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A New Derived Power Series Distribution

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ABSTRACT

Power Series Distributions are a family of probability distributions possessing the property that $P(x) = \frac{a_x \theta^x}{f(\theta)}$. Several authors contributed to deriving new power series distributions and their properties.

In this paper, an attempt is made to propose a new derived power series distribution using the ratio of two Poisson distributions with parameters μ_1 and μ_2 ($< \mu_1$). Its properties like moment generating function, probability generating function, characteristic function, recurrent relation, moments, skewness, kurtosis are derived and illustrated with suitable example.

Keywords and Phrases: Poisson distribution, Ratio of two Poisson's and Properties

1. INTRODUCTION

Noack (1950) identified the family of Power series distribution, which encompasses many discrete distributions. It has a broader application, such as determining decreasing failure rate, derived asymptotic variance and covariance, data that have a frequency of observed zeros significantly higher than predicted by models based on the standard parametric family of discrete distributions and the ratio of the power series is flexible when compared to the existing methods.

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Definition 1.1: A random variable X is said to follow a Poisson distribution if it possesses the probability that

$$P(X=x) = \frac{e^{-\mu}\mu^{x}}{x!}\ ;\ x = 0,1,2,\ldots \text{ and } \mu > 0;\quad P(X=x) = 0;\ \text{otherwise} \qquad \ldots (1)$$

Definition 1.2: A discrete random variable X is said to belong to the family of Power Series distribution (PSD), if its probability mass function is given by

$$P(X=x) = \frac{a_x\theta^{x}}{f(\theta)},\ a_x \geq 0;\ x = 0, 1, 2, \ldots;\ \theta > 0 \text{ and } f(\theta) = \sum_{x=0}^{\infty} a_x\theta^{x}. \qquad \ldots (2)$$

Definition 1.3: A discrete random variable X is said to be a Power Series distribution, if it is expressed as the ratio of two power series distributions in the form

$$\frac{1+\sum_{i=1}^{\infty} a_i s^{i}}{1+\sum_{i=1}^{\infty} b_i s^{i}} = \sum_{i=1}^{\infty} d_i s^{i}\ ; \qquad \ldots (3)$$

where, $d_i = a_i - \sum_{j=1}^{i} b_j\, d_{i-j}$; i = 1, 2, 3, ... with d_0 =1 and $a_i = \left(\frac{a_i^*}{a_0}\right)(\theta_1)^i$ and $b_j = \left(\frac{b_j^*}{b_0}\right)(\theta_2)^j$

In this paper, a new power series distribution is derived using two Poisson distributions with parameters μ_1 and μ_2 by considering their ratio and is presented below.

2. DERIVED POWER SERIES DISTRIBUTION

Let us consider the two random variables X and Y follows Poisson distribution with parameter μ_1, and μ_2, where ($\mu_1 > \mu_2$), with probability generating functions

$$P(X=s) = \sum_{x=0}^{\infty} \frac{e^{-\mu_1}\mu_1^{x}}{x!} s^{x},\ |s| \leq 1, \qquad \ldots (4)$$

$$P(Y=s) = \sum_{y=0}^{\infty} \frac{e^{-\mu_2}\mu_2^{y}}{y!} s^{y},\ |s| \leq 1. \qquad \ldots (5)$$

Then, the ratio of Probability generating functions of (4) and (5) is given by

$$\frac{P(X=s)}{P(Y=s)} = e^{-\mu_1+\mu_2} \sum_{i=0}^{\infty} d_i s^{i} = \sum_{i=0}^{\infty} p_i s^{i}. \qquad \ldots (6)$$

where,

$$p_i = e^{-\mu_1+\mu_2} d_i \text{ and } \quad d_i = a_i - \sum_{j=1}^{i} b_j\, d_{i-j}\ ;\ i=1,2,\ldots, d_0 = 1;\ a_i = \frac{\mu_1^{i}}{i!} \text{ and } b_j = \frac{\mu_2^{j}}{j!}. \ldots (7)$$

