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Mean Time to System Failure Analysis of Probabilistic System with Provision of Switch Repair and Operating Time Threshold

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Abstract

In this paper a stochastic model for a standby system is developed. The system works with the proviso of preventive maintenance and random switch. The system is restored by a server available in the system. The model is evaluated using semi-Markov process and regenerative point technique of stochastic theory. The numerical results are obtained using laws of Weibull distribution. The numerical results highlights the importance of the switch restoration and the operating time threshold for the standby on system performance.

Keywords: Standby System, Semi-Markov process, Performance Measures, Time Threshold.

1. INTRODUCTION

The preventive maintenance includes set of activities to correct the faults that can take the form of major failures in near future and hence make the system running smoothly for longer periods of time. It is the long term investment for maintaining the reliability and availability of systems. This concept is studied by various authors including (Ruiz-Castro, 2015), (Garg & Kadyan, 2016), (Levitin et al., 2020). Further, the cold standby redundancy is a common technique which is widely used for improving system reliability and availability. The system models with possible failures of cold-standby are studied by (Osaki & Nakagawa, 1971), (Subramanian et al., 1976), (Bhardwaj et al., 2017), (Chen et al., 2018). The switching devices are vital for a cold standby system. They are responsible for keeping the system working by putting the standby units into operation. This issue is highlighted in some studies such as (Sharma et al., 2019) and (Shekhar et al., 2020). The present research work investigates the possibility of cold standby and switching failure synchronously.

The current paper extends the research work on a cold standby system model developed by (Bhardwaj & Kaur, 2019). The stated study debated only on the cost-benefit of the standby system. Another aspect associated is the reliability of the system, as in some cases it is urgently required. Keeping this aspect in mind, here we evaluated the reliability of the system in terms of mean time to system failure. The system starts along two identical units with one unit as operating and other reserved as a cold standby. The standby unit may or may not found fit for operation when needed. Therefore inspection is done to check its operational fitness. Also for the operating unit, a maximum time threshold, known as maximum operation time (MOT), is set so as to do the preventive maintenance. If it fails before reaching MOT then directly goes under repair. Furthermore, the switching mechanism or the switch, needed to switch the standby into operation at the failure of operating unit, is also subject to failure. It is rectified by the single server which is available in the system to perform all remedial activities. The system model is developed with the help of semi-Markov process and evaluated using regenerative point technique. The various system performance measures including MTSF, steady state availability, busy periods of server and expected number of repairs, preventive maintenances, inspections and the system profit are obtained using Weibull distribution and numerical results are presented in tabular form.

2. NOTATIONS AND SYMBOLS

O	The unit is operative and in normal mode
CS	The unit is in cold-standby mode
CS_{nso}	Cold-standby unit not switched on
CS_{NSO}	Continuously in cold-standby mode from previous state
p/q	Probability that switch is working/failed
a/b	Probability that repair/ replacement is feasible after inspection
F_{ui}/F_{UI}	Failed unit under inspection /under inspection continuously from previous state
F_{wi}/F_{WI}	Failed unit waiting for inspection / waiting for inspection continuously from previous state

F_{ur} / F_{UR}	Failed unit under repair / under repair continuously from previous state
F_{wr} / F_{WR}	Failed unit waiting for repair / waiting for repair continuously from previous state
S_{ur} / S_{UR}	Failed switch under repair / under repair continuously from previous state
S_{wr} / S_{WR}	Failed switch waiting for repair / waiting for repair continuously from previous state
u_{pmt} / U_{PMT}	Unit under Preventive Maintenance (PM)/ under PM continuously from previous state
w_{pmt} / W_{PMT}	Unit waiting for PM/ waiting for PM continuously from previous state
$z(t) / Z(t)$	pdf/ cdf of failure time of unit
$g(t) / G(t)$	pdf / cdf of inspection time upon cold standby failure
$f(t) / F(t)$	pdf / cdf of repair time of unit
$h(t) / H(t)$	pdf / cdf of repair time of switch
$s(t) / S(t)$	pdf/ cdf of failure time of cold standby unit (max. redundancy time)
$o(t) / O(t)$	pdf/ cdf of maximum operation time
$p_m(t) / P_m(t)$	pdf/ cdf of preventive maintenance time
$q_{ij}(t) / Q_{ij}(t)$	pdf/ cdf of direct transition time from regenerative state S_i to S_j or failed state S_j without visiting any other regenerative state in $(0,t]$
$q_{ij.kr}(t) / Q_{ij.kr}(t)$	pdf/cdf of first passage time from regenerative state S_i to S_j or failed state S_j visiting state S_k, S_r once in $(0,t]$
$\mu_i(t)$	Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state
$W_i(t)$	Probability that the server is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
$[s]/[c]$	Symbol for Laplace-Stietjes convolution/Laplace convolution

