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Oblong Difference Mean Prime Labeling of Some Path Graphs

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Abstract

The labeling of a graph, we mean assign some integers to the vertices or edges (or both) of the graph. In this paper we developed a new kind of labeling, called oblong difference mean prime labeling. Here the vertices of the graph are labeled with oblong numbers and the edges are labeled with mean of the absolute difference of the end vertex values. Here the g c d of a vertex of degree greater than one is defined as the g c d of the labels of the incident edges. If the g c d of each vertex of degree greater than one is 1, then the graph admits oblong difference mean prime labeling. Here we characterize some path related graphs for oblong difference mean prime labeling.

Keywords : Graph labeling, oblong numbers, prime graphs, prime labeling, path graph.

1. INTRODUCTION

In this paper we deal with graphs that are simple, finite and undirected. The symbol V and E denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge

set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.
A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3],[4],[5],[6] and [7]. Some basic concepts are taken from Frank Harary [1]. In this paper we investigated the oblong difference mean prime labeling of some path graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common divisor of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form n(n+1).The oblong numbers are 2, 6, 12, 20, --------------.

2. MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

f : V(G) → {2,6,12,20 ,------------------,p(p+1)} by f(v_i) = $i(i+1)$, for every i from 1 to p and define a 1-1 mapping f^*_{odmpl} : E(G) → set of natural numbers N by $f^*_{odmpl}(uv)$ = | $\frac{f(u)-f(v)}{2}$ |. The induced function f^*_{odmpl} is said to be an oblong difference mean prime labeling, if the g c d of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong difference mean prime labeling is called an oblong difference mean prime graph.

Theorem: 2.1 The path P_n admits oblong difference mean prime labeling.

Proof : Let G = P_n and let v_1,v_2,----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = n-1.

Define a function f : V → {2,6,12,-----------------,$n(n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,n.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\, v_{i+1})$ = (i+1), i = 1,2,------------,n-1

Clearly f^*_{odmpl} is an injection.

g c d of (v_{i+1}) = g c d of { $f^*_{odmpl}(v_i\, v_{i+1})$, $f^*_{odmpl}(v_{i+1}\, v_{i+2})$ }

= g c d of { (i+1) , (i+2) }

= 1, i = 1,2,---------------,n-2

So, g c d of each vertex of degree greater than one is 1.

Hence P_n , admits oblong difference mean prime labeling.

Theorem 2.2 Two tuple graph of path P_n, admits oblong difference mean prime labeling, when n is not a multiple of 3.

Proof : Let G = $T^2(P_n)$ and let v_1, v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-2.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_{2i-1}\ v_{2i})$ = 2i, i = 1,2,------------,n

$f^*_{odmpl}(v_{2i-1}\ v_{2i+1})$ = 4i+1, i = 1,2,------------,n-1

$f^*_{odmpl}(v_{2i}\ v_{2i+2})$ = 4i+3, i = 1,2,------------,n-1

Clearly f^*_{odmpl} is an injection.

g c d of (v_{2i+1}) = g c d of { $f^*_{odmpl}(v_{2i+2}\ v_{2i+1})$, $f^*_{odmpl}(v_{2i+1}\ v_{2i+3})$ }

= g c d of { 2i+2,4i+5 }

= 1, i = 1,2,------------,n-2

g c d of (v_{2i+2}) = g c d of { $f^*_{odmpl}(v_{2i}\ v_{2i+2})$, $f^*_{odmpl}(v_{2i+1}\ v_{2i+2})$ }

= g c d of { 2i+2,4i+3 } = 1, i = 1,2,------------,n-2

g c d of (v_1) = g c d of {2, 5} =1.

g c d of (v_2) = g c d of {2,7} =1.

g c d of (v_{2n-1}) = g c d of { $f^*_{odmpl}(v_{2n-3}\ v_{2n-1})$, $f^*_{odmpl}(v_{2n-1}\ v_{2n})$ }

= g c d of { 2n, 4n-3} = 1.

g c d of (v_{2n}) = g c d of { $f^*_{odmpl}(v_{2n-2}\ v_{2n})$, $f^*_{odmpl}(v_{2n-1}\ v_{2n})$ }

= g c d of {2n, 2n-1} = 1.

So, g c d of each vertex of degree greater than one is 1.

Hence $T^2(P_n)$,admits oblong difference mean prime labeling.

Theorem 2.3 P_n^2 admits oblong difference mean prime labeling, when n ≡ 0(mod4)

Proof : Let G = P_n^2 and let v_1, v_2,----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = 2n-3.

Define a function f : V → {2,6,12,-----------------,$n(n+1)$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,n.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\ v_{i+1})$	= (i+1),	i = 1,2,------------,n-1
$f^*_{odmpl}(v_i\ v_{n+1-i})$	= $\frac{n^2+3n-2ni-4i+2}{2}$,	i = 1,2,------------,$\frac{n-2}{2}$
$f^*_{odmpl}(v_{i+1}\ v_{n+1-i})$	= $\frac{n^2+3n-2ni-6i}{2}$,	i = 1,2,------------,$\frac{n-2}{2}$

Clearly f^*_{odmpl} is an injection.

g c d of (v_1) = g c d of $\{f^*_{odmpl}(v_1\ v_2), f^*_{odmpl}(v_1\ v_n)\}$

= g c d of { 2, $\frac{n^2+n-2}{2}$ }= 1.

g c d of (v_n) = g c d of { $f^*_{odmpl}(v_1\ v_n)$, $f^*_{odmpl}(v_2\ v_n)$ }

= g c d of {$\frac{n^2+n-2}{2}$, $\frac{n^2+n-6}{2}$ }= 1

g c d of (v_{i+1}) = 1, i = 1,2,------------,n-2

So, g c d of each vertex of degree greater than one is 1.

