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A Note on Compound Rayleigh -Triangular Lifetime Distribution

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Abstract

The process control is capable of producing better products for the given specifications and the manufacturer decides to relax certain constraints accordingly on Men / Material / Machines. Assume that the process parameter (quality characteristic) follows triangular distribution. When the product is used, the resulting lifetime distribution of such a product is Compound Rayleigh -Triangular Lifetime Distribution.

In this paper, reliability, hazard and cumulative hazard functions of Compound Rayleigh -Triangular Lifetime Distribution were derived. The properties were illustrated with a suitable example.

Keywords: Rayleigh Distribution, Triangular Distribution, reliability properties, product control and process control.

1. INTRODUCTION

Let the measurable quality characteristic X of the product be normally distributed with mean μ and variance σ^2. Let U, L be the upper and lower specification limits. Based on the observations of the production process over time, suppose it can be concluded that the process is capable of producing better products, meeting the specification, U-L > 6 σ. Let us assume that the manufacturer decides to relax certain constraints on the process by fine-tuning the operations of either one or more of the three M's (material, men and machines), leading to shifts in μ which follow triangular distribution in $[\mu_L, \mu_U]$, where $\mu_U = U - 3\sigma$ and $\mu_L = L + 3\sigma$. Using the above assumptions, Compound Exponential Lifetime Distributions-I, Compound Exponential Lifetime Distributions-II and Compound Rayleigh Lifetime

Distributions-I and their properties were derived and were presented in Sirisha and Swamy (2013), (2014), (2020) and Sirisha and Jayasree (2018).

Let the distribution of product's lifetime follow Rayleigh with α as the parameter.

It can be observed that the increase in the shift in μ from the process mean μ_o, will result in a decrease in the lifetime of the product. Hence, the increase in the absolute deviation of μ from μ_o results in an increase in the reciprocal of the expected lifetime $\sqrt{2/\pi}\,(1/\alpha)$, and the same is represented by the re $\sqrt{2/\pi}$ n $(1/\alpha) = c + mU = V$, where c, m >0 and U = | μ – μ_o | which represents absolute deviation of μ from μ_o.

Let T be the lifetime of the product produced under the above framework, then the characteristic features of Compound Rayleigh -Triangular Lifetime Distribution are

a) The density function of T is

$$f^*(t) = \begin{cases} 2(m\delta)^{-2}\int_c^{c+m\delta}(m\delta+c-\upsilon)\dfrac{t\pi\upsilon^2}{2}e^{-1/2\left(\frac{t^2\upsilon^2\pi}{2}\right)}dv & ; \quad 0<t<\infty \\ & \quad c<v<c+m\delta, \\ 0 & ; \quad elsewhere. \end{cases} \quad \ldots (1.1)$$

Note: The pdf f*(t) can also be expressed in terms of incomplete Gamma distribution function Γ, as follows

$$f^*(t) = \begin{cases} \dfrac{4(c+m\delta)}{m^2\delta^2t^2\sqrt{\pi}}[\Gamma(3/2,\omega_1)-\Gamma(3/2,\omega_2)] - \dfrac{8}{m^2\delta^2t^3\pi}[\Gamma(2,\omega_1)-\Gamma(2,\omega_2)] & ; \quad 0<t<\infty, \\ 0 & ; \quad \text{elsewhere}. \end{cases} \quad (1.2)$$

where $\omega_1 = t^2c^2\pi 4^{-1}$; $\omega_2 = t^2(c+m\delta)^2\pi 4^{-1}$. … (1.3)

b) The expected lifetime is given by

$$E^*(T) = 2(m\delta)^{-2}\,[(c+m\delta)\log(1+m\delta c^{-1})-m\delta] \quad \ldots (1.4)$$

For all m, c>0, the expected lifetime of the Compound Rayleigh -Triangular Lifetime Distribution is less than that of the Rayleigh distribution.

c) The variance of T is given by

$$V^*(T) = 8(m\delta)^{-2}\pi^{-1}[m\delta c^{-1}-\log(1+m\delta c^{-1})] - 4(m\delta)^{-4}[(c+m\delta)\log(1+m\delta c^{-1})-m\delta]^2 \quad \ldots (1.5)$$

d) The distribution function F*(t) is

$$F^*(t) = 1 - 2(c+m\delta)(m\delta)^{-2}\, t^{-1}\, \pi^{-1/2}\, [\Gamma(1/2, \omega_1) - \Gamma(1/2, \omega_2)] + 4\,(m\delta)^{-2}\, t^{-2}\, \pi^{-1}\, [\Gamma(1,\omega_1) - \Gamma(1,\omega_2] \quad \ldots \quad (1.6)$$

where ω_1, ω_2 are as in (1.3)

All the above properties are presented in Sirisha (2009).