3. MODEL DEVELOPMENT

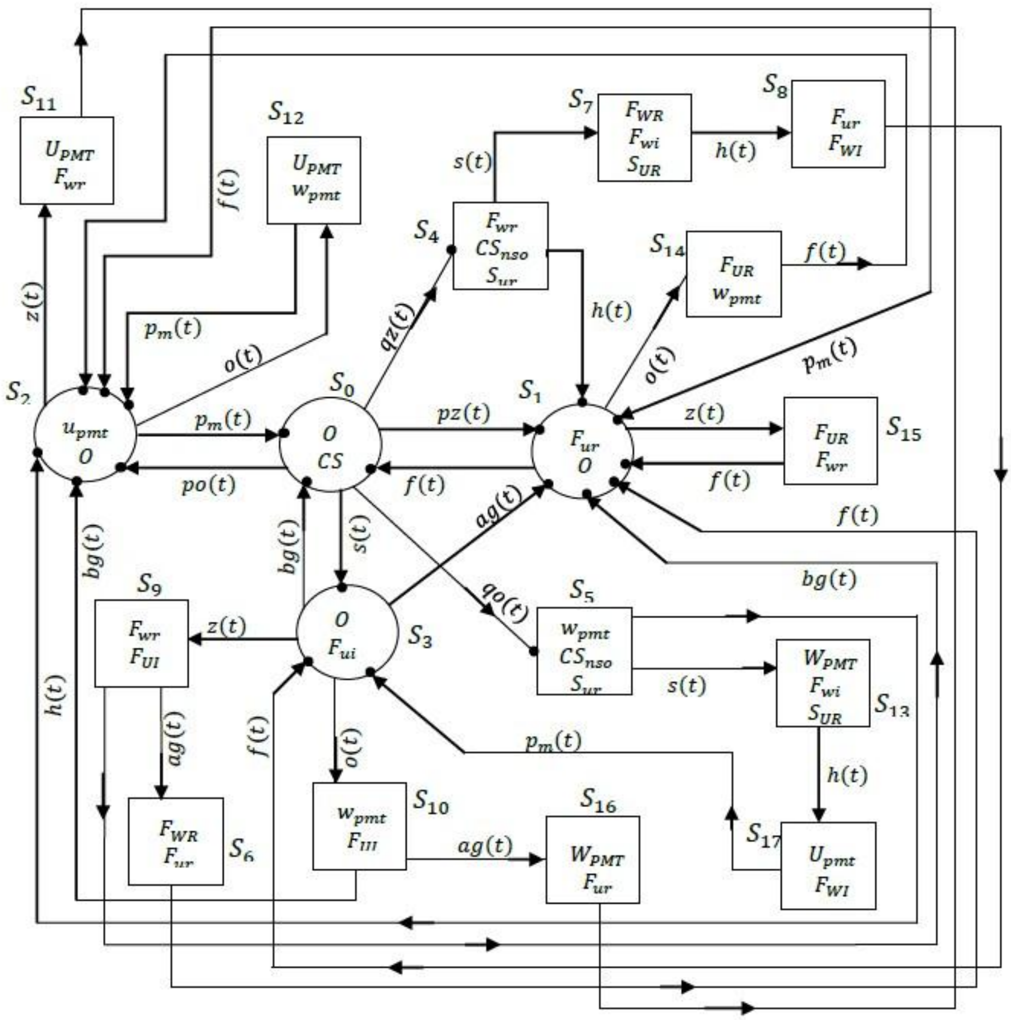

Figure 1: System State Transition Diagram (Bhardwaj & Kaur, 2019)

3.1. State Transition Diagram

Considering all the possible transitions and the re-generative points, a systematic state transition diagram is constructed as shown in figure 1.

