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A Novel Intra Lot Relational Forward Chain Sampling Plans with an Emphasis on Zero Defective – ILRFChSP (0,C_k)

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Abstract

This research article portrays a novel algorithm to control lot quality through forward chaining of sample inspection within the lot and is termed as Intra Lot Relational Forward Chain Sampling Plans ILRFChSP (0,C_k). This new sampling plan emphasis zero defective in the production process while gives an allowance of chaining index k depending on the number of defectives permitted during sampling inspection. If the products are costly or destructive then during inspection zero defective is emphasized during inspection. However if any C_k defective are found in the sample, then subsequent k random samples are inspected from the same lot and if all the k samples are resulted in zero defective then only the lot is accepted otherwise rejected. This gives a better convex effect on the OC curve and at the same time gives pressure on the producer to maintain the quality of the lot. The new efficiency measures of sampling plans such as operating characteristics function, average sample number, etc., are derived and provided. The Designing Procedure for ILRChSP(0,C_k) is given and the parameters are determined. Tables are constructed to facilitate quality control practitioners to select and implement the sampling plans.

Keywords: Intra Lot, Relational Chain Sampling Plans, Consumer, Producer, AQL, LQL.

Introduction

In Industries, to control quality of the finished or partly finished product of costly items Chain Sampling Plans are recommended. Prof. Dodge(1955) has introduced the ChSP-1 plans and many authors have contributed to Chain Sampling Plans. However ChSP-1 chain sampling plans algorithm chains the results of preceding lots only. Many quality control practitioners are short of independent chain sampling plans focusing on the individual lot. In many production processes if the items are costly or destructive then the current lot should be given more importance not succeeding or preceding lots. Hence to bridge the gap a new research is being carried out in which the sampling protocol chains sample results of only the current lot. Thus a novel Intra Lot Chain Sampling Plans are being developed and the relevant measures are derived.

Devaarul S and Vijila M (2017) have developed a new relational chain sampling plans which has a tremendous application in industries. On the improvement of RChSP(0,i), a new two sided relational chain sampling plans are being developed by Devaarul S and Jothimani K (2020). Devaarul, S and Edna K (2010) have developed a new two sided complete chain sampling plans for costly situations.

But many quality control practitioners argue that if zero defectives are not maintained then chaining should be done within the same lot and not on preceding lots. Hence , a novel Intra lot chain sampling plans are being developed by relating the results of sample with that of number of succeeding samples to be drawn from the same lot.

This novel chain sampling plans give more pressure on the producer and at the same time an advantage through inspection of more samples in the same lot which may lead to acceptance of the lot. This new sampling plan will offset the disadvantages of chain sampling plans due to Dodge (1955) and others. Hence on the improvement of Chain Sampling Plans, the new sampling plans known to be Intra Lot Relational Forward Chain Sampling Plans is being developed for the first time and is termed as ILRFChSP(0,C_k).

In this new sampling protocol, if there is no defective unit more than the first acceptance number during the sample inspection then the lot is accepted immediately. Suppose more than c_1 defectives occur in the first sample, then in the same lot succeeding one sample is randomly taken and the inspection is called forward chaining. If the succeeding sample has zero defective then the lot is accepted otherwise rejected. Suppose more than c_2 defectives occur in the first sample, then in the same lot succeeding two samples are randomly taken and if the succeeding samples has zero defective then the lot is accepted otherwise rejected. In general, more than c_k defectives occur in the first sample, then in the same lot succeeding k samples are randomly taken and if the succeeding samples have zero defective then the lot is accepted otherwise rejected. Hence the inspection protocol relates the number of defectives of current sample with that of future samples within the same lot for making a unique decision. Since the chaining is done within the same lot it is termed as Intra Lot RFChSP. This new Intra Lot F Relational Chain Sampling plan gives more protection to the consumer when the products are costly or destructive. If the quality is not maintained then pressure is given to the producer through the inspection index k.

Comparison between ordinary chain and intra lot relational forward chain sampling plans shows better discrimination of bad lots whenever there is increase in non-conformities. And at the same time there is evident of high probability of acceptance when the quality of the lot is maintained with the prescribed acceptance number. The pattern of number of defectives between each sample is negligible in a steady state process and independency exists between the samples or lots. These plans are more efficient with respect to destructive products and cost of inspection. In Chain Sampling plans due to Dodge(1955), if the lot is not accepted based on occurrence of one defective during the inspection then the entire inspection protocol depends on the previous lots results. There is no pressure on the producer in terms of current lot sampling inspection. Hence to eradicate the disadvantage, a novel Intra Lot Relational Forward Chain Sampling plans are developed.