2. RELIABILITY ASPECTS OF COMPOUND RAYLEIGH -TRIANGULAR LIFETIME DISTRIBUTION

Some properties of the lifetime (T) with particular reference to reliability, are derived.

Lemma 2.1: The reliability function R*(t) is

$$R^*(T) = 2(c+m\delta)\,(m\delta)^{-2}\, t^{-1}\, \pi^{-1/2}\, [\Gamma(1/2, \omega_1) - \Gamma(1/2, \omega_2)] + 4(m\delta)^{-2}\, t^{-2}\, \pi^{-1}\, [\Gamma(1, \omega_1) - \Gamma(1, \omega_2)] \quad \ldots \quad (2.1)$$

Proof: From definition R*(t) = 1 – F*(t) and eq (1.6)

$$= 2\,(c+m\delta)\,(m\delta)^{-2}\, t^{-1}\, \pi^{-1/2}\, [\Gamma(1/2, \omega_1) - \Gamma(1/2, \omega_2)] - 4(m\delta)^{-2}\, t^{-2}\, \pi^{-1}\, [\Gamma(1, \omega_1) - \Gamma(1, \omega_2)]$$

Lemma 2.2: The Hazard function of the distribution is given by

$$h^*(t) = \left(\frac{4(c+m\delta)(m\delta)^{-2}\, t^{-2}\, \pi^{-1/2}\, [\Gamma(3/2, \omega_1) - \Gamma(3/2, \omega_2] - 8(m\delta)^{-2}\, t^{-3}\, \pi^{-1}\, [\Gamma(2, \omega_1) - \Gamma(2, \omega_2)]}{2(c+m\delta)(m\delta)^{-2}\, t^{-1}\, \pi^{-1/2}\, [\Gamma(1/2, \omega_1) - \Gamma(1/2, \omega_2] - 4(m\delta t)^{-2}\, \pi^{-1}\, [\Gamma(1, \omega_1) - \Gamma(1, \omega_2)]} \right) \quad \ldots \quad (2.2)$$

Proof: we know that h*(t) = f*(t) / R*(t)

Proof follows from eq (1.2) and eq (2.1)

Lemma 2.3: The cumulative hazard function H*(t) is

$$H^*(t) = \int_0^t \frac{4(c+m\delta)(m\delta)^{-2}\, x^{-2}\, \pi^{-1/2}\, [\Gamma(3/2, \omega_1) - \Gamma(3/2, \omega_2] - 8(m\delta)^{-2}\, x^{-3}\, \pi^{-1}\, [\Gamma(2, \omega_1) - \Gamma(2, \omega_2)]\, dx}{2(c+m\delta)(m\delta)^{-2}\, x^{-1}\, \pi^{-1/2}\, [\Gamma(1/2, \omega_1) - \Gamma(1/2, \omega_2] - 4(m\delta x)^{-2}\, \pi^{-1}\, [\Gamma(1, \omega_1) - \Gamma(1, \omega_2)]} \quad \ldots \quad (2.3)$$

Proof: we know that $H^*(t) = \int_0^t h^*(x)\,dx$

Proof follows from eq (2.2)

Note: For given parameters c, m and δ, the values of incomplete Gamma integrals can be obtained from Abromowitz and Stegun (1972).

