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Reliability Properties of a Mixture Distribution

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Abstract

Mixture distributions are a class of probability distributions, having lot of applications in real time scenario. In this paper, some reliability properties of a mixture of Lindley and Lomax distribution proposed by Swarnalatha et al (2022), were derived and presented.

Keywords: Reliability function, Hazard function and Lindley and Lomax mixture distribution.

1. INTRODUCTION

Swarnalatha et al (2022), proposed a mixture distribution with Lindley and Lomax defined as follows

Definition: Let X be random variable satisfies the Lindley distribution with probability p_1 $(=\frac{1}{\theta+1})$ and Lomax distribution with probability p_2 $(=\frac{\theta}{\theta+1})$ generates a mixture distribution with probability density function

$$f(x;\theta) = \frac{\theta^2}{(\theta+1)^2(x+\theta)^2}\left[1+\theta+(1+x)(x+\theta)^2 e^{-\theta x}\right]; x>0, \theta>0. \tag{1}$$

Its mean and variance are

$$\frac{\theta+2}{\theta(\theta+1)^2} - \frac{\theta^2}{\theta+1} \text{ and } \frac{1}{\theta^2(\theta+1)^4} M_1$$

where

$$M_1 = [-\theta^8 - 2\theta^7 - \theta^6 + 2\theta^5 + 6\theta^3 - 9\theta^2 + 10\theta + 12]$$

2. RELIABILITY PROPERTIES

The reliability properties of the mixture distribution like, reliability function R(t), hazard rate h(x), cumulative hazard rate H(t) are derived and its graphical presentation of Hazard rate function for different values of parameter θ are also presented.

a) Distribution function:

$$F(t,\theta) = \int_0^t f(x)dx$$

$$= \int_0^t \frac{\theta^2}{(\theta+1)^2(x+\theta)^2}\left[1+\theta+(1+x)(x+\theta)^2 e^{-\theta x}\right]dx$$

$$= \frac{\theta^2}{(\theta+1)}\int_0^t (x+\theta)^{-2}dx + \frac{\theta^2}{(\theta+1)^2}\int_0^t (1+x)\, e^{-\theta x}dx$$

On simplification,

$$F(t,\theta) = 1 - \frac{e^{-\theta t}}{\theta+1} - \frac{\theta t e^{-\theta t}}{(\theta+1)^2} - \frac{\theta^2}{(\theta+1)(t+\theta)};\ t>0, \theta>0. \tag{2}$$

b) The reliability function $\boldsymbol{R(t,\theta)}$**:**

$$R(t,\theta) = 1 - F(t,\theta)$$

$$= \frac{e^{-\theta t}}{\theta+1} + \frac{\theta t e^{-\theta t}}{(\theta+1)^2} + \frac{\theta^2}{(\theta+1)(t+\theta)} \tag{3}$$

c) Hazard function:

$$h(x) = \frac{f(x,\theta)}{R(x,\theta)}$$

$$= \frac{\frac{\theta^2}{(\theta+1)^2(x+\theta)^2}\left[1+\theta+(1+x)(x+\theta)^2 e^{-\theta x}\right]}{\frac{e^{-\theta x}}{\theta+1} + \frac{\theta x e^{-\theta x}}{(\theta+1)^2} + \frac{\theta^2}{(\theta+1)(x+\theta)}}$$

On simplification,

$$= \frac{\boldsymbol{\theta^2[(1+x)(x+\theta)e^{-\theta x} + (\theta+1)(x+\theta)^{-1}]}}{\boldsymbol{(\theta+1)(x+\theta)e^{-\theta x} + \theta x e^{-\theta x}(x+\theta) + \theta^2(\theta+1)}} \tag{4}$$

d) Cumulative Hazard rate function:

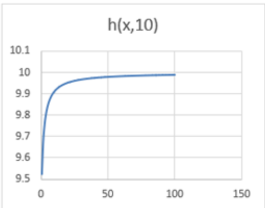
$$H(t) = \int_0^t h(x)dx$$
$$= \int_0^t \frac{\boldsymbol{\theta^2[(1+x)(x+\theta)e^{-\theta x} + (\theta+1)(x+\theta)^{-1}]}}{\boldsymbol{(\theta+1)(x+\theta)e^{-\theta x} + \theta x e^{-\theta x}(x+\theta) + \theta^2(\theta+1)}} dx \tag{5}$$

3. GRAPHICAL ANALYSIS:

The graphical representation of Hazard rate function.

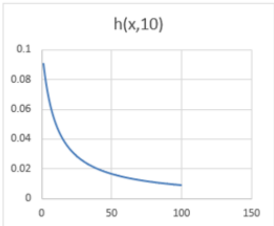
For θ=10,

Lindley



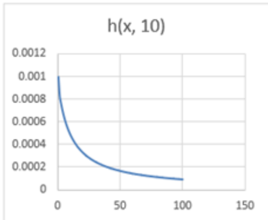
Graph 3.1(*a*)

Lomax



Graph 3.1(b)

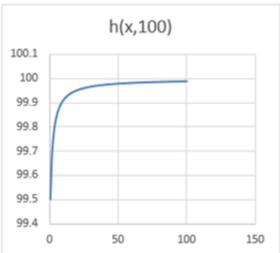
Lindley and Lomax mixture



Graph 3.1(c)

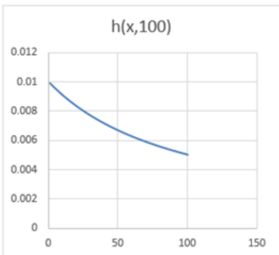
For θ=100,

Lindley



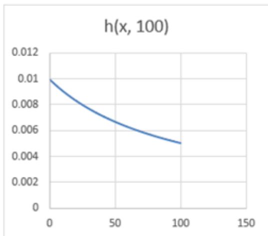
Graph 3.2(*a*)

Lomax



Graph 3.2(b)

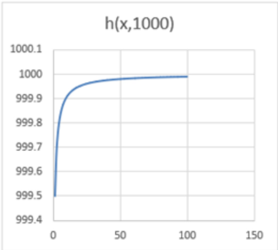
Lindley and Lomax mixture



Graph 3.2(c)

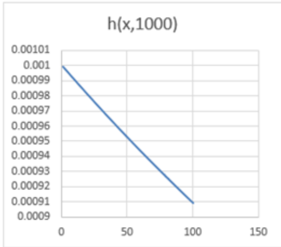
For θ=1000,

Lindley



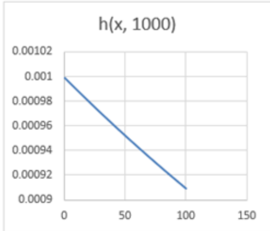
Graph 3.3(*a)*

Lomax



Graph 3.3(b)

Lindley and Lomax mixture



Graph 3.3(c)

Note:

1. As the time ‘x’ is increasing, the failure rate of Lindley and Lomax mixture distribution is decreasing.
2. The Hazard rate curves of LL and Lomax distributions appear to be in the same form.
3. As the parameter ‘θ’ is increasing, the failure rate curve of Lindley and Lomax mixture distribution is tending to a straight line.
4. It can be used as lifetime distribution of components.

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