Many quality control practitioners insist that compromise in quality is not advisable since in many industries defectives may lead to severe loss. Hence non-conformity is to be maintained in the production process so as to sustain in the competitive market. But Zero defective in any production industries is like a mirage in quality control sections. However acceptance criteria with zero acceptance number plans are more emphasized which may give pressure on the producer to maintain the quality of the lot. Dodge and Stephens (1966) found that sampling plans does not discriminate between good and bad lots whenever the acceptance constant is zero. To overcome this drawback, Prof. Dodge recommended Chain sampling plans instead of zero acceptance single sampling plans. But the consumers are not satisfied even if the producer maintains the specified quality level due to chaining of the past lots result. Hence to protect the producer and at the same time to satisfy the consumer, Intra Lot Relational Chain Forward Sampling Plans are being developed by the authors.

Rebecca Jebaseeli and Jemmy Joyce (2013) have studied chain sampling plan for variable fraction non-conformities. Balamurali S and M. Usha (2013) have designed optimal Variables Chain Sampling Plan by Minimizing the Average Sample Number. Vijayaraghavan R and K. M. Sakthivel (2011) have used Bayesian methodology in studying Chain sampling inspection plans. Latha M and Jeyabharathi S (2014) have contributed towards Bayesian Chain Sampling Plans. Clark C.R (1955) has studied family of OC curves for chain sampling plans. Frishman & Fred (1960) have developed an extended chain sampling plans. Dodge H.F. and Stephens K.S. (1966) have contributed towards evaluation of OC of Chain sampling plans through Markov Chain approach.

Soundararajan. V (1978) has given procedure and tables for construction and selection of Chain sampling plans. Suresh K.K. and Devaarul . S (2002) have developed Mixed Sampling Plans with Chain Sampling as attribute plan. Devaarul S and Rebecca Edna (2012) have derived a new class of Complete Chain Sampling Plans (CChsp(0,1)). Jothikumar, J. and Raju, C. (1996) have developed Two stage chain sampling plans ChSP-(0,2) and ChSP-(1,2). Even though several authors have extended their research work by using Dodge’s chain sampling, literature is scarce in case of Intra Lot Relational Chain Sampling Plans. Hence an attempt has been made to design and develop ILRChsp(0, C_k) plans. The second section consists of algorithm for Intra Lot Relational Chain Sampling Plans. The related efficiency measures are derived and are given in section 3. Designing procedure is given in section 4. Constructions of tables and illustrations are given in the remaining sections with necessary theorems and proof.

Conditions for Application

1. The Production Process should be steady, stable and continuous.
2. The size of the Lot should be very large.
3. The chaining index k can be fixed on an agreement between producer and consumer.

2. Algorithm for Intra Lot Relational Forward Chain Sampling Plans

Determine the sample size and acceptance constant c_k by utilizing OC function.

Stage I

1. Draw a random sample of size n. Let it be first trial.
2. Count the number of defectives. Let it be d.
3. If $d \leq c_1$, then Accept the Lot.
4. If $d > c_k$, then Reject the Lot.
5. If $c_1+1 < d \leq c_2$, then go to II stage of inspection for forward chaining in the same lot
6. If $c_2+1 < d \leq c_3$, then go to III stage of inspection for forward chaining in the same lot.
7. If $c_{k-1}+1 < d \leq c_k$, then go to k^{th} stage of inspection for forward chaining in the same lot.

Stage II

8. Draw another sample of size n.
9. Count the number of defectives. Let it be d.
10. If d = 0, in the sample then accept the Lot. Otherwise reject it.

Stage III

11. Draw two consecutive samples of each size n.
12. Count the number of defectives in each sample. Let it be d.
13. If d = 0, in all the three samples then accept the Lot. Otherwise reject it.

.....In general,

Stage K

14. Draw k-1 such consecutive samples of each with size n.
15. Count the number of defectives in each sample. Let it be d.
16. If d = 0, in all the 'k' samples then accept the Lot. Otherwise reject it.

3. Theorem:

The Operating Characteristics function of Intra lot-RFChSP plan is given below

$$P_a(p) = P_n(d \le c_1) + P_n(c_1 + 1 < d \le c_2)[P_n(d=0)] + P_n(c_2 + 1 < d \le c_3))[P_n(d=0)]^2 + \ldots + \ldots + P_n(c_{k-1} + 1 < d \le c_k))[P_n(d=0)]^{k-1}$$

In general,

$$P_a(p) = P_n(d \le c_1) + \sum_{x:2}^{k} P_n(c_{x-1} + 1 < d \le c_x))[P_n(d=0)]^{x-1}$$

Proof:

Let n = Sample Size for each instance

d = Number of defectives in the sample

C_1= first acceptance number.

C_2 = second stage acceptance number.

C_k= k^{th} stage acceptance number.

$P_n(d \le c)$ = Utmost c Number of defectives in the sample.

$P_n(d=0)$ = Exactly zero defective in the sample.