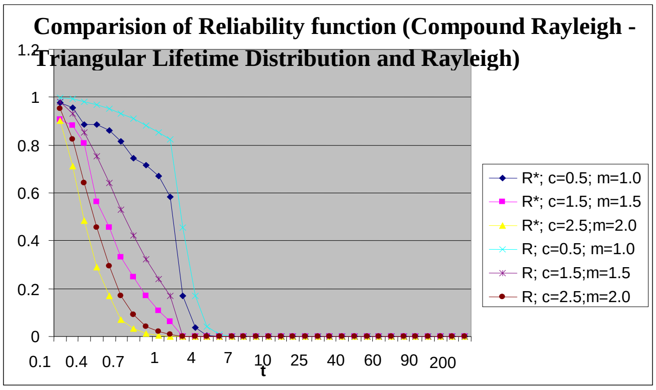
3. EXAMPLE: Consider the example pertaining to manufacturing of piston rings of an automotive engine (Montgomery (1991), pp.206), is taken for the purpose of illustration. The measurable quality characteristic X, is the inside diameter of the piston ring, is assumed to be normally distributed with mean μ, variance σ^2. U = 75 mm and L = 73 mm are the upper and lower specification limits respectively, then $\mu_o = 74.001$ mm and the estimated value of σ = 0.00989 mm. Here, U- L > 6 σ. Hence using the concept of modified control charts, one has $\mu_U = 74.97033$ mm, $\mu_L = 73.02967$ mm and δ = 0.97033 mm.

For different values of c & m the values of the reliability function and hazard function for Compound Rayleigh -Triangular Lifetime Distribution and conventional Rayleigh Distribution are represented as R*, h*, R, h respectively. These are tabulated in Table 3.1. Their respective graphs are presented in Graphs 3.1 and 3.2.

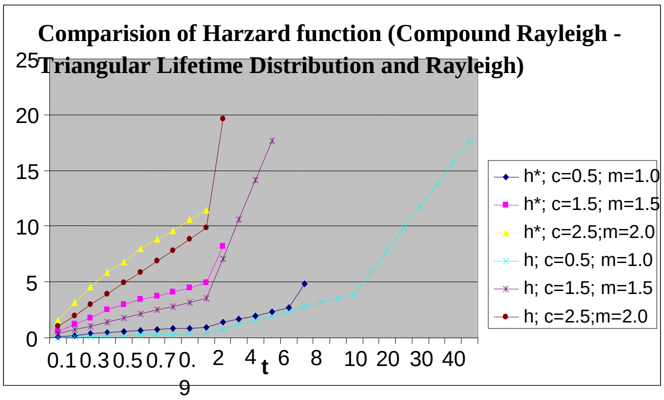
Table 3.1: Compound Rayleigh -Triangular Lifetime Distribution and Rayleigh Distribution

	c= 0.5000 and m=1.0000				c=1.5000 and m=1.5000				c=2.5000 and m=2.0000			
t	**R***	**h***	**R**	**h**	**R***	**h***	**R**	**h**	**R***	**h***	**R**	**h**
0.1	0.9773	0.1111	0.998	0.039	0.9066	0.579	0.9825	0.353	0.90156	1.588	0.9521	0.981
0.2	0.95409	0.1778	0.9922	0.079	0.88047	1.1956	0.9318	0.707	0.711785	3.119	0.8218	1.963
0.3	0.88715	0.3538	0.9825	0.118	0.80637	1.7946	0.853	1.06	0.48518	4.539	0.643	2.944
0.4	0.8846	0.4522	0.9691	0.157	0.5648	2.477	0.7538	1.413	0.28877	5.815	0.4561	3.925
0.5	0.85939	0.551	0.9521	0.196	0.45572	2.938	0.643	1.766	0.16788	6.76	0.2933	4.906
0.6	0.81415	0.64	0.9318	0.236	0.33006	3.412	0.5295	2.12	0.07218	7.975	0.171	5.888
0.7	0.746176	0.748	0.9083	0.275	0.24699	3.741	0.4209	2.473	0.0333	8.794	0.0904	6.869
0.8	0.7164	0.81	0.882	0.314	0.16989	4.087	0.3229	2.826	0.01387	9.594	0.0433	7.85
0.9	0.67	0.88	0.853	0.353	0.10953	4.443	0.2392	3.179	0.00461	10.629	0.0188	8.831
1	0.582814	0.97	0.8218	0.393	0.0619	4.91	0.171	3.533	0.0016	11.416	0.0074	9.813
2	0.169712	1.424	0.4561	0.785	0.000113	8.168	0.0009	7.065	0		0	19.63
3	0.0353	1.713	0.171	1.178	0		0	10.6	0		0	29.44
4	0.00616	1.974	0.0433	1.57	0		0	14.13			0	39.25
5	0.000687	2.299	0.0074	1.963	0		0	17.66			0	49.06
6	5.88E-05	2.72	0.0009	2.355	0		0	21.2			0	58.88
7	2.25E-06	4.844	0.0001	2.748	0		0	24.73			0	68.69
8	0		0	3.14	0		0	28.26			0	78.5
9	0		0	3.533	0		0	31.79			0	88.31

