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On The Pattern of Migration In The Household: An Explanation Through Binomial Law

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Abstract

Household characteristics such as age, size, occupation, socio-economic status, etc. play vital role in deciding the movement of adult members. A study of movement process at the household level is useful for the prediction of future size of the households as well as to study the imbalances in sex-ratio occurred due to such migration. Motivated by the fact that the data related to migration is influenced by the number of the adult members present in household, under certain assumptions some probability models (inflated binomial and beta binomial distributions) have been proposed to describe the migration pattern and it has been applied to the observed distribution of adult migrants from the households among fixed number of adult members present in household. Data have been taken from a survey conducted in the most flooded area of Bihar, India. Findings reveal that proposed model explains satisfactorily the pattern of rural out migration of adults from the household. Also we have observed that the risk of migration is increasing with increasing number of adult member in the household.

Introduction

Migration from the household is an important component of demography, is least studied as compared to fertility and mortality. Due to increasing need of individual as well as household, migration (internal and international) has become a more important concern for demographers and other social scientists. In developing countries particularly in India where about seventy per cent people still live in villages (Census

2011), migration from rural areas has become a major subject of interest for social scientists as well as program makers. In last few decades the pace and pattern of migration has been changed in India. Especially in last two decades the gap between rural and urban areas has steadily widened and it is more prevalent in some areas and states.

In recent years migration pattern has also been affected by growing spatial economic inequalities. Keeping the diverse nature of migration in mind, most of the studies have been done on different groups of internal migrants and the poorest segment of the population was focused most. Studies conducted in the past to formulate the migration process to predict and explain the migration patterns within and between the districts, states and nations. These types of studies are unable to provide sufficient explanation for the tremendous regional and local heterogeneity. They also ignore the decision making process of migrating individuals (Singh and Yadava, 1981).

Although, macro level studies have their own importance since this approach describes, aggregate flow of rate of migration and identifies factors influencing out migration (Banerjee, 1986), the behavioral parameters of process can be explained through micro level studies i.e., at the level of household or individual. Micro level studies have important implications for housing policies and also for the development of other sociological models related to families and communities (Pryor, 1975; Rossi, 1955). At the micro level, the topic of household is getting prominence in demography to understand demographic process. It is seen that a migrant household (with one or more persons involved in the process of migration in relation to do some job outside the village) may have different socio-economic and cultural characteristics through remittances besides providing good ideas, awareness and environments than a non-migrant household (Yadava, 2010).

A household, especially in Indian context is a basic socio-economic unit for the integrated rural development. Household characteristics (age, size, occupation, socio-economic status, etc.) play a vital role in decision of its members to move or not to move. A study of movement process at the household level is also useful for the prediction of future size of the households as well as to study the imbalances in sex-ratio occurred due to such migration. Migration from a household can take place in three ways. The first type of household is one from where only an adult male migrate alone leaving his wife and children in the village. Another type of household is one from where an adult male migrates with his wife and children. Third kind of household is one from where a male migrates with his wife and children along with some other member(s) of the household either migrating with him or to another place of destination (Yadava, 2010).

The relative importance of socio-economic and demographic factors which are influencing migration varies with the level of socio-economic and educational development of the area. Several attempts have been done in the past to study the pattern of rural out migration through the use of probability models in different socio-economic conditions (Iwunor,1995; Sharma, 1985; Singh and Yadava, 1981; Yadav et. al. 1991). Singh and Yadava (1981) introduced the idea that number of migrants from a household is a random variable and follows a Poisson distribution and the risk of migration varies from household to household and follows a Pearson type-III distribution. Sharma (1984) proposed a probability model to describe the distribution of households according to total number of male migrants aged 15 years and above follows negative binomial distribution.

Yadava, Singh and Kumar (1989) proposed a joint probability function by taking the distribution of males migrants aged 15 years and above as negative binomial distribution and of associated migrants (wife, children, relative and friends) as displaced geometric distribution. Yadava and Yadava (1988) proposed a mixture of two displaced geometric distribution under the assumption that migration occur in clusters and both type of migration may take place from the same household. Assuming that the number of person migrate in cluster follows the inflated logarithmic series distribution a migration model for the total number of migrants has been derived by Yadava, Singh and Kumar (1991) as modified negative binomial distribution with three parameters. After that no studies have been found on this issue in India as well as anywhere in the world. Although some studies have been conducted to formulate the pattern of number of migrants from a household, but very few studies had paid attention on the effect of number of adult members on migration process in the household.