The Lot may be accepted in the following cases:

Case(i). In the first trial, the lot will be accepted if the number of defective $d \le C_1$

The corresponding probability of acceptance is $P_n(d \leq C_1)$.

Case (ii). Suppose in the first trial if, $C_1 + 1 < d \leq C_2$ then the lot will be accepted in the following another trial with (d=0).

The corresponding probability of acceptance is $P_n(C_1 + 1 < d \leq C_2). [P_n (d=0)]]$

Case (iii) Suppose in the first trial if $C_2 + 1 < d \leq C_3$ then the lot will be accepted in the following another two trials with (d=0).

The corresponding probability of acceptance is $P_n(C_2 + 1 < d \leq C_3). [P_n (d=0)]]^2$

……

In general (k) Suppose in the first trial if (d=k) then the lot will be accepted in the following another k-1 trials with (d=0)

Case(i), Case(ii), ...etc., are all mutually exclusive events. Therefore by addition theorem on Probability we get,

$$P_a (p) = P_n (d \leq c_1) + P_n (c_1 + 1 <d \leq c_2)[P_n (d=0)] + P_n (c_2 + 1 <d \leq c_3))[P_n (d=0)]^{2} + \ldots..+ \ldots+ P_n (c_{k-1} + 1 <d \leq c_k))[P_n (d=0)]^{k-1}$$

In general,

$$P_a(p) = P_n (d \leq c_1) + \sum_{x:2}^{k} P_n (c_{x-1} + 1 < d \leq c_x))[P_n (d=0)]^{x-1}$$

Note: For Costly products and practical reasons it is advisable to have smaller index k. Since the chaining is done within the same lot, it is termed as Intra Lot Forward Chain Sampling Plans.

1. AVERAGE SAMPLE NUMBER

ASN is defined as the expected number of units required to make a unique decision on the lots.

$$ASN = n.P_n (d \leq c_1) + (n+n) P_n (c_1 + 1 <d \leq c_2)[P_n (d=0)] + (n+n+n) P_n (c_2 + 1 <d \leq c_3))[P_n (d=0)]^{2} + \ldots..+ \ldots+ (n+n+n\ldots(k)\text{times } n)P_n (c_{k-1} + 1 <d \leq c_k))[P_n (d=0)]^{k-1}$$

$$ASN = n P_n (d = 0) + (n+n)P_n (d=1)[P_n (d=0)] + (n+n+n) P_n [d = 2][P_n (d = 0)]^2 +\ldots\ldots+ (n+n+n\ldots(k)\text{times } n)P_n (d = k)[P_n (d = 0)]^{k-1}$$

$$ASN = \sum_0^{k-1}((x + 1) * n) \quad P_n(Ck - 1 < d \leq Ck)[P_n(d = 0)]^x$$

Proof:

Case (i)

During the sampling inspection if the lot is accepted based on the first trial then the expected number of units inspected is

$$n\, P_n(d=0).$$

Case (ii)

Suppose if the lot is not accepted during the first trial, then another sample is to be taken. The corresponding expected number of units inspected is

$$(n+n)P_n\,(d=1)[P_n\,(d=0)]$$

….

Case(k)

If the number of defective is k units in the first trial, then k such samples are to be taken for making a unique decision. The expected number of units inspected is

$$(n+n+n\ldots(k)\text{times } n)P_n\,(d = k)[P_n\,(d = 0)]^{k-1}$$

Case (i), Case(ii), etc., are mutually exclusive cases. Hence we get

$$ASN = n\, P_n\,(d = 0) + (n+n)P_n\,(d=1)[P_n\,(d=0)] + (n+n+n)\, P_n\,[d = 2][P_n\,(d = 0)]^2 + \ldots\ldots + (n+n+n\ldots(k)\text{times } n)P_n\,(d = k)[P_n\,(d = 0)]^k$$

$$ASN = \sum_0^{k-1}((x+1)*n) \quad P_n(Ck-1 < d \le Ck)[P_n(d = 0)]^x$$

Hence the Proof.

Note: It is found that whenever the quality deteriorates, the sample size increases and hence the cost of inspection may be higher. This may pressurizes the producer to maintain the quality products in order to avoid additional expenses.

2. AVERAGE OUTGOING QUALITY

$$AOQ = (p)\,[\,P_n\,(d \le c_1\,) + \sum_{x:2}^{k} P_n\,(c_{x-1} + 1 < d \le c_x)\,)[P_n\,(d=0)]^{x-1}$$

Designing ILChSP(0,k) plans indexed through AQL.

Step 1: Determine the process average from the production process.

Let it be p_1= AQL

Step 2: Let α be the producer risk.

