Study of Reliability Parameters of a System Having Two Main Units and a Redundant Associate Unit

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Abstract

In order to increase the reliability of any system, the concept of redundancy and maintenance are often incorporated. The system performs better if the combination of these two techniques is resorted to. We know that fault occurrence in any system is inevitable because of stochastic characteristics of the system and operating environment. However, the magnitude of the faults can be controlled and their occurrence can be delayed by employing suitable mechanisms as envisaged through system evaluation studies. Arnold [1], Kumar and Kapoor [2] investigated a similar reliability model after incorporating the concept of coverage in the context of Computer Systems. Rander et al [4], studied the cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby unit. In the present paper, we analyze a system having two separate main units along with associate units taken in standby mode because of its vulnerability to failures. In the end, the result is interpreted with the help of table and graphs which is prepared with the help of ‘C’ program.

Keywords: Regenerative Point, MTSF, Availability, Busy Period
1. SYSTEM DESCRIPTION ABOUT THE MODEL:

The system consists of three units namely two main units M and B along with associate unit A. An additional associate unit A is also dispensed with owing to its repeated failure. The system stops operation when either M fails or B fails or both of A fail. The associate units are dependents upon main units for functioning and the system is operable when both the main units and either of the associate units are in operable. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end the results are elaborated with the help of graphs and expected profit is also calculated.

Figure: State transition diagram
2. ASSUMPTIONS USED IN THE MODEL:

a. The system consists of two main units and associate units in standby configuration.
b. The associate unit A works with the help of main units M and B.
c. There is a single repairman which repairs the failed units on priority basis.
d. After random period of time the whole system goes to preventive maintenance.
e. All units work as new after repair.
f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.
g. Switching devices are perfect and instantaneous.

3. SYMBOLS AND NOTATIONS:

- $p_{i,j}$ = Transition probabilities from $S_i$ to $S_j$
- $\mu_i$ = Mean sojourn time at time $t$
- $E_0$ = State of the system at epoch $t=0$
- $E$ = Set of regenerative states $S_0, S_1, S_2, S_3$
- $q_{i,j}(t)$ = Probability density function of transition time from $S_i$ to $S_j$
- $Q_{i,j}(t)$ = Cumulative distribution function of transition time from $S_i$ to $S_j$
- $\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$
- $\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$
- $B_i(t)$ = Repairman is busy in the repair at time $t$ / $E_0 = S_i \in E$
- $r_1/r_2/r_3$ = Constant repair rate of Main unit M /associate Unit B /associate Unit C
- $\alpha / \beta / \gamma$ = Failure rate of Main unit M / Unit A / Unit B
- $g_1 / g_2 / g_3$ = Probability density function of repair time of Main unit M / associate Unit A / Unit B
- $G_i / G_2 / G_3$ = Cumulative distribution function of repair time of Main unit M / Unit A / Unit B
- $a(t)$ = Probability density function of preventive maintenance
- $b(t)$ = Probability density function of preventive maintenance completion time
- $\bar{A}(t)$ = Cumulative distribution functions of preventive maintenance
\( \bar{B}(t) \) = Cumulative distribution functions of preventive maintenance completion time.

\( \mathcal{S} \) = Symbol for Laplace -stieltjes transforms.

\( \mathcal{C} \) = Symbol for Laplace-convolution.

4. **SYMBOLS USED FOR STATES OF THE SYSTEM:**

- \( M_0 / M_g / M_r \) -- Main unit ‘M’ under operation/good and non-operative mode/ repair mode
- \( A_0 / A_g / A_r \) -- Associate Unit ‘A’ under operation/repair/good/standby and non-operative mode
- \( B_0 / B_g / B_r \) -- Associate Unit ‘B’ under operation/repair/ good and non-operative mode

- \( P.M. \) -- System under preventive maintenance.
- \( S.D. \) -- System under shut down mode.