Motivated by the fact that the data related to migration is influenced by the number of adult members in a household and keeping this in mind some models have been developed for the study of the distribution of migrants which will be a powerful device to explain changes and variation in the population. In the present study two probability models have been proposed to describe the phenomenon and have been applied to the observed distribution of adult migrants from the households for fixed number of adult members in the household.

Construction of models

Suppose the number of adult members in the household be n. Let p be the probability of migration of an adult person in the household and X is the total number of migrated adult persons out of n adult persons. Thus X is a random variable which denotes the number of migrants for fixed number of adult members.

The model is developed under the consideration that each adult person of the household is either a migrant or non-migrant. Let us define for i^{th} (i=1,2,…, n) person lives in the household, a random variable z_i taking value 1 if the person is migrated and 0 otherwise. Thus this is a Bernoulli variable, now if we assume that migration of persons live in the household are independent of each other and having same probability p then total number of migrated persons X from the household is nothing but sum of independent Bernoulli variables and hence follows a binomial distribution. Therefore, the distribution of X may be given by

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \qquad ; \quad 0 \le p \le 1 \qquad (1)$$

where, $x = 0, 1, 2 \ldots\ldots\ldots\ldots n.$

Model-I

In this model, we consider that the population consist a high proportion of households having no migrants. Due to more observations with zero counts, the frequency of zero cells is inflated and the resulting over dispersion cannot be modeled accurately with the simple binomial model. In such scenario an inflated binomial model may be used. Assume that the proportion of households prone to the migration be α, and $(1-\alpha)$ proportion have no migrants in the household. Therefore the probability density function of zero inflated binomial model is

$$P[X=x]=\begin{cases}(1-\alpha)+\alpha(1-p)^n & \text{for } x=0\\ \alpha\binom{n}{x}p^x(1-p)^{n-x} & \text{for } x=1,2,3,\ldots\ldots\ldots,n\end{cases} \quad (2)$$

The zero class data can be partitioned lacking households having no migrants (denoted by X_{00}) and the households have any migrants but no response were recorded (denoted by X_{01}). X_{00} is estimated by (1-α) and X_{01} and estimated as (N_0-1+α), where N_0 is the proportion of zeroth cell frequency.

Model-II

In the model-I, we have assumed that probability of migration of adult members '*p*' from a household is fixed for all. But in reality, '*p*' is affected by a number of factors and therefore assumption of *p* being constant for all households seems to be questionable. Thus, it seems more logical to consider *p* as a random variable following some distribution *g(p)*. Beta distribution of first kind with parameters (*a, b*) is a suitable distribution for risk of migration '*p*', since '*p*' the risk varies from 0 to 1 and beta distribution possess the property of flexibility, and capability of accommodating wide range of variability. The probability density function of Beta distribution is:

$$g(p)=\frac{1}{\beta(a,b)}p^{(a-1)}(1-p)^{(b-1)}; \qquad 0\le p\le 1; \qquad a,b>0 \quad (3)$$

Thus, the joint distribution of *x* and *p* is given by

$$P[X=x\cap P=p]=P[X=x/p]\times g(p)=\binom{n}{x}p^x(1-p)^{n-x}\frac{1}{\beta(a,b)}p^{(a-1)}(1-p)^{b-1} \quad (4)$$

and the marginal distribution of *x* is given as

$$P[X=x]=\binom{n}{x}\frac{\beta(a+x,b+n-x)}{\beta(a,b)}; \qquad a,b>0 \quad 5)$$

where, $x = 0, 1, 2\ldots\ldots\ldots\ldots\ldots n$

The above marginal distribution of *X* (5) is known as beta-binomial distribution and it is a natural extension of binomial model under the consideration for random nature of '*p*' in the population. The parameters *a* and *b* are its shape parameter. If someone is interested in getting a single value (like *p*) for comparing the migration of two places, one may take mean i.e. $\frac{\hat{a}}{\hat{a}+\hat{b}}$ as an estimate of average number of migrants at the household level. Also with this distribution one can know the distribution of risk of migration which cannot be obtained directly.

