International Journal of Statistics and Systems
ISSN 0973-2675 Volume 18, Number 2 (2023), pp. 95-104

https://dx.doi.org/10.37622/IJSS/18.2.2023.95-104

Swift Decision Chain Sampling Plans (SDChSP) for Variable Quality Characteristics

Dr. S. DevaArul[1] and M. Shanthi[2]

*[1]Associate Professor & Head, Department of Statistics,
Government Arts College, Coimbatore, Tamilnadu, India.
[2]Research Scholar, Department of Statistics,
Government Arts College, Coimbatore, Tamilnadu, India.*

Abstract:

This research article presents the algorithm and methodology of swift decision on the incoming lots through Chain Sampling Inspection for Variable Quality Characteristics. In industries speedy decision is necessary while continuous lots are waiting for either acceptance or rejection. Producers face difficult situations to hold the lots in the production area. The Quality Control Professionals insist for speedy decision on the lots. Hence few conditions are imposed on chain sampling inspection for making swift decision on lots. The new algorithm is capable of sentencing the lot in a speedy manner and the efficiency measures such as operating characteristics function, average outgoing quality, average sample number and average total inspection are derived. The new OC function is capable of partitioning good and bad lots more effectively. Tables are constructed and illustration are given for various quality levels for easy implementation in industries.

Keyword: Swift decision chain sampling, Operating Characteristic (OC) function and variables.

Introduction:

The chain sampling inspection procedure was introduced by Prof. Dodge (1955a). The chain sampling plan by variable will be useful, when the testing environment required small samples with variable criteria. In the manufacturing Industries, if the items can be grouped into batches, then single, double, chain, link, repetitive group sampling plans may be used according to the necessity of quality control division. Prof. Dodge has developed partial and single sided chain sampling plans. The decision on lots due to chain sampling plans depends on either past or future lots results. In the modern industries, quality control professionals insist for fast decision and speedy sentencing of the lots due to several factors. Hence an attempt has been made to develop swift decision on the lots using modified chain sampling plans.

Dodge (1955), have studied chain sampling plan (ChSP-1) advanced shape of the operating characteristic curve. Govindaraju and Kuralmani (1993) have studied sampling plans with auxiliary variables. Govindaraju and Balamurali (1998) have studied the shape of OC curve of known sigma sampling plans. Rebecca Jebaseeli Edna and Jemmy Joyce (2013) have designed chain sampling plans by variables using stochastic differential equation and determined new operating characteristic function. Raju and Vidhya (2017) have constructed tables with acceptable quality level and limiting quality level for chain sampling plans. Devaarul and Vijila (2017) have designed and developed relational chain sampling plans for attribute quality characteristics and given the comparison of the relational and ordinary chain sampling plan. Vijila and Devaarul (2017) have specified the construction and selection of double-sided chain sampling plans indexed with outgoing quality limit. Vijila and Devaarul (2017) have presented the designing and selection of chain sampling plans indexed through inflection point and also discussed about the maximum allowable proportion defective for chain sampling plans.

Rebecca Jebaseeli Edna and Jemmy Joyce (2018) have specified modified chain sampling plan for attributes using minimum angle criterion. Jeyadurga and Balamurali (2019) have discussed about the chain sampling plan under Gamma Poisson distribution by determining the parameters using two points on the operating characteristic curve.

Milky Mathew and Rajeswari (2019) have mentioned the two-sided complete Bayesian chain sampling plans considering the beta-geometric as the prior distribution. Rebecca Jebaseeli Edna and Jemmy Joyce (2020) have also discussed about the sampling plans process potential index and least tangent angle. Devaarul and Jothimani (2020) have studied the two-sided chain sampling plans properties. Muhammad Farouk et.al. (2020) have specified the two-sided complete chain sampling plan operates with few acceptance criterion.

Rebecca Jebaseeli Edna and Jemmy Joyce (2022) have designed two-sided chain sampling inspection plan with process potential measure. Waqar Hafeez and Nazrina Aziz (2022) have specified the Bayesian two-sided group chain sampling plan for binomial distribution using beta prior through quality regions. Rebecca Jebaseeli Edna et.al. (2022) have developed a new attribute and variable relational complete chain sampling plan for special circumstances.