t	R*	h*	R	h	R*	h*	R	h	R*	h*	R	h
10	0		0	3.925	0		0	35.33			0	98.13
15	0		0	5.888	0		0	52.99			0	147.2
20	0		0	7.85	0		0	70.65			0	196.3
25	0		0	9.813	0		0	88.31			0	245.3
30	0		0	11.78	0		0	106			0	294.4
35	0		0	13.74	0		0	123.6			0	343.4
40	0		0	15.7	0		0	141.3			0	392.5
45	0		0	17.66	0		0	159			0	441.6
50	0		0	19.63	0		0	176.6			0	490.6
60	0		0	23.55	0		0	212			0	588.8
70	0		0	27.48	0		0	247.3			0	686.9
80	0		0	31.4	0		0	282.6			0	785
90	0		0	35.33	0		0	317.9			0	883.1
100	0		0	39.25	0		0	353.3			0	981.3
150	0		0	58.88	0		0	529.9			0	1472
200	0		0	78.5	0		0	706.5			0	1963



Graph 3.1



Graph 3.2

4. CONCLUSION:

i) From the graphs we can observe that Compound Rayleigh -Triangular Lifetime Distribution has increasing failure rate (IFR) distribution.

ii) From the tables we can indicate that Compound Rayleigh -Triangular Lifetime Distribution has increasing failure rate (IFR) distribution.

iii) If the manufacturer would like to relax one are more of the conditions imposed on material, machines and men in a situation, when U-L > 6σ.The Compound Rayleigh -Triangular Lifetime Distribution can be used rather than the conventional Rayleigh distribution.

iv) The reliability function R*(t) is

$$R^*(T) = 2(c+m\delta)\,(m\delta)^{-2}\,t^{-1}\,\pi^{-1/2}\,[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2)] + 4(m\delta)^{-2}\,t^{-2}\,\pi^{-1}\,[\Gamma(1,\omega_1) - \Gamma(1,\omega_2)]$$

v) The Hazard function of the distribution is given by

$$h^*(t) = \frac{4(c+m\delta)(m\delta)^{-2}\,t^{-2}\,\pi^{-1/2}\,[\Gamma(3/2,\omega_1) - \Gamma(3/2,\omega_2] - 8(m\delta)^{-2}\,t^{-3}\,\pi^{-1}\,[\Gamma(2,\omega_1) - \Gamma(2,\omega_2)]}{2(c+m\delta)(m\delta)^{-2}\,t^{-1}\,\pi^{-1/2}\,[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2] - 4(m\delta t)^{-2}\,\pi^{-1}\,[\Gamma(1,\omega_1) - \Gamma(1,\omega_2)]}$$

vi) The cumulative hazard function H*(t) is

$$H^*(t) = \int_0^t \frac{4(c+m\delta)(m\delta)^{-2}\,x^{-2}\,\pi^{-1/2}\,[\Gamma(3/2,\omega_1) - \Gamma(3/2,\omega_2] - 8(m\delta)^{-2}\,x^{-3}\,\pi^{-1}\,[\Gamma(2,\omega_1) - \Gamma(2,\omega_2)]}{2(c+m\delta)(m\delta)^{-2}\,x^{-1}\,\pi^{-1/2}\,[\Gamma(1/2,\omega_1) - \Gamma(1/2,\omega_2] - 4(m\delta x)^{-2}\,\pi^{-1}\,[\Gamma(1,\omega_1) - \Gamma(1,\omega_2)]}\,dx$$

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