Estimation

Model-I

The moment estimates of the parameters α and *p* of the proposed model can be obtained as follows:

$$E(X) = \alpha np \tag{6}$$

$$E(X^2) = \alpha np(np + 1 - p - \alpha np) + (\alpha np)^2 \tag{7}$$

Let $\mu_1^{'}$ and $\mu_2^{'}$ denotes the first two raw moments about zero for data in hand. Replacing $E(X)$ and $E(X^2)$ by $\mu_1^{'}$ and $\mu_2^{'}$ in above equations we get two equations with two unknowns α and *p* as given below:

$$\mu_1^{'} = \alpha np \tag{8}$$

and $$\mu_2^{'} = \alpha np(np + 1 - p - \alpha np) + (\alpha np)^2 \tag{9}$$

With these two equations (8) and (9), estimates of *p* and α can be obtained easily.

Model-II

The moment estimates of the parameters *a* and *b* can be obtained as follows

$$E(X) = \frac{na}{(a+b)} \tag{10}$$

$$E(X^2) = \frac{na[n(1+a)+b]}{(a+b)(a+b+1)} \tag{11}$$

As mentioned above replacing $E(X)$ and $E(X^2)$ by $\mu_1^{'}$ and $\mu_2^{'}$ in above equations we get two equations with two unknowns *a* and *b* as given below:

$$\mu_1^{'} = \frac{na}{(a+b)} \tag{12}$$

and $$\mu_2^{'} = \frac{na[n(1+a)+b]}{(a+b)(a+b+1)} \tag{13}$$

Substituting the value of $b = \left(\frac{n-\mu_1^{'}}{\mu_1^{'}}\right)a$ from the equation (12) in the above equation and separating the coefficients for *a* we have

$$a\left[\mu_2^{'} - n\mu_1^{'} + \left(\frac{n-\mu_1^{'}}{\mu_1^{'}}\right)(\mu_2^{'} - \mu_1^{'})\right] = n\mu_1^{'} - \mu_2^{'} \qquad (14)$$

or

$$a = \frac{n\mu_1^{'2} - \mu_1^{'}\mu_2^{'}}{n(\mu_2^{'} - \mu_1^{'}) - n\mu_1^{'2} + \mu_1^{'2}} \qquad (15)$$

after solving this we can get the estimate of *a* and the using this estimate and equation (12) *b* can be estimated easily.

Application of the Models

The models has been applied to the primary data taken from a survey entitled "Migration and Related Characteristics-A Case Study of North-Eastern Bihar" conducted during October 2008 to March 2009. Data had been collected using a multistage random sampling procedure. This analysis is based on the completed information collected from 664 households. The households with inadequate and/or incomplete information have been excluded.

Discussions and Conclusion

In this section we have discussed about the estimate of parameters and fitting of the proposed probabilistic models. Since, in this study two models have been proposed for fixed number of adult migrant members in the household so that after obtaining the estimate of parameters for different household sizes (adult members of the household), we obtained the estimated frequencies for both the models. Table 1 shows the distribution of households according to the number of adult members and the number of migrants. Tables 2-5 show the expected frequencies along with the observed frequencies for household size 5 to 8 respectively of Koshi river basin in Bihar. Here household size refers to the number of adult members (>15 years of age) in the house. Estimate of parameters, the value of χ^2 with degree of freedom and average risk of migration from a household size are given in the respective tables. The value of χ^2 shown in the tables clearly indicate that both the models describe the distribution of number of migrants for fixed sizes of households satisfactorily well. The advantage of model-I is that the parameters involved in the model have physical meaning such that *p* provides the risk of migration at the household level whereas (1-α) gives the proportion of households where migration does not occur. From the tables it can be easily seen that the risk of migration increases with the increasing size of households and also the proportion of households having migrants increases with increasing household size.