3.2. States of System

Regenerative states

$S_0 = (O, CS),\ S_1 = (F_{ur}, O),\ S_2 = (u_{pmt}, O),\ S_3 = (O, F_{ui}),$

$S_4 = (F_{wr}, CS_{nso}, S_{ur}),\ S_5 = (w_{pmt}, CS_{nso}, S_{ur})$

Non-Regenerative states

$S_6 = (F_{WR}, F_{ur}),\quad S_7 = (F_{WR}, F_{wi}, S_{UR}),\quad S_8 = (F_{ur}, F_{WI}),$
$S_9 = (F_{wr}, F_{UI}),\quad S_{10} = (w_{pmt}, F_{UI})$,

$S_{11} = (U_{PMT}, F_{wr}),\ S_{12} = (U_{PMT}, w_{pmt}),\quad S_{13} = (W_{PMT}, F_{wi}, S_{UR}),$
$S_{14} = (F_{UR}, w_{pmt}),\quad S_{15} = (F_{UR}, F_{wr}),\ S_{16} = (W_{PMT}, F_{ur}),\ S_{17} = (u_{pmt}, F_{WI})$

3.3. Transition Probabilities

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt = \tilde{Q}_{ij}(0)$$

$$p_{01} = \int_0^\infty pz(t)\bar{S}(t)\bar{O}(t)dt,\ p_{02} = \int_0^\infty po(t)\bar{Z}(t)\bar{S}(t)dt,\ p_{03} = \int_0^\infty s(t)\bar{Z}(t)\bar{O}(t)dt,$$

$$p_{04} = \int_0^\infty qz(t)\bar{S}(t)\bar{O}(t)dt,\ p_{05} = \int_0^\infty qo(t)\bar{Z}(t)\bar{S}(t)dt,\ p_{10} = \int_0^\infty f(t)\bar{Z}(t)\bar{O}(t)dt,$$

$$p_{1,14} = \int_0^\infty o(t)\bar{F}(t)\bar{Z}(t)dt,\ p_{1,15} = \int_0^\infty z(t)\bar{F}(t)\bar{O}(t)dt,\ p_{20} = \int_0^\infty p_m(t)\bar{O}(t)\bar{Z}(t)dt,$$

$$p_{2,11} = \int_0^\infty z(t)\bar{P}_m(t)\bar{O}(t)dt,\ p_{2,12} = \int_0^\infty o(t)\bar{P}_m(t)\bar{Z}(t)dt,\ p_{30} = \int_0^\infty bg(t)\bar{Z}(t)\bar{O}(t)dt,$$

$$p_{31} = \int_0^\infty ag(t)\bar{Z}(t)\bar{O}(t)dt,\ p_{39} = \int_0^\infty z(t)\bar{G}(t)\bar{O}(t)dt,\ p_{3,10} = \int_0^\infty o(t)\bar{Z}(t)\bar{G}(t)dt,\ p_{41} = \int_0^\infty h(t)\bar{S}(t)dt,$$

$$p_{47} = \int_0^\infty s(t)\bar{H}(t)dt,\quad p_{52} = \int_0^\infty h(t)\bar{S}(t)dt,\quad p_{5,13} = \int_0^\infty s(t)\bar{H}(t)dt,\quad p_{61} = \int_0^\infty f(t)dt,\quad p_{78} = \int_0^\infty h(t)dt,$$

$$p_{83} = \int_0^\infty f(t)dt,\quad p_{91} = \int_0^\infty bg(t)dt,\quad p_{96} = \int_0^\infty ag(t)dt,\quad p_{10,2} = \int_0^\infty bg(t)dt,\quad p_{10,16} = \int_0^\infty ag(t)dt,$$

