Z_{810}	7.2 – 4.1i	Z_{910}	5.7 – 6.4i	Z_{1010}	8.1 – 589.4i

Conclusions

The following conclusions are evident from results presented above.

Self Impedance

- The self impedance remains same for fixed operating frequency and fixed length of element.
- The self impedance of the elements is dependent on frequency of operation. As frequency is varied the corresponding self impedances of the different elements of the antenna also vary.
- The self impedances of the active element which is of the length 0.5 λ and the element with length of 0.45 λ are having marginally higher values compared to other elements for that corresponding operating frequency.
- As the operating frequency is increased the self impedance decreases for all elements.
- For a fixed operating frequency the self impedance decreases with decrease in length of antenna where length is less than 0.45 λ

Mutual Impedance

- For a fixed operating frequency and fixed length of elements the mutual impedance decreases from first element to last element.
- The mutual impedances remain same for fixed lengths of elements and fixed spacing independent of frequency.

- For a fixed operating frequency the mutual impedances decrease with decrease in lengths of the elements.
- The variation of self impedance with lengths of elements for a fixed frequency and for different frequencies is shown in Fig. 2 and Fig. 3 below respectively.

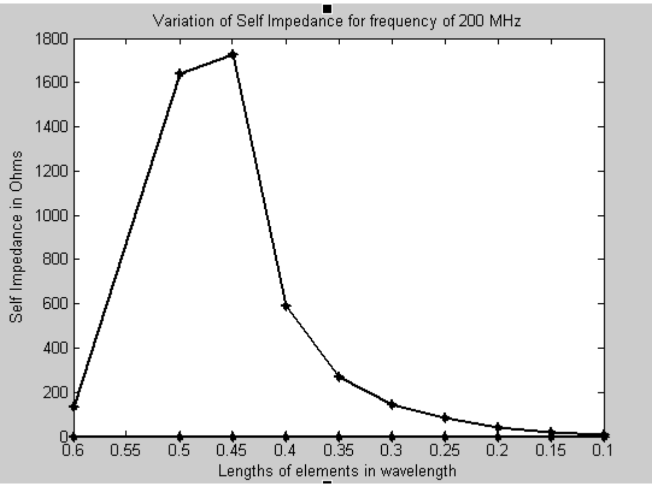

Figure 2: Variation of Self impedance with lengths.

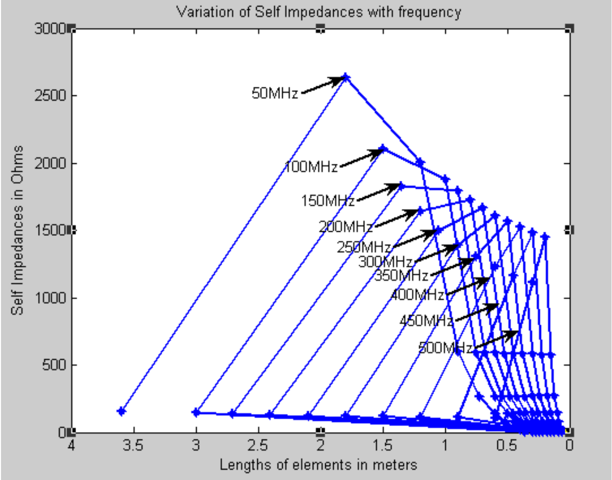

Figure 3: Variation of Self impedance with frequency.

The variation of Mutual Impedances with the spacing between the elements and lengths of elements is shown in Fig. 4 below. Note that Z_{12}= Z_{21}, Z_{13}=Z_{31}, Z_{23}=Z_{32}……, Z_{910}=Z_{109}.

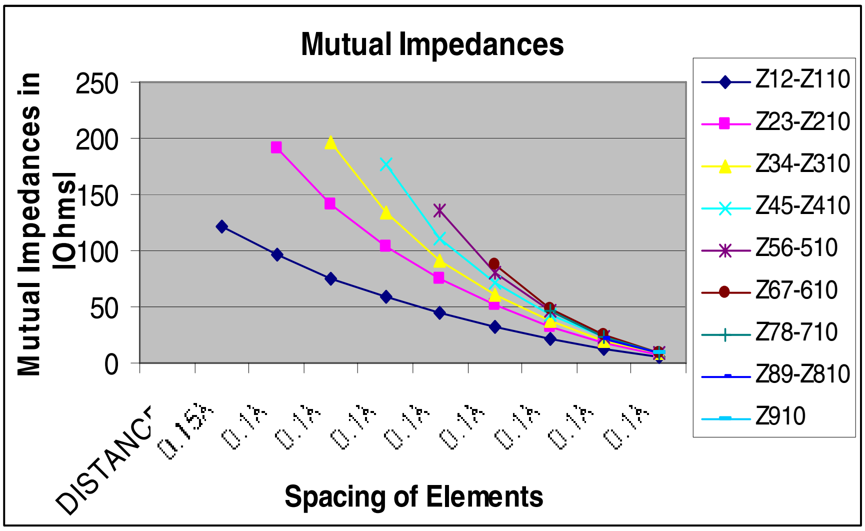

Figure 4: Variation of Mutual Impedances.

References

[1] Constantine A. Balanis, Antenna Theory, John Wiley & Sons, 1997

[2] L.V. Blake, Antennas, John Wiley & Sons, New York, 1966

[3] G.A. Thiele and E. P. Ekelnman Jr., Design of Formulas for Vee Dipoles, IEEE Transactions on Antennas Propagation., Vol. AP-28, N0. 4, pp, 588-590, July 1980.

[4] Dr. G.S.N. Raju, Antennas & Wave Propagation, Pearson Education, 2005

[5] Sommerfield A. Partial differential equations, Academic Press, New York, 1949.

[6] Banos, A., Dipole Radiation in the presence of a conducting half space, Peragamon Press, Oxford England, 1966.

[7] King R. W. P., Theory of Linear Antennas, Harvard University Press, Cambridge, Mass, 1956.

[8] Bhattacharya B.K., Input resistances of horizontal electric and vertical magnetic dipoles over a homogeneous ground, IEEE Transactions, Antennas & Propagation, Vol. AP-11, No.3 pp. 261-266, May 1963.

[9] King. R.W.P. and T.T. Wu, Currents, Charges near fields of antennas, Radio Sci., Vol. 69D, No. 3, pp 429-446, March, 1965.

[10] M. T. Maa, Theory and Applications of Antenna Arrays, John Wiley & Sons, 1974

[11] Edward C. Jordan, Keith G. Balmin, Electro-Magnetic waves and radiating systems, PHI, 2001

[12] Carter P.S., Circuit relations in radiating systems and application to antenna problems, Proceedings, IRE, Vol. 20, No.6, pp, 1004-1041, June, 1932.

Appendix

The generalized sine and cosine integrals appearing in equations (3) to (25) are expressed as [10]

$$S(b,x)=\int_0^x \frac{\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A1}$$

$$C(b,x)=\int_0^x \frac{1-\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A2}$$

$$\overline{C}(b,x)=\int_0^x \frac{\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A3}$$

$$S_s(b,x)=\int_0^x \frac{\sin y\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A4}$$

$$S_c(b,x)=\int_0^x \frac{\cos y\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A5}$$

$$C_s(b,x)=\int_0^x \frac{\sin y\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A6}$$

$$C_c(b,x)=\int_0^x \frac{(1-\cos y)\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A7}$$

$$\overline{C}_c(b,x)=\int_0^x \frac{\cos y\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A8}$$

The numerical integrations are carried out using MATLAB in all the analysis carried out for impedance calculations.

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