Formulation of Swift Decision Chain Sampling Plans for variable:

Let, N = Lot size, n = Sample size

$\bar{x}$ = Mean where, $\bar{x}_i = \frac{\sum x_i}{n}$

σ = Standard Deviation where, $\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$

U = Upper Specification Limit, L = Lower Specification Limit

i = Preceding i samples

1-α = Producer's Risk
β = Consumer's Risk
p_1 = Accepting Quality Level
p_2 = Limiting Quality Level
w = Variable factor such that lot is accepted, if $\bar{x}$ + wσ $\leq$ U (or) $\bar{x} + w\sigma \geq$ L
Where, $\frac{K}{i}$ = w (swift decision factor)

Algorithm to sentence a lot-SDChSP for variable:

Step 1: Let U be the upper specification given by the consumer and σ is the known process standard deviation.
Step 2: Draw a random sample of size n from the lot. Measure the required quality characteristics using appropriate scale. Let it be x_1, x_2, x_3, ……… x_n
Step 3: Determine the sample mean $\bar{x}$ and swift decision factor w.
Step 4: If $\bar{x} \leq$ U – wσ, accept the lot.
Step 5: If $\bar{x} >$ U – wσ, go to next step.
Step 6: Accept the current lot provided preceding i lots have been accepted on the criterion that $\{\bar{x} \leq \text{U} - w\sigma\}$. Otherwise reject the lot.
Step 7: Suppose L be the lower specification given by the consumer σ is the known process standard deviation.
Step 8: Draw a random sample of size n. Measure the quality characteristics using appropriate scale. Let it be x_1, x_2, x_3, ……… x_n.
Step 9: Determine the sample mean $\bar{x}$ and swift decision factor w.
Step 10: If $\bar{x} \geq$ L – wσ, accept the lot.
Step 11: If $\bar{x} <$ L – wσ, go to next step.
Step 12: Accept the current lot provided preceding i lots have been accepted on the criterion that,
if $\{\bar{x} \geq \text{L} - w\sigma\}$. Otherwise reject the lot.

Measures of Sampling Plans:

1. Operating Characteristic function:

The OC function for the given U is
$P_a(p) = P[\bar{x} \leq U - w\sigma] + P[\bar{x} > U - w\sigma]\{P[\bar{x} \leq U - w\sigma]\}^i$
The lot will be accepted if the following cases are satisfied.
Case (i): if $\bar{x} \leq$ U – wσ
Case (ii): if $\bar{x} >$ U – wσ and $\{\bar{x} \leq U - w\sigma\}^i$
Case (i) and Case (ii) are mutually exclusive events. Therefore,
$P_a(p) = P(i) + P(ii)$
$= P[\bar{x} \leq U - w\sigma] + P[\bar{x} > U - w\sigma]\{P[\bar{x} \leq U - w\sigma]\}^i$
$= P_n[\bar{x} \leq U - w\sigma] + P_n[\bar{x} > U - w\sigma]\{P_n[\bar{x} \leq U - w\sigma]\}^i$
$= \int_{-\infty}^{U-W\sigma} f(\bar{x})\, dx + \int_{\bar{x} > U - W\sigma}^{\infty} f(\bar{x})\, dx . \int_{-\infty}^{(U-W\sigma)^i} f(\bar{x})\, dx$

2. Average Outgoing Quality [AOQ]

AOQ = p. P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i}

AOQ = p.[$\frac{N-n}{N}$] P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i}

Proof:

AOQ = p.P_a(p) ($\frac{N-n}{N}$) + 0(1-P_a(p)) (($\frac{N-n}{N}$)

= p.P_a(p) ($\frac{N-n}{N}$)

AOQ = p.P_a(p) (1-$\frac{n}{N}$)

Sample size is very small in the proportion of the lot $\frac{n}{N} \sim 0$

So, we get,

AOQ = p.P_a(p)

Then, AOQ = p. (P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i})

3. Average Sample Number (ASN):

ASN = n (P(i)+P(ii))

= n (P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i})

P(i) and P(ii) probability of the decision on the basis of the sample

P(i) = probability of acceptance of the current lot

P(ii) = probability of acceptance of the current lot provided the conditional probability is satisfied.

4. Average Total Inspection:

ATI = n + (1-P_a(p)) (N-n)

= n P_a(p) + N (1-P_a(p))

= n. (P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i}) + N (1-(P[$\bar{x} \leq$ U – wσ] +

P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i})

Designing the Swift Decision Chain Sampling Plan through AQL:

The parameters of Swift Decision Chain sampling plan for known sigma are (n, k, w). The parameters of n and w (w=$\frac{k}{i}$) are determined based on the following designing procedure.

Step: 1 Let the probability of acceptance of the fraction defective p_1 is 1-α and P_a (p_1) $\geq 1 - \alpha$, Assume α = 0.05.

Step: 2 Now determine the sample of size n and variable factor w for the given index i, from the following equation.

