An Innovative Approach of Dimension Reduction of Large Scale Control System using Bio-inspired Bacterial Foraging Optimization Technique

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Abstract

The authors proposed here a new technique for dimension reduction of linear time-invariant large scale control system. In this paper, an algorithm for model reduction of LTIC system is proposed using combined advantages of stability equation method and error minimization by Bacterial Foraging optimization (BFO). In this algorithm, coefficients of denominator polynomial of reduced order transfer function model are obtained by stability equation method and the numerator coefficients are determined by minimizing the integral square error (ISE) between the transient responses of original high order system and reduced order model using BFO technique. The proposed algorithm in this paper is illustrated through numerical example and a comparison has been made with some existing techniques. The method preserves the stability in the reduced model of original system. It is also observed that my proposed method gives low ISE than existing methods. The method is simple, rugged and computer oriented. The proposed algorithm has been implemented in Mat-lab 2010 environment on a Pentium-IV computer and c.p.u time required is being less than 30 sec.

Keywords: Model reduction, Stability, Bacterial Foraging optimization, Integral square error.

1. Introduction

The modern bio or nature inspired soft-computing search and optimization algorithms include a variety of population-based swarm intelligence algorithms, such as Particle...
Swarm Optimization [1-2], Bacterial Foraging Optimization [3], Ant Colony Optimization Algorithms [4], Genetic Algorithm [5], Biogeography Based Optimization, Mimetic Optimization, Snake-Pray Optimization, etc. These can be applied to various real life optimization problems. In the recent years, these algorithms are extensively used in pioneer areas of engineering such as, control systems, aerospace, biomedical, power systems, mechanical and civil applications. Swarm Intelligence has recently emerged as a family of bio-inspired algorithms [6-7], especially known for their ability to produce cost effective, fast and reasonably accurate solutions to the complex large problems. Further, PSO is a population-based continuous optimization technique proposed by Kennedy and Eberhart [1], inspired by social behavior of bird flocking or fish schooling. A recent bio-inspired soft-computing optimization technique, namely Bacterial Foraging optimization (BFO) has been proposed by Passino [3] for optimization problems and further incorporated by Mishra [8] for optimization. In this approach, the foraging behavior of E. coli bacteria in human intestine, including locating, handling, and ingesting food is mimicked. The technique consists of the four stages namely chemotactic, swarming, reproduction, and elimination & dispersal, which are modeled to tackle optimization problems.

2. Description of Proposed Dimension Reduction Method

The method consists of two steps for reducing the order of high order original system, as stated below:

**STEP-1:** Determination of the denominator coefficients of reduced order model \( G(s) \) using Stability Equation Method

For stable high order system \( G(s) \), the denominator \( D(s) \) of the high order system is decomposed in the even and odd components in the form of stability equations as:

\[
D_{even}(s) = \sum_{i=0,2,4} d_i s^i = d_0 \prod_{i=1}^{m_1} (1 + \frac{s^2}{z_i^2}) \quad (1)
\]

\[
D_{odd}(s) = \sum_{i=1,3,5} d_i s^i = d_0 \prod_{i=1}^{m_2} (1 + \frac{s^2}{p_i^2}) \quad (2)
\]

where, \( m_1 \) and \( m_2 \) are integer parts of \( n/2 \) and \( (n-1)/2 \), respectively and \( z_1^2 < p_1^2 < z_2^2 < p_2^2 \ldots \ldots \)

Now, by neglecting the factors of larger magnitudes of \( z_i^2 \) and \( p_i^2 \) in equation (1) and (2), the stability equations for \( r^{th} \) order system are obtained as below :

\[
D_{even}^r(s) = d_0 \prod_{i=1}^{m_3} (1 + \frac{s^2}{z_i^2}) \quad (3)
\]

\[
D_{odd}^r(s) = d_4 s \prod_{i=1}^{m_4} (1 + \frac{s^2}{p_i^2}) \quad (4)
\]

where, \( m_3 \) and \( m_4 \) are the integer parts of \( r/2 \) and \( (r-1)/2 \) respectively.
An Innovative Approach of Dimension Reduction of Large Scale Control System

After adding two reduced order model stability equations given in equation (3) and (4) and then proper normalizing it, the r\textsuperscript{th} order denominator \( D_r(s) \) of reduced model can be obtained as:

\[
D_r(s) = D_{even}^r(s) + D_{odd}^r(s) = \sum_{i=0}^{r-1} d_i s^i + s^r
\]  

(5)

Thus, the denominator polynomial is now known as:

\[
D_r(s) = f_0 + f_1 s + f_2 s^2 + \cdots + f_{r-1} s^{r-1} + s^r
\]  

(6)

**STEP-2:** Determination of The numerator coefficients of the reduced order model \( R(s) \) By Bacterial Foraging (BFO) Method.