In general, for i =n,

$$d_n = a_n - b_1 d_{n-1} - b_2 d_{n-2} - b_3 d_{n-3} \dots\dots - b_n d_0$$

$$= \left(\frac{\mu_1{}^n}{n\,!}\right) - \sum_{j=1}^{n} \frac{\mu_2{}^j}{j\,!} d_j \quad = \frac{(\mu_1 - \mu_2)^n}{n\,!} \qquad \dots (8)$$

Theorem: Let p_0, p_1, p_2, … be the set of probabilities generated by a power series distribution (6) with $p_i = e^{-\mu_1 + \mu_2} d_i$, where $d_i = \frac{(\mu_1 - \mu_2)^n}{n\,!}$; ($\mu_1 > \mu_2$) defines a proper probability distribution.

Proof: We have $\mu_1 > \mu_2$ therefore, $d_i = \frac{(\mu_1 - \mu_2)^n}{n\,!} > 0$ for i = 1, 2, ... and d_0 =1

We have P [X = n] = p_n then from the axiom of certainty,

$\sum_{n=0}^{\infty} p_n = e^{-\mu_1 + \mu_2} \sum_{n=0}^{\infty} d_n$;

$$= e^{-(\mu_1 - \mu_2)} \sum_{n=0}^{\infty} \frac{(\mu_1 - \mu_2)^n}{n\,!}\ ;\ \mu_1 > \mu_2$$

$$= e^{-(\mu_1 - \mu_2)} .\ e^{(\mu_1 - \mu_2)}\ ;\ \left(\sum_{n=0}^{\infty} \frac{(\mu_1 - \mu_2)^n}{n\,!} = e^{(\mu_1 - \mu_2)}\right)$$

$\therefore \sum_{n=0}^{\infty} p_n = 1$

Hence, the p_i's are proper probabilities of a distribution. □

3. PROPERTIES OF POISSONO-POISSON POWER SERIES DISTRIBUTION

In this section a set of properties of Poissono-poisson derived power series distribution are presented.

a) Characteristic Function of the distribution is $\phi_X(t) = e^{-(\mu_1 - \mu_2)(e^{it} - 1)}$ … (9)

b) Probability Generating Function of the distribution is $P_X[s] = e^{(\mu_1 - \mu_2)(s-1)}$; $|s| \le 1$ … (10)

c) Moment Generating Function of distribution is $M_x(t) = e^{-(\mu_1 - \mu_2)(e^{t} - 1)}$ … (11)

d) The moments of the distribution can be obtained from the Probability Generating Function as

$$\mu_1^1 = E(x) = \left| \frac{d}{ds} P[X = s] \right|_{s=1} = (\mu_1 - \mu_2) \qquad \dots (12)$$

$$\mu_2 = V(X) = \left| \frac{d^2}{ds^2} P[X = s] \right|_{s=1} + \frac{d}{ds} P[X = s] \Big|_{s=1} - [E(X)]^2 = (\mu_1 - \mu_2) \qquad \dots (13)$$

$\mu_3^1 = (\mu_1 - \mu_2)^3 + 3(\mu_1 - \mu_2)^2 + (\mu_1 - \mu_2)$ … (14)

$\mu_3 = (\mu_1 - \mu_2)$ … (15)

$\mu_4^1 = (\mu_1 - \mu_2)^4 + 6(\mu_1 - \mu_2)^3 + 7(\mu_1 - \mu_2)^2 + (\mu_1 - \mu_2)$ … (16)

$\mu_4 = 3(\mu_1 - \mu_2)^2 + (\mu_1 - \mu_2)$ … (17)

e) Skewness of distribution is $\beta_1 = (\mu_1 - \mu_2)^{-1}$ … (18)

f) Kurtosis of the distribution is $\beta_2 = 3 + (\mu_1 - \mu_2)^{-1}$ … (19)

g) Sum of two independent Poissono-poisson derived power series variates also follows a Poissono-poisson distribution. From (11)

$$M_{X1+\ldots+Xn}(t) = M_{X1}(t). \ldots M_{Xn}(t) = e^{(\theta_1+\theta_2+\cdots\theta_n)(e^t-1)} \quad \ldots (20)$$

where, $\theta_1 = (\mu_1 - \mu_2), \theta_2 = (\mu_3 - \mu_4), \ldots\ldots, \theta_n = (\mu_{n+1} - \mu_{n+2})$;

h) The recurrence relation between the probabilities can be obtained as

$$P(X = x+1) = \left[\frac{\mu_1 - \mu_2}{x+1}\right]. [P(X = x)] \quad \ldots (21)$$