**Up states:** \( S_0 = (M_0, A_0, B_0, A_r); S_2 = (M_0, A_0, A_r, B_0) \)

**Down States:** \( S_1 = (M_r, A_g, B_g, A_g); S_3 = (M_g, A_g, B_g, A_g); S_4 = (S.D.); S_5 = (P.M.) \)

5. **TRANSITION PROBABILITIES:**

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. \( Q_{01}(t) = \int_0^t \alpha e^{-\gamma \alpha \beta \gamma} \bar{A}(t) dt \)
2. \( Q_{02}(t) = \int_0^t \beta e^{-\gamma \alpha \beta \gamma} \bar{A}(t) dt \)
3. \( Q_{03}(t) = \int_0^t \gamma e^{-\gamma \alpha \beta \gamma} \bar{A}(t) dt \)
4. \( Q_{04}(t) = \int_0^t g_1(t) dt \)
5. \( Q_{20}(t) = \int_0^t \alpha e^{-\gamma \alpha \beta \gamma} g_2(t) dt \)
6. \( Q_{24}(t) = \int_0^t (\alpha + \beta + \gamma)e^{-\gamma \alpha \beta \gamma} \bar{G}(t) dt \)
7. \( Q_{30}(t) = \int_0^t g_3(t) dt \)
8. \( Q_{40}(t) = \int_0^t g_4(t) dt \)
9. \( Q_{50}(t) = \int_0^t b(t) dt \)
10. \( Q_{05}(t) = \int_0^t \alpha e^{-\gamma \alpha \beta \gamma} dt \)
Where \( x_1 = \alpha + \beta + \gamma \), Now letting \( t \to \infty \), we get \( \lim_{t \to \infty} Q_{ij}(t) = p_{ij} \)

11. \( p_{01} = \int_0^\infty \alpha e^{-(\alpha+\beta+\gamma)t} A(t) \, dt = \frac{\alpha}{x_1} [1 - a^*(x_1)] \),

12. \( p_{02} = \int_0^\infty \beta e^{-(\alpha+\beta+\gamma)t} A(t) \, dt = \frac{\beta}{x_1} [1 - a^*(x_1)] \),

13. \( p_{03} = \int_0^\infty \gamma e^{-(\alpha+\beta+\gamma)t} A(t) \, dt = \frac{\gamma}{x_1} [1 - a^*(x_1)] \),

14. \( p_{05} = \int_0^\infty a(t) e^{-(\alpha+\beta+\gamma)t} \, dt = a^*(x_1) \)

15. \( p_{10} = \int_0^\infty g_1(t) \, dt = 1 \),

16. \( p_{20} = \int_0^\infty e^{-(\alpha+\beta+\gamma)t} g_2(t) \, dt = g_2^*(x_1) \),

17. \( p_{24} = \int_0^\infty (\alpha + \beta + \gamma) e^{-(\alpha+\beta+\gamma)t} g_2(t) \, dt = 1 - g_2^*(x_1) \),

18. \( p_{30}(t) = \int_0^\infty g_3(t) \, dt = 1 \)

19. \( p_{40}(t) = \int_0^\infty g_4(t) \, dt = 1 \),

20. \( p_{50}(t) = \int_0^\infty b(t) \, dt = 1 \)

21. \( p_{10} = p_{30} = p_{40} = p_{50} = 1 \) \([6.1-6.21]\)

It is easy to see that

\[ p_{01} + p_{02} + p_{03} + p_{05} = 1 \quad p_{20} + p_{24} = 1 \) \([6.22-6.23]\)

And mean sojourn time are given by

24. \( \mu_0 = \frac{1}{x_1} [1 - a^*(x_1)] \),

25. \( \mu_i = \int_0^\infty g_i(t) \, dt \).
26. $\mu_2 = \frac{1}{x_1} [1 - g^*_2(x_1)]$