Step 3: Determine the parameters of ILRFChSP such that

$$P_a(p_1) \geq 1 - \alpha$$

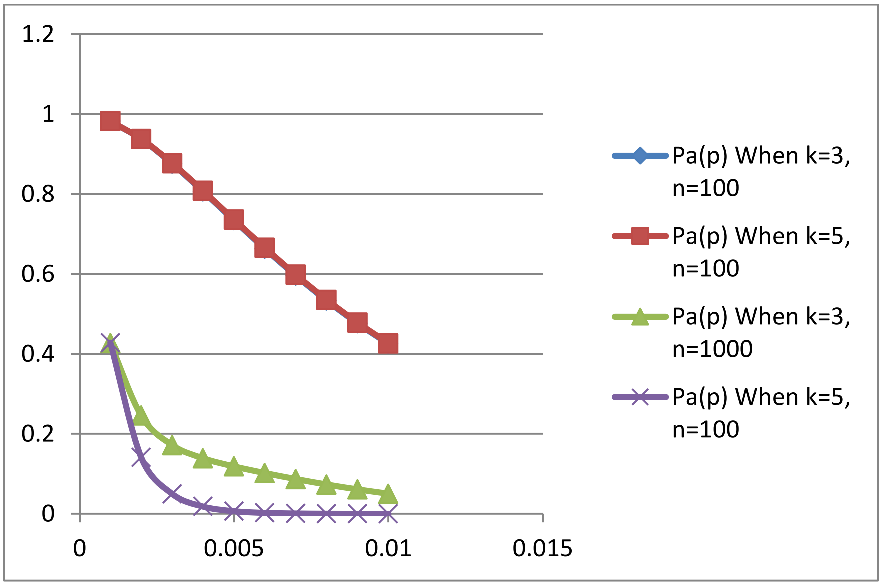
Step 4:

$$P_a(p) = P_n(d \leq c_1) + \sum_{x:2}^{k} P_n(c_{x-1} + 1 < d \leq c_x)\,)[P_n(d=0)]^{x-1} \geq 1-\alpha.$$

Step 5: The required tables are constructed for various values of AQL.

Tables are being constructed using standard quality levels for selection of ILRChSP indexed through AQL, LQL, IQL, Mintan method, crossover point, through MAPD, etc.

Figure1 : Comparison of OC values for various sample sizes and chain index k.



Interpretation: From the figure one can interpret that whenever the chaining index k is increased then there is better shouldering effect on the OC curve. Whereas if the chaining index k is reduced then the OC curve tends towards concave downwards.

Table 1: Values of Probability of acceptance. For various values of chaining index k=3 with $c_1=0, c_2=1, c_3=2$ and for various values of chaining index k=5 with $c_1=0, c_2=1, c_3=2$, $c_4=3$, $c_5=4$

AQL p	Pa(p) When k=3, n=100	Pa(p) When k=5, n=100
0.001	0.982226	0.982329
0.002	0.937316	0.937826
0.003	0.876009	0.877069
0.004	0.806412	0.807952
0.005	0.734252	0.736092
0.006	0.663329	0.665269
0.007	0.595982	0.597859
0.008	0.533504	0.535207
0.009	0.476468	0.477941
0.01	0.424977	0.426204

Note: If the chaining index is three, then the there will be three acceptance numbers. From the table one can find that if chaining index k increased then there is small increase in the Probability of acceptance.

Table 2: Comparison of OC values with k and n. For various values of chaining index k=5 with $c_1=0, c_2=1, c_3=2, c_4=3, c_5=4$

AQL p	Pa(p) When k=3, n=100	Pa(p) When k=5, n=100	Pa(p) When k=3, n=1000	Pa(p) When k=5, n=1000
0.001	0.982226	0.982329	0.42664	0.427867
0.002	0.937316	0.937826	0.245498	0.140758
0.003	0.876009	0.877069	0.171562	0.049962
0.004	0.806412	0.807952	0.138648	0.018195
0.005	0.734252	0.736092	0.118219	0.006656
0.006	0.663329	0.665269	0.101614	0.002434
0.007	0.595982	0.597859	0.086609	0.00089
0.008	0.533504	0.535207	0.072899	0.000325
0.009	0.476468	0.477941	0.060603	0.000118
0.01	0.424977	0.426204	0.04983	0.000011

Illustration: A production process is known to have incoming lot quality level p = 0.001. Determine the sample size when the chaining index is k=5 at 98% Probability of acceptance for a ILRFChSP.

Solution:

Since the chaining index is k=5. Let us have $\mathbf{c_1=0, c_2=1, c_3=2, c_4=3, c_5=4}$

It is given that p_1= AQL = 0.001

From Table(1), the required sampling size is n=100.

Summary

In this paper a novel algorithm is developed and the corresponding efficiency measures are derived. It is found that when the chaining index k is increased for the constant sample size, the probability of acceptance increases. This is an indication of reduced risk for the producer. However if the lot quality deteriorates then the probability of acceptance decreases with an increase in the sample size. Hence consumer can trust the newly developed sampling plan and can be implemented in modern industries.

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