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Construction and Comparison of Mixed Sampling Plans Having Double Sampling Plan as Attribute Plan

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Abstract

This paper presents the procedure for constructing the mixed sampling plan using MAPD as a quality standard. Having the Double sampling plan as attribute plan, the plans are constructed when MAPD and AQL are given. The plan indexed through MAPD is compared with the plan indexed through AQL. Tables are constructed for easy selection of the plan.

Key words and Phrases: acceptable quality level, maximum allowable percent defective, operating characteristic, tangent intercept.

1. Introduction

Mixed sampling plan is a two stage sampling procedure involving variables inspection in the first stage and attributes inspection in the second stage if the variables inspection of the first sample does not lead to acceptance. Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

Maximum allowable percent defective (MAPD - p_*), introduced by Mayer (1967) and studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p_*' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing, a selection procedure for single sampling plan indexed with MAPD and $R=p_t/p_*$ is provided by Soundararajan (1975)

One of the desired properties of an OC curve is that the decrease of Pa (p) should be slower for lesser values of ‘p’ (i.e. for $p \leq$ standard quality level) and faster for greater values of ‘p’. If we set p_* as the quality standard, the above property of the OC curve is obtained since p_* corresponds to the inflection point of the OC curve and hence

$$d^2Pa(p)/dp^2 = 0 \text{ for } p = p_*$$

$$d^2Pa(p)/dp^2 < 0 \text{ for } p < p_*$$

$$\text{and } \quad d^2Pa(p)/dp^2 > 0 \text{ for } p > p_*$$

The mixed sampling plans have been designed under two cases of significant interest. In the first case the sample size n_1 is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The procedure for designing the mixed sampling plans to satisfy the above-mentioned conditions was provided by Schilling (1967). Using Schilling’s procedure, Devaarul (2003) have constructed tables for mixed sampling plans having double sampling plan as an attribute plan indexed through AQL and IQL. Radhakrishnan and Sampath Kumar (2005, 2006a, 2006b, 2006c) have made contributions to mixed sampling plans.

In this paper, using the operating procedure of mixed sampling plan (independent case) with Double sampling plan (DSP) as attribute plan, when $n_{1,2} = n_{1,2}$, tables are constructed for the mixed sampling plan indexed through MAPD and AQL separately. The plan indexed through MAPD is compared with the plan indexed through AQL and the conclusions are derived. Suitable suggestions are also provided for future research.

2. Glossary of Symbols

The symbols used in this paper are as follows:

- p : submitted quality of lot or process
- $P_a(p)$: probability of acceptance for given quality ‘p’
- p_1 : the submitted quality such that $P_a(p_1) = 0.95$ (also called AQL)
- p_* : maximum allowable percent defective (MAPD)
- h_* : relative slope at ‘p_*’
- $n_{1,1}$: sample size for variable sampling plan
- $n_{1,2}$: first sample size for attribute sampling plan
- $n_{2,2}$: second sample size for attribute sampling plan
- β_j : probability of acceptance for lot quality ‘p_j’
- β_j' : probability of acceptance assigned to first stage for percent defective ‘p_j’
- β_j'' : probability of acceptance assigned to second stage for percent defective ‘p_j’
- $z(j)$: ‘z’ value for the j^{th} ordered observation
- k : variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

3. Formulation of Mixed Sampling Plan with DSP as Attribute Plan

The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit 'U'. By symmetry a parallel discussion can be made use for lower specification limits. Also it is suggested that the mixed sampling plan with DSP in the case of single sided specification (U), standard deviation (σ) known can be formulated by the parameters $n_{1,2}$, $n_{2,2}$, c_1, c_2 and k. By giving the values for the parameters an independent plan for single sided specification, S.D. known would be carried out as follows:

- Determine the parameters with reference to Average sample number (ASN) and Operating Characteristic (OC) curves
- Take a random sample of size $n_{1,1}$ from the lot assumed to be large
- If a sample average $\bar{X} \leq A = U - k\sigma$, accept the lot
- If the sample average $\bar{X} > A = U - k\sigma$, take another sample of size $n_{1,2}$ from each lot and observe the number of non-conforming units, d_1.
- If the number of defectives $d_1 \leq c_1$, accept the lot.
- If the number of defectives $d_1 > c_2$, reject the lot.
- If $c_1 < d_1 \leq c_2$, take another sample of size '$n_{2,2}$' from the same lot and count the number of defectives d_2.
- If $d_1 + d_2 \leq c_2$, accept the lot, otherwise reject the lot