27. $\mu_3 = \int_0^\infty G_3(t)dt$

28. $\mu_4 = \int_0^\infty G_4(t)dt$

29. $\mu_5 = \int_0^\infty B(t)dt$  \[6.24-6.29\]

We note that the Laplace-stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$ i.e. $\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t) dt = L\{Q_{ij}(t)\} = q_{ij}^*(s)$  \[6.30\]

31. $\tilde{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+x_1)t} A(t) dt = \frac{\alpha}{s + x_1} [1 - a^*(s + x_1)]$

32. $\tilde{Q}_{02}(s) = \int_0^\infty \beta e^{-(s+x_1)t} \bar{A}(t) dt = \frac{\beta}{s + x_1} [1 - a^*(s + x_1)]$

33. $\tilde{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+x_1)t} \bar{A}(t) dt = \frac{\gamma}{s + x_1} [1 - a^*(s + x_1)]$

34. $\tilde{Q}_{05}(s) = \int_0^\infty e^{-(s+x_1)t} a(t) dt = a^*(s + x_1)$

35. $\tilde{Q}_{10}(s) = \int_0^\infty e^{-st} g_1(t) dt = g_1^*(s)$

36. $\tilde{Q}_{20}(s) = \int_0^\infty e^{-(s+x_1)t} g_2(t) dt = g_2^*(s + x_1)$

37. $\tilde{Q}_{24}(s) = \int_0^\infty x_1 e^{-(s+x_1)t} \bar{G}_2(t) dt = \frac{x_1}{s + x_1} [1 - g^*_2(s + x_1)]$

38. $\tilde{Q}_{30}(s) = \int_0^\infty e^{-st} g_3(t) dt = g_3^*(s)$

39. $\tilde{Q}_{40}(s) = \int_0^\infty e^{-st} g_4(t) dt = g_4^*(s)$

40. $\tilde{Q}_{50}(s) = \int_0^\infty e^{-st} b(t) dt = b^*(s)$  \[6.31-6.40\]
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We define $m_{i,j}$ as follows:

$$m_{i,j} = -\left(\frac{d}{ds} \tilde{Q}_{i,j}(s)\right)_{s=0} = -Q'_{i,j}(0)$$  [6.41]

It can to show that $m_{01} + m_{02} + m_{03} + m_{05} = \mu_0; m_{20} + m_{24} = \mu_2$  [6.42-6.43]

Where $\alpha + \beta + \gamma = x_1$

6. **MEAN TIME TO SYSTEM FAILURE**-

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t)$$
$$\pi_2(t) = Q_{20}(t)$$

$$\pi_0(t) + \pi_2(t) = Q_{03}(t) + Q_{05}(t)$$

$$\pi_0(t) + \pi_2(t) = Q_{24}(t)$$  [7.1-7.2]

Taking Laplace-stieltjes transform of above equations and writing in matrix form.

We get

$$\begin{bmatrix} 1 & -\tilde{Q}_{02} \\ -\tilde{Q}_{20} & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{01} + \tilde{Q}_{03} + \tilde{Q}_{05} \\ \tilde{Q}_{24} \end{bmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{02} \\ -\tilde{Q}_{20} & 1 \end{vmatrix} = 1 - \tilde{Q}_{20}\tilde{Q}_{02}$$  [7.3]

And $N_1(s) = \begin{vmatrix} \tilde{Q}_{01} + \tilde{Q}_{03} + \tilde{Q}_{05} - \tilde{Q}_{02} \\ \tilde{Q}_{24} \end{vmatrix} = (\tilde{Q}_{01} + \tilde{Q}_{03} + \tilde{Q}_{05} + \tilde{Q}_{02}\tilde{Q}_{24})$

Now letting $s \to 0$ we get

$$D_1(0) = 1 - p_{02}p_{20}$$  [7.4]

The mean time to system failure when the system starts from the state $S_0$ is given by

$$MTSF = E(T) = -\left(\frac{d}{ds} \tilde{\pi}_0(s)\right)_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$$  [7.6]

To obtain the numerator of the above equation, we collect the coefficients of relevant of $m_{i,j}$ in $D_1'(0) - N_1'(0)$.

