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Heat Transfer in a Power Law Fluid with Variable Thermal Conductivity

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Abstract

We examine the heat transfer in a power law fluid near a moving wall with variable thermal conductivity. The thermal conductivity depends on temperature and time. The governing nonlinear partial differential equations are transformed into a nonlinear ordinary differential equation and then solve numerically using shooting method. The result is presented as temperature profile and this shows the influence of the variable thermal conductivity and power law exponent on the heat flow when the viscous dissipation is important.

Keyword : Power law Fluid, thermal conductivity.

Introduction

The study of heat transfer in non Newtonian power law fluid is gaining the attention of scientists and researchers due to its interesting application in chemical and engineering processes. Ibrahim et al (2005) investigated the method of similarity reduction for problems of radiative and magnetic field effect on free convection and mass transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using lie group method.They also presented the numerical results for the non-uniform m agnetic field.

Yurusoy and Pakdemirli(1999)examine the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behaviuor increases. They also compared the results with that of Newtonian fluid. Makinde (2005) examined the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. The plate is maintained at a uniform temperature with uniform species concentration and the fluid is considered to be gray, absorbing – emitting. The coupled non-linear momentum, energy and concentration equation governing the problem is obtained and made similar by introducing a time dependent length scale. The similarity equations are then solved numerically by using superposition method.

Chung (2006) examined the nonlinear stability of steady flow and temperature distribution of a Newtonian fluid in a channel heated from below and the viscosity is a function of temperature.

Howell et al. (1997) examined momentum and heat transfer on a continuously moving surface in a power law fluid .They examined the momentum and heat transfer occurring in the laminar boundary layer on a continuously moving and stretching two dimensional surface in non Newtonian fluid. Their results in clued situation when then velocity is nonlinear and when the surface is stretched linearly.

Hassanien et al. (1998) investigated the flow and heat transfer in a power law fluid over a non-isothermal stretching sheet. They presented a boundary layer anlysis for the problem of flow and heat transfer from a power law fluid to a continuous stretching sheet with variable wall temperature. They performed parametric studies to investigate the effect of non-Newtonian flow index, generalized pandtl number, power law surface temperature and surface mass transfer. Their result showed that friction factor and heat transfer depend strongly on the flow parameter.

Hassanien (1992) examined the problem of a non-Newtonian viscoelastic fluid obeying the Walter's model with heat transfer over a continuous surface in parallel over a continuous surface in parallel stream using finite difference method.

Hassanien and Gorla (1990) investigated the problem of the flow and heat transfer in a non-Newtonian fluid with micro – rotation past a stretching porous sheet.

In this paper, we examine the heat transfer in a power law fluid with variable thermal conductivity; the thermal conductivity is a function of temperature and time.And the result is presented as temperature profiles for various values of the power law index and heat flow coefficient.

Mathematical Formulation

Consider an unsteady flow of a non-Newtonian power law fluid near a flat plate with variable thermal conductivity. The infinte plate lies at rest for t<0 on the y=0 plane in x-y space.In the volume described by y>0 exists a power law fluid of semi – infinite extent which is at rest at t<0.At t=0, the plate is suddenly accelerated to a constant velocity V, entirely in the x – direction. Let u and v be the velocity components along the x and y direction respectively. The relevant governing equations are:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation

$$\rho \frac{\partial u}{\partial t} = - \frac{\partial \tau_{yx}}{\partial y} \tag{2}$$

Energy Equation

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial \mathrm{y}}\left(\mathrm{K}\,(\mathrm{T},\mathrm{t}) \frac{\partial T}{\partial y} \right) - \tau_{\mathrm{yx}} \frac{\partial u}{\partial y} \tag{3}$$