In the table 2 and 3 where the distribution of migrants among the households of size 5 and 6 are given, the value of χ^2 cannot be presented due to the degree of freedom comes out to be zero. However, the expected frequencies are very close to the observed frequencies. Hence we have calculated the Kolmogorov-Smirnov Statistics (KS-statistics) to see the suitability of the proposed models. KS-statistics shows that in case of household size 5, beta-binomial gives better fit whereas in case of household size 6 inflated binomial model gives better fit. Rest other table 4 and 5 provide the distribution of migrants among the households of size 7 and 8, here in this table chi-square value also given which shows the suitability of models.

Another important advantage of using the model-II may be that as soon as we get the estimates of *a* and *b*, an estimated distribution of *p* for the population can be obtained. It is worthwhile to note that the model provides a way to study the distribution of *p* which cannot be studied otherwise since *p* itself is unobservable. Table 6 gives the expected and observed proportion of zeroth cell of the distribution of migrants in the household. When household size is increasing the proportion of household having migrants but say no migration is decreasing. All the values of parameters of the proposed models have been shown in table 7 at a glance. Results clearly indicate that probability of migrating of a person increases with the size of household. Tcha (1995) had shown that the effect of the size of the household on the migration decision is ambiguous. When size of the household increases, the adult members of household are more likely to migrate. Figure 1 shows the distribution of risk of migration which is not observable directly. For lower household size the risk of migration is left skewed and leptokurtic than higher household size and as size of household increases the distribution of risk of migration becomes flatter and flatter. It shows clearly that the location measure of migration for lower household size is less than the higher household size.

Table 1: Distribution of the number of migrants according to the household size

Number of migrants	Household Size						Total
	<=4	**5**	**6**	**7**	**8**	**9+**	
0	176	123	58	26	14	4	401
1	29	33	33	23	10	19	147
2	2	9	9	10	11	16	57
3	-	2	2	7	6	12	29
4	-	-	1	1	2	12	16
5	-	-	-	-	-	8	8
6	-	-	-	-	-	5	5
7	-	-	-	-	-	1	1
Total no. of household	**207**	**167**	**103**	**67**	**43**	**77**	**664**
Total no. of migrants	**33**	**57**	**61**	**68**	**58**	**212**	**489**
Migrants Per household	**0.159**	**0.341**	**0.592**	**1.015**	**1.349**	**2.753**	**0.736**

Table 2: Expected & observed frequency distribution of migrants in household with size 5

Number of migrants	Observed number of households	Expected number of households	
		Inflated Binomial	**Beta Binomial**
0	123	128.5	122.5
1	33	30.4	34.2
2	9	8.1	10.3
3	2		
Total	**167**	**167**	**167**
Mean = 0.3413 **Variance = 0.4045**		p = 0.1053 α = 0.5404	a = 0.94 b = 12.77
K-S Statistic		D = 0.032541	D = 0.004124
		p(0.05)=0.1052	p(0.01)=0.1261

Table 3: Expected & observed frequency distribution of migrants in the household with size 6

<table>
<tr><th rowspan="2">Number of migrants</th><th rowspan="2">Observed number of households</th><th colspan="2">Expected number of households</th></tr>
<tr><th>Inflated Binomial</th><th>Beta Binomial</th></tr>
<tr><td>0</td><td>58</td><td>59.5</td><td>58.8</td></tr>
<tr><td>1</td><td>33</td><td>29.1</td><td>31.3</td></tr>
<tr><td>2</td><td>9</td><td rowspan="3">14.4</td><td rowspan="3">12.9</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>4</td><td>1</td></tr>
<tr><td>Total</td><td>103</td><td>103</td><td>103</td></tr>
<tr><td colspan="2">Mean = 0.5922
Variance = 0.6493</td><td>p = 0.1377
α = 0.7168</td><td>a = 2.18
b = 19.93</td></tr>
<tr><td colspan="2" rowspan="2">K-S Statistics</td><td>D= 0.023281</td><td>D = 0.011835</td></tr>
<tr><td>p(0.05)=0.134</td><td>p(0.01)= 0.1606</td></tr>
</table>

Table 4: Expected & observed frequency distribution of migrants in the household with size 7