4. Construction and Selection of Mixed Sampling Plan having DSP as attribute plan indexed through MAPD

Schilling (1967) has given the procedure for constructing the mixed sampling plans when a point on OC curve and n_1 are known. With DSP as attribute plan to satisfy (p_*, β_*), $n_{1,1}$, c_1 and c_2 on the OC curve, the procedure is given as follows:

- Assume that the mixed plan is independent
- Split the probability of acceptance (β_*) determining the probability of acceptance that will be assigned to the first stage. Let it be β_*'.
- Decide the sample size $n_{1,1}$ (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as

  $A = U - k\sigma = U - [z(p_*) + \{z(\beta_*')/\sqrt{n_{1,1}}\}]\sigma$, where z (t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$
- Now determine β_*'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_*'' = (\beta_* - \beta_*') / (1-\beta_*')$
- Determine the appropriate second stage sample of size $n_{1,2}$ (1^{st} sample size for the attribute plan) , $n_{2,2}$ (2^{nd} sample size for the attribute plan), c_1 and c_2 from

$$Pa(p) = \beta_*'' \quad \text{for } p = p_*.$$

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with DSP as attribute plan indexed through MAPD.

Construction of Tables

The probability of acceptance under Poisson model is given by

$$Pa(p) = \sum_{r=0}^{c_1} \frac{e^{-n_1p}(n_1p)^r}{r!} + \left[\sum_{k=c_1+1}^{c_2} \frac{e^{-n_1p}(n_1p)^k}{k!} \left\{\sum_{r=0}^{c_2-k} \frac{e^{-n_2p}(n_2p)^r}{r!}\right\}\right]$$

For $n_1=n_2=n$ (say), the inflection point (p_*) is obtained by using $d^2Pa(p)/dp^2 = 0$ and $d^3Pa(p)/dp^3 \neq 0$. The relative slope of the OC curve $h_* = \left[\frac{-p}{Pa(p)}\right]\frac{dPa(p)}{dp}$ at $p = p_*$. The inflection tangent of the OC curve cuts the 'p' axis at $p_t = p_* + (p_*/h_*)$. The values of np_*, h_*, np_t and $R = p_t/p_*$ are calculated for different values of c_1 and c_2 for $\beta_*'=0.40$ using visual basic program and presented in Table 1.

Table 1: Various characteristics of the plan for $\beta_*' = 0.40$

c_1	c_2	β_*	β_*''	np_*	h_*	np_t	$R = p_t / p_*$
0	1	0.75	0.58	0.855	0.8163	1.9024	2.23
0	2	0.69	0.48	1.468	1.3538	2.5612	1.75
0	3	0.65	0.42	2.087	1.8361	3.2236	1.54
0	4	0.63	0.38	2.708	2.2661	3.9030	1.44
0	5	0.62	0.37	3.262	2.5480	4.5422	1.39
1	2	0.71	0.52	1.763	1.1602	3.2825	1.86
1	3	0.67	0.45	2.266	1.6237	3.6616	1.62
1	4	0.64	0.40	2.813	2.0880	4.1602	1.48
1	5	0.62	0.37	3.365	2.4912	4.7158	1.40
1	6	0.61	0.35	3.925	2.8416	5.3063	1.35
2	3	0.68	0.47	2.853	1.4995	4.7556	1.67
2	4	0.66	0.43	3.181	1.8297	4.9195	1.55
2	5	0.64	0.40	3.580	2.2009	5.2066	1.45
2	6	0.62	0.37	4.065	2.6064	5.6246	1.38
2	7	0.61	0.35	4.573	2.9699	6.1128	1.34
3	4	0.65	0.42	4.087	1.8918	6.2474	1.53
3	5	0.65	0.42	4.166	2.0061	6.2427	1.50
3	6	0.64	0.40	4.437	2.2894	6.3751	1.44
3	7	0.62	0.37	4.850	2.6806	6.6593	1.37
3	8	0.61	0.35	5.292	3.0503	7.0269	1.33
4	5	0.64	0.40	5.243	2.1942	7.6325	1.46
4	6	0.64	0.40	5.276	2.2529	7.6179	1.44
4	7	0.63	0.38	5.485	2.4841	7.6930	1.40
4	8	0.63	0.38	5.661	2.6878	7.7672	1.37
4	9	0.62	0.37	5.977	2.9797	7.9829	1.34
5	6	0.63	0.38	6.427	2.5043	8.9934	1.40
5	7	0.63	0.38	6.440	2.5312	8.9842	1.39
5	8	0.63	0.38	6.485	2.6097	8.9700	1.38
5	9	0.63	0.38	6.590	2.7571	8.9802	1.36
5	10	0.62	0.37	6.833	3.0165	9.0982	1.33