With boundary and initial conditions,

$$\left.\begin{aligned} &u(0,t) = V \quad , \; u(\infty,t) = 0 \qquad t > 0 \\ &u(y,0) = 0 \qquad \frac{\partial u}{\partial y}(0,t) = \phi(t) \end{aligned}\right\} \tag{4}$$

$$T(0,t) = T_0 \quad , \quad T(\infty,t) = T_1 \quad T_0 > T_1 \tag{5}$$

where ,

$$\tau_{xy} = m\left(-\frac{\partial u}{\partial y}\right)^n \tag{6}$$

$$\phi(t) = -\frac{VA}{t^\alpha} \tag{7}$$

$$K(T,t) = \frac{G(1+T)}{t^{\frac{n-1}{n+1}}} \tag{8}$$

Note:
u is the x- component of velocity
v is the y-component of velocity
T is the temperature
ρ is the density
c is the specific heat
p is the pressure
τ_{yx} is the stress
t is the time
K is the thermal conductivity
G is the heat flow coefficient.

Method of Solution

We first seek the solution of the momentum together with the condition (4). Define a variable

$$\eta = \frac{Ay}{t^\alpha} \tag{9}$$

Such that

$$u(y,t) = Vf(\eta) \tag{10}$$

Using equation (6),(7),(9)and (10) in the momentum equation (2).Then, equation (2) together with the conditions (4) becomes,

$$-\alpha\eta f' - n\beta(-f')^{n-1} f'' = 0 \tag{11}$$

$$f(0) = 1, \quad f(\infty) = 0\,,\; f'(0) = -1$$

Solving (11), we obtain

$$f'(\eta) = -\left[\frac{n-1}{n\beta}\left(\frac{\alpha\eta}{2} + \frac{n\beta}{n-1}\right)\right]^{\frac{1}{n-1}} \tag{12}$$

And so, by (10),

$$\frac{\partial u}{\partial y} = -\frac{A}{t^{\alpha}}\left[\frac{n-1}{n\beta}\left(\frac{\alpha\eta}{2} + \frac{n\beta}{n-1}\right)\right]^{\frac{1}{n-1}} \tag{13}$$

Remark

The similarity solution exist for

$$\alpha = \frac{1}{n+1} \tag{14}$$

$$\beta = \frac{mA^{n+1}}{\rho} \tag{15}$$

Now, consider the energy equation (3), together with the condition (5).
Let.

$$T(y,t) = g(\eta) \tag{16}$$

Using equations (6),(8),(9),(14) and (16) in the energy equation (3),we have,

$$-\rho c\alpha\eta g' - A^2G(g'' + (g')^2 + gg'') - mA^{n+1}\left[\frac{n-1}{n\beta}\left(\frac{\alpha\eta}{2} + \frac{n\beta}{n-1}\right)\right]^{\frac{1}{n-1}} = 0 \tag{17}$$

$$g(0) = 1\,,\; g(\infty) = 0 \tag{18}$$

We resolve equation (17) together with (18) into system of equations as follows:
Let,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \eta \\ g \\ g' \end{pmatrix} \tag{19}$$

We consider,

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ \dfrac{-\rho c \alpha x_1 x_3 - mA^{n+1}\left[\dfrac{n+1}{n\beta}\left(\dfrac{\alpha x_1}{2} - \dfrac{n\beta}{n-1}\right)\right]^{\frac{1}{n-1}} - A^2 G x_3^2}{A^2 G(1+x_2)} \end{pmatrix} \tag{20}$$

Together with the initial conditions,

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -\Gamma \end{pmatrix} \tag{21}$$

Problem (20) together with the initial condition (19) is solved numerically using shooting method for various values of the power law exponent and the heat flow coefficient respectively. The result is presented as temperature profiles in figures 1 and 2.

Values of $g'(0)$ satisfying (17) and (18) are as given in Tables 1 and 2 below;