P [$\bar{x} \leq$ U – wσ] + P [$\bar{x} >$ U – wσ] {P [$\bar{x} \leq$ U – $w\sigma$]}^{i} $\geq$ 0.95

The above equation is nonlinear with two unknowns; hence a computer program is written to solve the equations and tables are constructed for easy selection of the sampling plans.

Table 1: The table gives the values of n and the variable factors $\frac{k}{i} = w$ for the parameters of Swift Decision Chain Sampling Plan for variables indexed through AQL at 95% Probability of acceptance.

Swift Decision Chain Sampling Plan through AQL					
$\boldsymbol{p_1}$	**i**	**k**	**w**	**N**	$\boldsymbol{p_{a(p_1)}}$
0.0009	1	2.6888	2.6888	8	0.950027
0.0809	2	3.0746	1.5373	51	0.950039
0.0973	3	3.5824	1.1941	70	0.94999
0.0939	4	4.8954	1.2239	140	0.949997
0.097	5	4.9808	0.99616	13	0.949965

Illustration 1:
Obtain the SDChSP by variable if the known AQL = 0.0809 when the index i=2

Solution:
From table 1, When i = 2, the sample size n = 51 and the variable factor w = 1.5373.
The Operating Procedure is as follows:
Step 1: It is given that i=2, n=51 and w= 1.5373.
Step 2: Draw a random sample of size 51 from the lot. Measure the required quality characteristics using appropriate scale. Let it be $x_1, x_2, x_3, \ldots\ldots\ldots x_n$
Step 3: Determine the sample mean $\bar{x}$.
Step 4: If $\bar{x} \leq$ U – 1.5373σ, accept the lot.
Step 5: If $\bar{x} >$ U – 1.5373σ, go to next step.
Step 6: Accept the current lot provided preceding 2 lots have been accepted on the criterion that $\{\bar{x} \leq \text{U} - 1.5373\sigma\}$. Otherwise reject the lot.
Step 7: Suppose L be the lower specification given by the consumer σ is the known process standard deviation.
Step 8: Draw a random sample of size 51. Measure the quality characteristics using appropriate scale. Let it be $x_1, x_2, x_3, \ldots\ldots\ldots x_n$.
Step 9: Determine the sample mean $\bar{x}$.
Step 10: If $\bar{x} \geq$ L – 1.5373σ, accept the lot.
Step 11: If $\bar{x} <$ L – 1.5373σ, go to next step.
Step 12: Accept the current lot provided preceding 2 lots have been accepted on the criterion that,
if $\{\bar{x} \geq \text{L} - 1.5373\sigma\}$. Otherwise reject the lot.

Designing the Swift Decision Chain Sampling Plan through LQL:
The parameters of Chain sampling plan for known sigma are (n, k, i, w). The parameters of n and w (w=$\frac{k}{i}$) are determined based on the designing procedure.

Step: 1 Let the probability of acceptance of the fraction defective p_2 is β and $P_a(p_2) \leq \beta$, Assume $\beta = 0.10$

Step: 2 Now determine the sample of size n and variable factor w for the given index i, from the following equation.

$$P[\bar{x} \leq U - w\sigma] + P[\bar{x} > U - w\sigma]\{P[\bar{x} \leq U - w\sigma]\}^i \leq 0.10$$

The above equation is nonlinear with two unknowns; hence computer program is written to solve the equations and tables are constructed.

Table 2: The table gives the values of n and the variable factors $\frac{k}{i} = w$ for the parameters of Swift Decision Chain Sampling Plan for variables indexed through LQL.

Swift Decision Chain Sampling Plan through LQL					
p_2	**i**	**K**	**w**	**N**	$p_{a(p_2)}$
0.002	1	2.2286	2.2286	9	0.099996
0.012	2	3.50895	1.75448	10	0.100012
0.0871	3	3.68863	1.229543	125	0.100014
0.0997	4	4.48062	1.120155	70	0.099975
0.0992	5	4.99924	0.999848	21	0.100014

Illustration 2:
Obtain the SDChSP by variable if the known LQL = 0.0871 when the index i=3.

Solution:
From table 2, when i=3, the sample size is n = 125 and the variable factor w = 1.226543.

Some properties of SDChSP for Variable:
Property 1: When i=1, the SDChSP for variable is converges to ordinary chain sampling plan

Proof: The OC function of SDChSP is
$P_a(p) = P[\bar{x} \leq U - w\sigma] + P[\bar{x} > U - w\sigma]\{P[\bar{x} \leq U - w\sigma]\}^i$
Where $w = \frac{k}{i}$, put i=1, $w = \frac{k}{1} = k$

Therefore, the probability of acceptance is

$P_a(p) = P\,[\bar{x} \le U - k\sigma] + P\,[\bar{x} > U - k\sigma]\; P\,[\bar{x} \le U - k\sigma]$

Table 3: Values of Operating Characteristic function based on property 1.