Remarks:

i. The mean and variance of the distribution are same and is the difference between the parameters of the numerator and denominator Poisson distributions considered, i.e.,

Mean = Variance = $(\mu_1 - \mu_2)$.

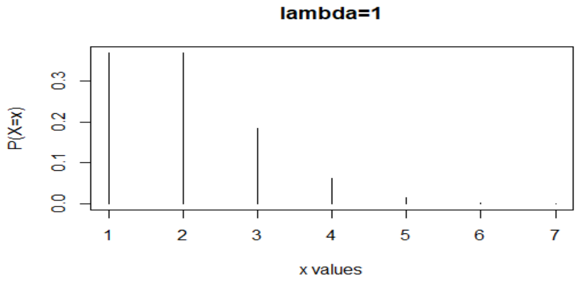
ii. The shape of the distribution is Poisson if $1 \le \mu_1 - \mu_2 \le 2$.

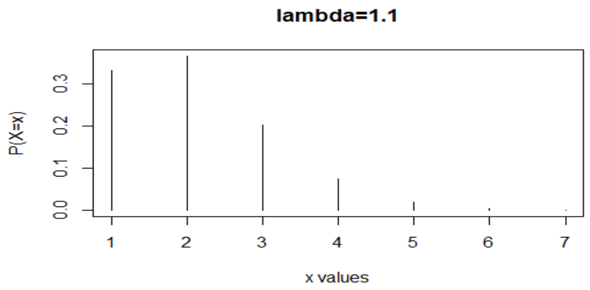
iii. The distribution is a positively skewed $\gamma_1 = (\mu_1 - \mu_2)^{-1/2}$ because $\mu_1 > \mu_2$.

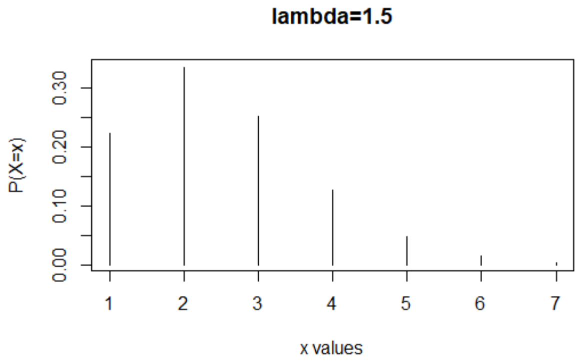
iv. The distribution is platy-kurtic because $\gamma_2 = (\mu_1 - \mu_2)^{-1}$ where $1 \le \mu_1 - \mu_2 \le 2$.

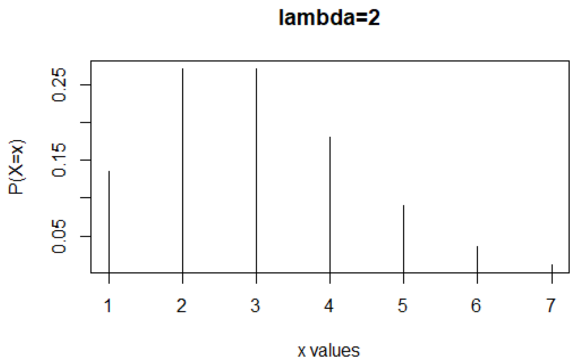
4. GRAPHICAL REPRESENTATION OF THE DISTRIBUTION

The graphical representation of Poissono-poisson derived power series distribution, for values of parameters $1 \leq \mu = \mu_1 - \mu_2 \leq 2$ are presented below.









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