<table>
<tr><th rowspan="2">Number of migrants</th><th rowspan="2">Observed number of households</th><th colspan="2">Expected number of households</th></tr>
<tr><th>Inflated Binomial</th><th>Beta Binomial</th></tr>
<tr><td>0</td><td>26</td><td>26.6</td><td>25.5</td></tr>
<tr><td>1</td><td>23</td><td>20.5</td><td>23.1</td></tr>
<tr><td>2</td><td>10</td><td>13.6</td><td>12.2</td></tr>
<tr><td>3</td><td>7</td><td rowspan="2">6.3</td><td rowspan="2">6.2</td></tr>
<tr><td>4</td><td>1</td></tr>
<tr><td>Total</td><td>67</td><td>67</td><td>67</td></tr>
<tr><td colspan="2">Mean=1.0149
Variance=1.0893</td><td>p=0.1814
α =0.7994</td><td>a=3.2623
b=19.2379</td></tr>
<tr><td colspan="2">$\chi^2_{0.05}(1)$</td><td>1.7445</td><td>0.9249</td></tr>
</table>

Table 5: Expected & observed frequency distributions of migrants in the household with size 8

Number of migrants	Observed number of households	Expected number of households	
		Inflated Binomial	Beta Binomial
0	14	11.9	11.9
1	10	10.1	14.3
2	11	10.9	9.7
3	6	10.1	7.1
4	2		
Total	**43**	**43**	**43**
Mean=1.3488 **Variance=1.4365**		p=0.2356 α =0.8178	a=4.0326 b=19.8850
$\chi^2_{0.05}(1)$		0.7808	1.9488

Table 6: Expected & observed proportion of Zeroth cell of the distributions of migrants in the household

Household size	Zeroth cell proportion [N_0]	proportion of households having migrants [X_{00}=(1-α)]	proportion of households having migrants but respond no (zero migrants) [X_{01}=(N_0-1+α)]
5	0.74	0.46	0.28
6	0.56	0.28	0.28
7	0.39	0.20	0.19
8	0.33	0.18	0.14

Table 7: Parameters of models according to the number of adult members in the household

Number of adult members in the household	Inflated Binomial Distribution		Beta Binomial Distribution	
	α	***p***	***a***	***b***
5	0.5404	0.1053	0.94	12.77
6	0.7168	0.1377	2.18	19.93
7	0.7994	0.1814	3.26	19.23
8	0.8178	0.2356	4.03	19.88

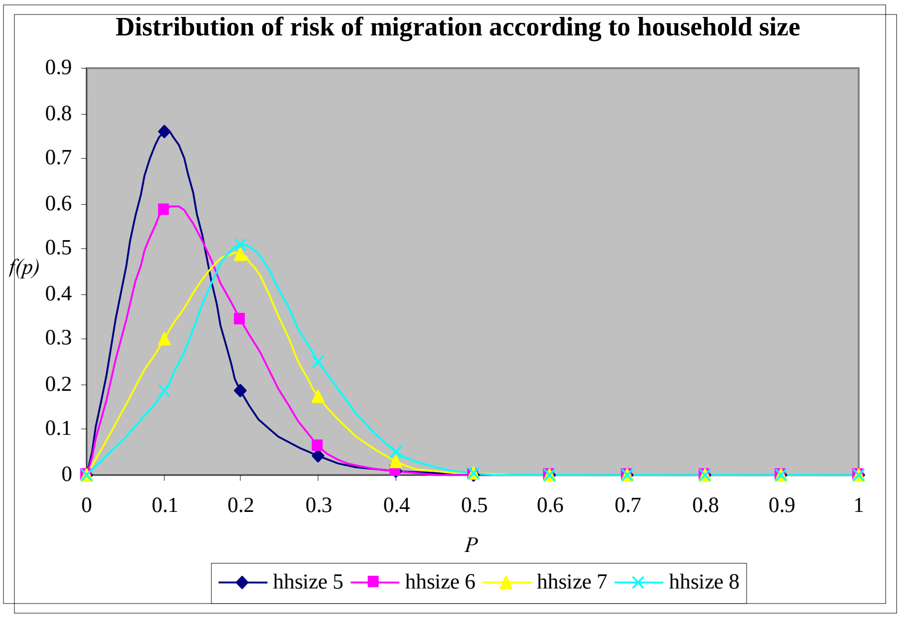

Figure 1: Distribution of risk of migration according to the household size

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