OC Function for when i=1					
p1	**Pa(p)**				
	n=70, k=3.8289	**n=90, k=3.8285**	**n=110, k=3.8282**	**n=130, k=3.8279**	**n=150, k=3.8275**
0.000045	0.9268	0.9714	0.9939	1.0000	0.9928
0.000046	0.9008	0.9518	0.9821	0.9970	0.9996
0.000047	0.8715	0.9278	0.9645	0.9869	0.9981
0.000048	0.8391	0.8995	0.9415	0.9703	0.9887
0.000049	0.8039	0.8672	0.9134	0.9474	0.9720
0.00005	0.7659	0.8311	0.8805	0.9187	0.9483
0.000051	0.7253	0.7916	0.8431	0.8844	0.9181
0.000052	0.6824	0.7487	0.8015	0.8449	0.8817
0.000053	0.6372	0.7027	0.7558	0.8005	0.8394
0.000054	0.5899	0.6537	0.7063	0.7514	0.7916
0.000055	0.5405	0.6019	0.6533	0.6980	0.7386
0.000056	0.4893	0.5476	0.5968	0.6403	0.6806
0.000057	0.4363	0.4907	0.5372	0.5788	0.6180
0.000058	0.3816	0.4315	0.4745	0.5135	0.5508
0.000059	0.3253	0.3700	0.4090	0.4446	0.4795
0.00006	0.2675	0.3065	0.3407	0.3724	0.4042
0.000061	0.2083	0.2409	0.2698	0.2971	0.3251
0.000062	0.1477	0.1735	0.1965	0.2187	0.2424

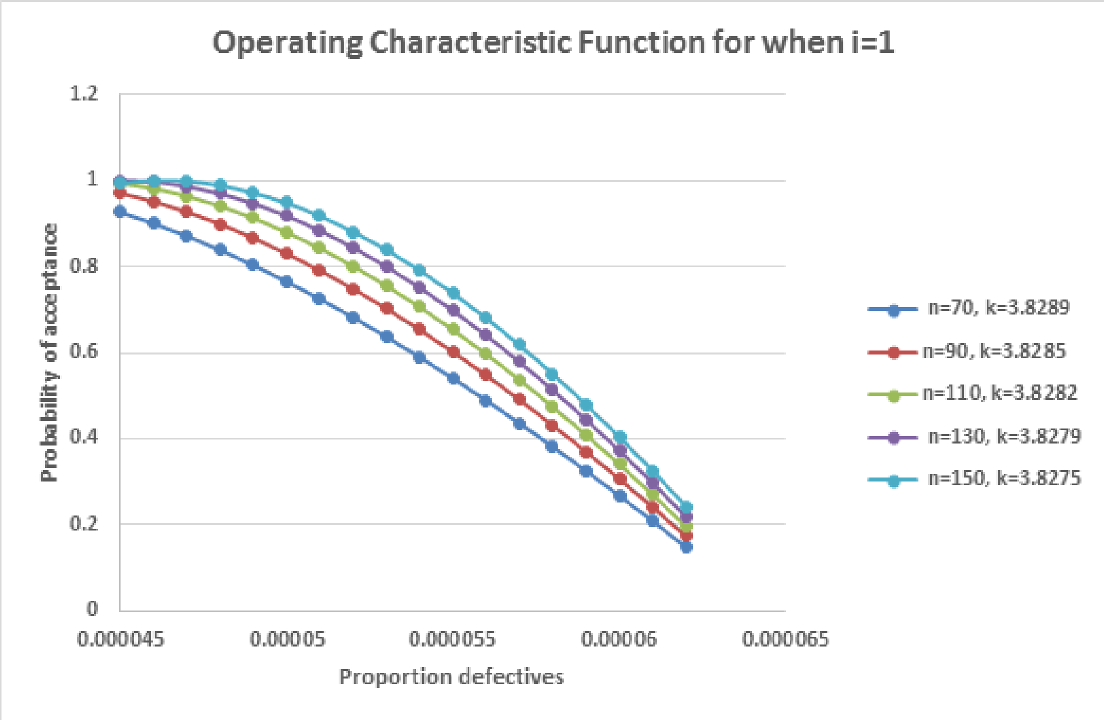

Figure 1: The OC Curve based on property 1.