Hence P_n^2 ,admits oblong difference mean prime labeling.

Theorem 2.4 Total graph of path P_n, admits oblong difference mean prime labeling when n is even.

Proof: Let G = $T(P_n)$ and let v_1, v_2,----------------,v_{2n-1} are the vertices of G.

Here $|V(G)|$ = 2n-1 and $|E(G)|$ = 4n-5.

Define a function f : V → {2,6,12,-----------------,$(2n-1)2n$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n-1.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\ v_{i+1})$ = (i+1), i = 1,2,------------,2n-2

$f^*_{odmpl}(v_1\ v_{2n-1})$ = $2n^2$-n-1

$f^*_{odmpl}(v_{i+1}\ v_{2n-i})$ = $2n^2$+n-2ni-2i-1, i = 1,2,------------,n-2

$f^*_{odmpl}(v_{i+1}\ v_{2n-i})$ = $2n^2$-n-2ni-i-1, i = 1,2,------------,n-2

Clearly f^*_{odmpl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-3

g c d of (v_1) = g c d of $\{f^*_{odmpl}(v_1 v_2), f^*_{odmpl}(v_1 v_{2n-1})\}$

= g c d of { 2, $2n^2$-n-1} = 1.

g c d of (v_{2n-1}) = g c d of { $f^*_{odmpl}(v_{2n-2}\ v_{2n-1})$, $f^*_{odmpl}(v_{2n-1}\ v_1)$ }

= g c d of { 2n-1, $2n^2$-n-1}= g c d of { 2n-1, 2n-2}= 1.

So, g c d of each vertex of degree greater than one is 1.

Hence $T(p_n)$, admits oblong difference mean prime labeling.

Theorem 2.5 Duplicate graph of path P_n, admits oblong difference mean prime labeling.

Proof: Let G = D(P_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here $|V(G)|$ = 2n and $|E(G)|$ = 2n-2.

Define a function f : V → {2,6,12,-----------------,$(2n+1)2n$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\ v_{i+1})$ = i+1, i = 1,2,------------,n-1

$f^*_{odmpl}(v_{n+i}\, v_{n+i+1})$ = n+i+1, i = 1,2,------------,n-1

Clearly f^*_{odmpl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,n-2.

g c d of (v_{n+i+1}) = 1, i = 1,2,------------,n-2.

So, g c d of each vertex of degree greater than one is 1.

Hence D(P_n) ,admits oblong difference mean prime labeling.

Theorem 2.6 Strong duplicate graph of path P_n, admits oblong difference mean prime labeling, when n is even.

Proof: Let G = SD(P_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-2.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,2n.

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\, v_{i+1})$ = i+1, i = 1,2,------------,2n-1

$f^*_{odmpl}(v_i\, v_{2n+1-i})$ = $2n^2$+3n-2ni-2i+1, i = 1,2,------------,n-1

Clearly f^*_{odmpl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

g c d of (v_1) = g c d of {$f^*_{odmpl}(v_1 v_2)$, $f^*_{odmpl}(v_1 v_{2n})$}

= g c d of {2, $2n^2$+n-1} =1

g c d of (v_{2n}) = g c d of {$f^*_{odmpl}(v_{2n}\, v_{2n-1})$, $f^*_{odmpl}(v_{2n}\, v_1)$}

= g c d of { 2n, $2n^2$ +n-1}= g c d of {2n, n-1}

= g c d of { 2, n-1}= 1 , since n is even.

So, g c d of each vertex of degree greater than one is 1.

Hence SD(P_n) ,admits oblong difference mean prime labeling.

Theorem 2.7 Strong shadow graph of path P_n, admits oblong difference mean prime labeling when n is even.

Proof: Let G = $SD_2(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 5n-4.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

$$f(v_i) = i(i+1) \ , i = 1,2,\text{------},2n.$$

For the vertex labeling f, the induced edge labeling f^*_{odmpl} is defined as follows

$f^*_{odmpl}(v_i\, v_{i+1})$ = i+1, i = 1,2,------------,2n-1.

$f^*_{odmpl}(v_i\, v_{2n-i+1})$ = $2n^2+3n-2ni-2i+1$, i = 1,2,------------,n-1

$f^*_{odmpl}(v_i\, v_{2n-i})$ = $2n^2+n-2ni-2i$, i = 1,2,------------,n-1

$f^*_{odmpl}(v_{i+1}\, v_{2n-i+1})$ = $2n^2+3n-2ni-3i$, i = 1,2,------------,n-1

Clearly f^*_{odmpl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

g c d of (v_1) = g c d of {$f^*_{odmpl}(v_1 v_2)$, $f^*_{odmpl}(v_1 v_{2n})$} = g c d of {2, $2n^2$+n-1} =1

g c d of (v_{2n}) = g c d of {$f^*_{odmpl}(v_{2n}\, v_{2n-1})$, $f^*_{odmpl}(v_1\, v_{2n})$}

= g c d of {2n, $2n^2$+n-1}= g c d of {2n, n-1}

= g c d of {n-1,n+1}= g c d of {2, n-1}= 1 .

So, g c d of each vertex of degree greater than one is 1.

Hence $SD_2(P_n)$,admits oblong difference mean prime labeling.

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