$$p_{11,1} = \int_0^\infty p_m(t)dt,\quad p_{12,2} = \int_0^\infty p_m(t)dt,\quad p_{13,17} = \int_0^\infty h(t)dt,\quad p_{14,2} = \int_0^\infty f(t)dt,\quad p_{15,1} = \int_0^\infty f(t)dt,$$

$$p_{16,2} = \int_0^\infty f(t)dt,\quad p_{17,3} = \int_0^\infty p_m(t)dt,\quad p_{1,1,15} = p_{1,15}[c]p_{15,1},\quad p_{1,2.14} = p_{1,14}[c]p_{14,2},$$

$$p_{2,1.11} = p_{2,11}[c]p_{11,1},\ p_{2,2.12} = p_{2,12}[c]p_{12,2},\ p_{3,1.9} = p_{39}[c]p_{91},\ p_{3,1.9,6} = p_{39}[c]p_{96}[c]p_{61},$$

$$p_{3,2.10} = p_{3,10}[c]p_{10,2},\ p_{3,2.10,16} = p_{3,10}[c]p_{10,16}[c]p_{16,2},\ p_{4,3.7,8} = p_{47}[c]p_{78}[c]p_{83},$$

$$p_{5,3.13,17} = p_{5,13}[c]p_{13,17}[c]p_{17,3}$$

3.4. Mean Sojourn Times

The mean sojourn time in the state S_i is given by

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt$$

where T denotes the time to system failure.

$\mu_0 = \int_0^\infty \overline{Z}(t)\overline{O}(t)\overline{S}(t)dt$, $\mu_1 = \int_0^\infty \overline{F}(t)\overline{Z}(t)\overline{O}(t)dt$, $\mu_2 = \int_0^\infty \overline{P_m}(t)\overline{Z}(t)\overline{O}(t)dt$, $\mu_3 = \int_0^\infty \overline{G}(t)\overline{Z}(t)\overline{O}(t)dt$,

$\mu_4 = \mu_5 = \int_0^\infty \overline{H}(t)\overline{S}(t)dt$, $\mu_6 = \mu_8 = \mu_{14} = \mu_{15} = \mu_{16} = \int_0^\infty \overline{F}(t)dt$, $\mu_7 = \mu_{13} = \int_0^\infty \overline{H}(t)dt$, $\mu_9 = \mu_{10} = \int_0^\infty \overline{G}(t)dt$,

4. SYSTEM PERFORMANCE MEASURES

4.1. Reliability and Mean time to system failure (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state S_i to a failed state, regarding the failed state as an absorbing state, we have the following recursive relations:

$$\phi_0(t) = Q_{01}(t)[s]\phi_1(t) + Q_{02}(t)[s]\phi_2(t) + Q_{03}(t)[s]\phi_3(t) + Q_{04}(t) + Q_{05}(t)$$

$$\phi_1(t) = Q_{10}(t)[s]\phi_0(t) + Q_{1,14}(t) + Q_{1,15}(t)$$

$$\phi_2(t) = Q_{20}(t)[s]\phi_0(t) + Q_{2,11}(t) + Q_{2,12}(t)$$

$$\phi_3(t) = Q_{30}(t)[s]\phi_0(t) + Q_{31}(t)[s]\phi_1(t) + Q_{39}(t) + Q_{3,10}(t) \quad (1)$$

Taking LST of above relations (1) and solving for $\tilde{\phi}_0(s)$, we get the mean time to system failure

$$MTSF = \lim_{s\to 0} \frac{1-\tilde{\phi}_0(s)}{s} = \frac{\mu_0 + \{p_{01} + p_{03}p_{31}\}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3}{1 - p_{10}\{p_{01} + p_{03}p_{31}\} - p_{02}p_{20} - p_{03}p_{30}} \quad (2)$$