Table 1

G	n	g’(0)
0.2	1/4	-0.6742
0.2	1/2	-0.4562
0.2	3/4	-0.4777

Table 2

G	n	g’(0)
0.2	0.5	-0.4562
0.5	0.5	-0.6256
1.5	0.5	-0.7060

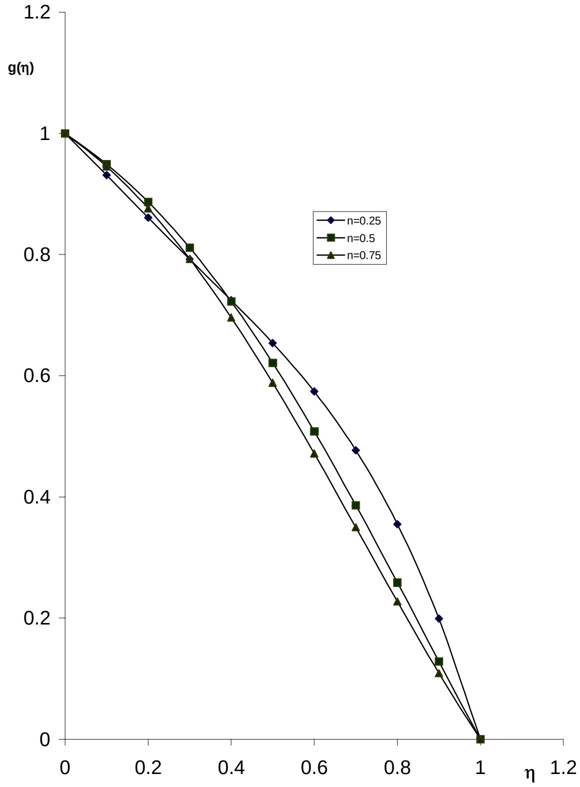

Figure 1: Temperature profile for various values of n and for affixed value of G=0.2.

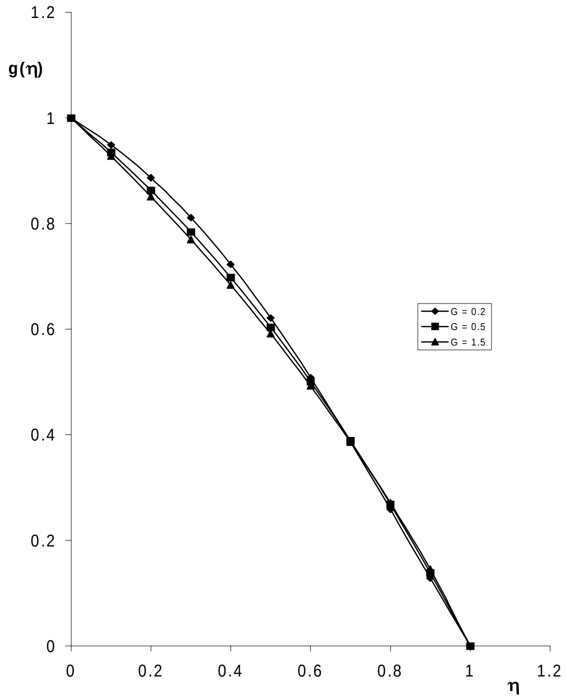

Figure 2: Temperature Profile for various values of values of G and for fixed value of n = 0.5

Discussion of Result and Conclusion

The temperature profile for various values of n; n = 0.25, 0.5, and 0.75 and for affixed value of G = 0.2 is as shown in figure 1. The profile shows that when the values of the power law exponent is increasing the rate at which the temperature $g(\eta)$ tends to zero increases, so, with increase in value of the power law exponent, the rate of heat flow increases.

The temperature profile for various values of G: G = 0.2, 0.5 and 1.5 for a fixed n; n= 0.5 is as shown in figure 2. As G, the flow rate coefficient increases the rate at which the temperature $g(\eta)$ tends to zero increases. So, with increase in the flow rate coefficient the rate of heat flow increases.

The fact that the temperature $g(\eta)$ tends to zero as η tends to infinity suggest that the fluid flow is a flow and the difference in the temperature distribution is due to the fact that during the fluid flow heat escape form the fluid surface into the surrounding air. So, the heat flowing in the fluid at a particular temperature and time is less than that entering the fluid by the amount which escapes from the fluid surface during the fluid flow and the heat flow per second along the direction decreases. Therefore at a particular time, the temperature gradient along the fluid flow direction is greatest where the heat flow is greatest. In conclusion, it is very obvious that variation in the temperature has a significant influence on the heat flow. Also, the power law exponent and heat flow coefficient has appreciable effect on the heat flow.

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