The Bacterial Foraging optimization (BFO) technique is used to minimize the objective function \( J \) which is the integral square error (ISE) between the transient responses of original high order model \( G(s) \) and reduced order model \( R(s) \). The ISE is defined by:

\[
ISE(J) = \int_0^\infty [ y(t) - y_r(t)]^2 dt
\]  

(7)

where \( y(t) \) and \( y_r(t) \) are the unit step responses of original high order system \( G(s) \) and reduced order system \( R(s) \), respectively. The parameters to be determined are the numerator polynomial coefficients \( e_i \) ( \( i = 0, 1, \ldots, (r-1) \)).

3. Control System Under Study

Consider a single-input single-output (SISO) 4\textsuperscript{th} order system described by transfer function \( G(s) \) in Equation (8):

\[
G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}
\]  

(8)

The objective is to derive second-order model, then the following steps are to be taken into consideration.

**Step-1:** Separating the even and odd components of denominator polynomial of the above original high order system in Eq. (8) and then get the stability equations as:

\[
D_{even}(s) = 24 + 35s^2 + s^4 = 24 \left(1 + \frac{s^2}{0.6997}\right) \left(1 + \frac{s^2}{34.3004}\right)
\]  

(9)

\[
D_{odd}(s) = 50s + 10s^3 = 50s(1 + \frac{s^2}{5})
\]  

(10)

Now, by neglecting the factors with large magnitude of \( z_i^2 \) and \( p_i^2 \) in \( D_{even}(s) \) in Eq.(9) and \( D_{odd}(s) \) in Eq. (10) respectively, the stability equation for 2\textsuperscript{nd} order model are obtained as follows:

\[
D_{even}^r(s) = 24 \left(1 + \frac{s^2}{0.6997}\right)
\]  

(11)

\[
D_{odd}^r(s) = 50s
\]  

(12)
Combining reduced order system stability equations given in equation (11) and equation (12) and thereafter normalizing, the denominator of the 2\textsuperscript{nd} order model is obtained as:

\[ D(s) = s^2 + 1.45771 + 0.6997 \]  

(13)

**Step-2:** The Bacterial Foraging (BFO) technique is used to minimize the objective function (ISE) and the numerator coefficients of reduced -order model are obtained as below:

\[ N(s) = 0.770451 s + 0.699846 \]  

(14)

Now, reduced model in terms of transfer function of 2\textsuperscript{nd} order is obtained as below

\[ R(s) = \frac{0.770451s+0.699846}{s^2+1.45771s+0.6997} \]  

(15)

with an ISE is $1.61172 \times 10^{-3}$.

**Table 1:** Typical parameters used by the BFO algorithm.

<table>
<thead>
<tr>
<th>Type of variables used</th>
<th>Values assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>8</td>
</tr>
<tr>
<td>p</td>
<td>2</td>
</tr>
<tr>
<td>Nc</td>
<td>8</td>
</tr>
<tr>
<td>Ns</td>
<td>3</td>
</tr>
<tr>
<td>Nre</td>
<td>6</td>
</tr>
<tr>
<td>Ned</td>
<td>3</td>
</tr>
<tr>
<td>ped</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Using Bacterial Foraging (BFO) technique, the step and impulse responses are obtained for high order system and reduced order model (second order) and are shown in Fig.1 and Fig. 2. The minimization of the objective function ISE through BFO optimization is carried out in Mat-Lab R2010b environment. The different parameter values used for the Bacterial Foraging Optimization (BFO) are listed in Table 1. The same parameters for BFO may not give the best results and so these can be changed according to the nature of problem.

Table 2 provides the comparative studies of the 2\textsuperscript{nd} order reduced model by different well known existing methods. An error performance index ISE is determined to measure the goodness of the reduced order model. It is observed that the proposed method for model order reduction gives the lowest value of ISE in comparison with other existing methods for same model as given in Table 2.

**Table 2**: Comparison of reduced order model by different methods.

<table>
<thead>
<tr>
<th>Methods of order reduction</th>
<th>Low order models</th>
<th>ISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Method</td>
<td>$0.770451s + 0.699846$</td>
<td>$1.611725 \times 10^{-3}$</td>
</tr>
<tr>
<td>Chen et al.</td>
<td>$0.6997(s + 1)$ \hspace{1cm} $0.770451s + 0.699846$</td>
<td>$2.665530 \times 10^{-3}$</td>
</tr>
<tr>
<td>Gutman et al.</td>
<td>$2(48s + 144)$ \hspace{1cm} $70s^2 + 300s + 288$</td>
<td>$45.592871 \times 10^{-3}$</td>
</tr>
<tr>
<td>Prasad and Pal</td>
<td>$(s + 34.2465)$ \hspace{1cm} $s^2 + 239.8082s + 34.3465$</td>
<td>$1.534272$</td>
</tr>
</tbody>
</table>

**Fig. 2**: Comparison of impulse responses of reduced and high order systems.
4. Conclusions
The proposed method based on BFO in this paper are illustrated through numerical example and compared with some existing techniques. This method preserves steady state value and stability in the reduced model of original system. It is also observed that my proposed method gives low value of ISE than existing methods. These methods are simple, rugged and computer oriented. The proposed algorithms have been implemented in Mat-lab 2010 environment on a Pentium-IV computer. The algorithm presented in this paper can be applied for power system transient studies as well as controller design.

References