The reliability of system model can be obtained as follows

$$R(t) = L^{-1}\left[\frac{1-\tilde{\phi}_0(s)}{s}\right] \quad (3)$$

5. SPECIAL CASE: WEIBULL DISTRIBUTION

As a special case Weibull density function with common shape parameter and different scale parameters is used as follows:

$$z(t)=\alpha\eta t^{\eta-1}\exp(-\alpha t^{\eta}),\ g(t)=\lambda\eta t^{\eta-1}\exp(-\lambda t^{\eta}),$$
$$f(t)=\beta\eta t^{\eta-1}\exp(-\beta t^{\eta}),\ h(t)=\gamma\eta t^{\eta-1}\exp(-\gamma t^{\eta}),$$
$$s(t)=\mu\eta t^{\eta-1}\exp(-\mu t^{\eta}),\ o(t)=\nu\eta t^{\eta-1}\exp(-\nu t^{\eta}),$$
$$p_m(t)=\omega\eta t^{\eta-1}\exp(-\omega t^{\eta}),\ \text{Where}\ t\ge 0\ \text{and}\ \alpha,\lambda,\beta,\gamma,\mu,\nu,\omega,\eta>0.$$

Table 1: Effect of various parameters on mean time to system failure

Failure rate (α)	MTSF (η=0.5)					
	p=0.4,q=0.6,a=0.3,b=0.7,β=0.6, γ=0.7,λ=0.3,μ=0.1,ν=0.02,ω=0.8	p=0.6,q=0.4	β=0.7	λ=0.5	ν=0.03	ω=1.0
0.01	602.79	753.98	605.67	638.84	423.60	603.68
0.02	423.15	528.22	425.28	445.82	318.56	423.76
0.03	318.24	396.47	319.91	333.53	250.50	318.68
0.04	250.26	311.16	251.62	261.05	203.29	250.60
0.05	203.11	252.04	204.24	210.96	168.91	203.37
Failure rate (α)	η=1.0					
	p=0.4,q=0.6,a=0.3,b=0.7,β=0.6, γ=0.7,λ=0.3,μ=0.1,ν=0.02,ω=0.8	p=0.6,q=0.4	β=0.7	λ=0.5	ν=0.03	ω=1.0
0.01	47.19	59.30	47.29	49.03	35.46	47.22
0.02	35.44	44.52	35.52	36.76	28.41	35.46
0.03	28.39	35.64	28.46	29.39	23.70	28.41
0.04	23.69	29.72	23.75	24.48	20.34	23.70
0.05	20.33	25.49	20.38	20.98	17.81	20.34
Failure rate (α)	η=2.0					
	p=0.4,q=0.6,a=0.3,b=0.7,β=0.6, γ=0.7,λ=0.3,μ=0.1,ν=0.02,ω=0.8	p=0.6,q=0.4	β=0.7	λ=0.5	ν=0.03	ω=1.0
0.01	20.19	25.52	20.23	21.48	15.61	20.20
0.02	15.60	19.74	15.64	16.57	12.84	15.61
0.03	12.83	16.26	12.87	13.61	10.99	12.84
0.04	10.98	13.92	11.02	11.63	9.67	10.99
0.05	9.65	12.25	9.69	10.21	8.66	9.66

6. DISCUSSION ON RESULTS

The behavior of MTSF w.r.t failure rate and varied values of shape parameter are shown in tables 1. The table shows a declining trend in MTSF as the failure rate of the unit (α) increases. We can also observe that the trends reverts as we increase the repair rate β from 0.6 to 0.7, inspection rate λ from 0.3 to 0.5, rate of PM ω from 0.8 to 1.0 while a decreasing trend can be seen when the rate ν changes from 0.8 to 1.0. These table also illustrates that as shape parameter (η) increases, MTSF and hence the system reliability decreases.

The results obtained here advocate the model's applicability in modern systems. When the system is new it has the maximum MTSF but it gradually decreases with system age. Here it is important to indicate that though the deterioration of system performance is unavoidable but it can be slowed down with more frequent preventive maintenance i.e. keeping the maximum operation time limit reasonably less. The numerical results reveals the high dependence of system performance on the failures of the standby and switch. Therefore adequate design and remedial strategies need to be implemented to make such systems more reliable.

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