Conclusion:

In this research article, a new type of swift decision chain sampling plan is developed. The probability of acceptance of the lots shows better discrimination of the good and bad lots. Based on the designing algorithm, tables are constructed for easy selection of the plan. It is found that, when i=1 the SDChSP will converges to ordinary chain sampling plans. This swift decision chain sampling plan requires small sample size hence inspection cost is smaller. Producer and consumer may be benefitted by this type of inspection which improves the product or lot quality.

Reference:

[1] DevaArul S and Jothimani K (2020), "Some properties of two-sided relational chain sampling plans", *International journal of statistics and applied mathematics, ISSN: 2456-1452, Maths* (2020), *5(4): pp.107-109.*

[2] DevaArul S and Vijila Moses (2017), "Design and development of relational chain sampling plans [RChSP (0,i)] for attribute quality characteristic", *Cogent Mathematics, ISSN: (Print) 2331-1835, pp.1-8 (2017).*

[3] Dodge (1955), Chain sampling inspection plan, *Industrial quality control, 11, pp.10-13.*

[4] Govindaraju and Balamurali (1998), Chain sampling plan for variable inspection, *Journal of applied Statistics, 25:1,pp. 103-109.*

[5] Govindaraju and Subramani (1993), Selection of chain sampling plans ChSP-1 and ChSP-(0,1) for given acceptance quality level and limiting quality level, *American Journal of Mathematics and Management Sciences, 13(1), pp.123-136.*

[6] Jeyadurga P and Balamurali S (2019), "Designing of chain sampling plan under gamma-poisson distribution", *International journal of engineering and advanced technology, ISSN;2249-8958, Volume-9, Issue-1S4 December* (2019).

[7] Milky Mathew C and Rajeswari M (2019), "Two-sided complete Bayesian chain sampling plan with beta geometric as prior distribution", *International journal for research in engineering application and management, ISSN: 2454-9150. Vol-05, May* (2019).

[8] Muhammad Farouk, et.al. (2020), "The application of lognormal distribution on the new two-sided group chain sampling plan", *Sains Malaysiana: pp.1145-1152,* (2020).

[9] Raju C and Vidya R (2017) "Chain sampling plan for desired Acceptable Quality Level (AQL) and Limiting Quality Level (LQL), *Proceeding of the 13th IMT-GT International Conference on Mathematics, Statistics and their Applications* (2017). *AIP Conference Proceedings 1905, 050036-1—050036-6.*

[10] Rebbeca Jebaseeli Edna K and Jemmy Joyce V (2020), "A Variable-Attribute testing plan done on process potential index and least tanget angle principle", *International Journal of Mechanical and production Engineering Research and Development, ISSN (P): 2249-6890; ISSN (E): 2249-8001, Vol. 10, Issue 3, Jun 2020, pp. 9067-9070, (2020).*

[11] Rebcca Jabaseeli Edna K and Jemmy Joyce, (2013) "Chain sampling plan for variables fraction non-conformities", *International Journal of pure and applied mathematics, Volume 87, No. 6, pp. 827-833* (2013).

[12] Rebecca Jebaseeli Edna K and Jemmy Joyce V (2018) "Selection of modified chain sampling plan through minimum angle criteria", *International Journal of Mechanical Engineering and Technology, Volume – 9, Issue 1. January (2018), pp.1102-1105.*

[13] Rebecca Jebaseeli Edna K, et.al. (2022), "A blended two-sided chain sampling plan created on process potential measure", *International Journal of Mechanical Engineering, ISSN:0974-5823, Vol.7, No.1, January* (2022).

[14] Rebecca Jebaseeli Edna K, et.al. (2022), "A new development of variable – attribute relational complete chain sampling plan (RCCSP) for quality control engineers'', *International Journal of Engineering Trends and Technology, ISSN:2231-5381, Volume 70, Issue 7, pp.319-328 July* (2022).

[15] Vijila M and Deva Arul S (2017), "Construction of two-sided complete chain sampling plans-CCHASP (0,1) indexed through AOQL", *International Journal of scientific research and management, Volume-5, pp. 7303-7307 (2017).*

[16] Vijila M and Deva Arul S (2017), "Designing and Selection of Complete Chain Sampling plans CChSP(0,1) Indexed through Inflection point", *International Journal of statistics and systems, ISSN 0973-2675, Volume 12, pp. 831-837, Number 4 (2017).*

[17] Waqar Hafeez and Nazrina Aziz (2022), Bayesian Two-sided Complete Group Chain Sampling Plan for Binomial Distribution using Beta Prior through Quality Regions, *Journal of ICT, 21, No.1 (January) 2022, pp: 51-59.*