Selection of the Plan

Table 1 is used to construct the plans when MAPD (p_*) and tangent intercept (p_t) are given. For any given values of c_1, p_t and p_*, one can find the ratio R= p_t/p_*. Corresponding to the value of c_1, find the value in Table1 under the column R, which is equal to or just greater than the specified ratio. Corresponding 'c_2' value is noted. From this c_1 and c_2 values one can determine $n_{2,2}$ value.

Example

Given $c_1 = 2$, $p_* = 0.08$, $p_t = 0.12$ and $\beta_*'=0.40$. Find the ratio $R=p_t/p_*=1.5$. Using Table 1, corresponding to $c_1 = 2$ select the value of R equal to or just greater than this ratio. The value of R is 1.55 which is associated with $c_1=2$ and $c_2= 4$. From this we can find that $n=np_*/p_* =3.181/0.08= 40$. The sampling plan is $n_{1,2} = 40$, $n_{2,2} = 40$, $c_1 =2$ and $c_2 = 4$.

5. Construction of Mixed Sampling Plan having DSP as attribute plan indexed through AQL

The general procedure given in Section 4 is used for designing the MSP having Double sampling as attribute plan indexed through AQL [for $\beta_1'' = (\beta_1 – \beta_1') / (1-\beta_1')$. For $n_{1,2} = n_{2,2} = n$ (say) , np_1 values are calculated for different values of c_1 and c_2 for $\beta_1'=0.40$ using visual basic program and presented in Table 1.

Selection of the Plan

Table 1 is used to construct the plans when AQL (p_1), c_1 and c_2 are given. For any given values of p_1, c_1 and c_2 one can determine 'n' value using $n = np_1/p_1$.

Example

Given $p_1=0.05$, $c_1=2$, $c_2 =5$ and $\beta_*'=0.40$. Using Table 1, we can find that $n=np_1/p_1=1.623/0.05=33$. The sampling plan is $n_{1,2} =33$, $n_{2,2} = 33$, $c_1=2$ and $c_2 =5$.

6. Comparison of DSP indexed through MAPD and AQL.

In this section Double Sampling plan indexed through MAPD is compared with Double Sampling Plan indexed through AQL.

Table 2: Comparison of plans

Given Values			through MAPD	through AQL
c_1	p_*	p_t	n, c_2	n, c_2
0	0.04	0.07	37, 2	53, 2
1	0.09	0.14	25, 3	30, 3
2	0.08	0.12	40, 4	45, 4
3	0.07	0.10	63, 6	70, 6
4	0.05	0.07	110, 7	132, 7
5	0.09	0.12	76,10	93,10

Comparison of OC curve

The OC curves for the said plans are constructed with parameters $n_{1,2} = 63$, $n_{2,2} = 63$, $c_1 = 3$ and $c_2 = 6$ (indexed through MAPD) and $n_{1,2} = 70$, $n_{2,2} = 70$, $c_1 = 3$ and $c_2 = 6$ and expressed in Figure 1

Figure 1

It is found from the OC curves that the probability of acceptance is more for lesser values of 'p' for the plan indexed through MAPD than the plan indexed through AQL.

7. Conclusion

It is concluded from the study that the second stage sample size required for Double Sampling Plan indexed through MAPD is less than that of the second sample size of the Double Sampling Plan indexed through AQL. The plan indexed through MAPD is better than the plan indexed through AQL in using Double Sampling Plan as an attribute plan in the construction of mixed sampling plans, because of less sampling cost. These plans are effective and flexible to the floor engineers and help them to decide their sampling plans on the floor itself and can take quick decisions to make the system very fast, effective and friendly. Different plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*' and β_1') and can also be compared for their efficiency.

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