Coeff. of $(m_{01} = m_{02} = m_{03} = m_{05}) = 1$
Coefficient of \( m_{20} \) = \( p_{02} \) \[7.7-7.8\]

From equation [7.6]

\[
MTSF = E(T) = \left[\frac{d}{ds} \hat{F}_o(s)\right]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}
\]

\[
= \frac{\mu_0 + \mu_2 P_{02}}{1 - P_{02} P_{20}}
\]

\[7.9\]

7. **AVAILABILITY ANALYSIS:**

Let \( M_i(t) \) \((i = 0,2)\) denote the probability that system is initially in regenerative state \( S_i \in E \) is up at time \( t \) without passing through any other regenerative state or returning to itself through one or more non regenerative states i.e. either it continues to remain in regenerative \( S_i \) or a non regenerative state including itself. By probabilistic arguments, we have the following recursive relations

\[ M_0(t) = e^{-(\alpha + \beta + \gamma)t} \bar{A}(t), \quad M_2(t) = e^{-(\alpha + \beta + \gamma)t} \bar{G}_2(t) \]

\[8.1-8.2\]

Recursive relations giving point wise availability \( A_i(t) \) given as follows:

\[ A_0(t) = M_0(t) + \sum_{i=1,2,3,5} q_{0i}(t) \quad \] \( A_1(t) = q_{10}(t) \quad \]

\[ A_2(t) = M_2(t) + \sum_{i=0,4} q_{2i}(t) \quad \] \( A_3(t) = q_{30}(t) \quad \]

\[ A_4(t) = q_{40}(t) \quad \] \( A_5(t) = q_{50}(t) \quad \]

\[8.3-8.8\]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

\[ q_{6 \times 6} \{ A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^* \} = [M_0^*, 0, M_2^*, 0, 0, 0]^T \]

\[8.9\]

Where \( q_{6 \times 6} = \)

\[
\begin{bmatrix}
1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & -q_{05}^* \\
-q_{10}^* & 1 & 0 & 0 & 0 & 0 \\
-q_{20}^* & 0 & 1 & 0 & -q_{24}^* & 0 \\
-q_{30}^* & 0 & 0 & 1 & 0 & 0 \\
-q_{40}^* & 0 & 0 & 0 & 1 & 0 \\
-q_{50}^* & 0 & 0 & 0 & 0 & 1 
\end{bmatrix}_{6 \times 6}
\]
Therefore \( D_2(s) = \begin{bmatrix} 1 & -q_{01} & -q_{02} & -q_{03} & 0 & -q_{05} \\ -q_{10} & 1 & 0 & 0 & 0 & 0 \\ -q_{20} & 0 & 1 & 0 & -q_{24} & 0 \\ -q_{30} & 0 & 0 & 1 & 0 & 0 \\ -q_{40} & 0 & 0 & 0 & 1 & 0 \\ -q_{50} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 6} \)

\[ = 1 - q_{01} q_{10} - q_{02} q_{20} + q_{24} q_{40} - q_{03} q_{30} - q_{05} q_{50} \]

If \( s \to 0 \) we get \( D_2(0) = 0 \) which is true \[8.10\]

Now \( N_2(s) = \begin{bmatrix} M_0^* & -q_{01} & -q_{02} & -q_{03} & 0 & -q_{05} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ M_2^* & 0 & 1 & 0 & -q_{24} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 6} \)

Solving this Determinant, we get

\[ N_2(s) = M_0^* + M_2^* q_{02}^* \] \[8.11\]

If \( s \to 0 \) we get

\[ N_2(0) = \mu_0 + \mu_2 p_{02} \] \[8.12\]

To find the value of \( D_2'(0) \) we collect the coefficient \( m_{ij} \) in \( D_2(s) \) we get

Coeff. of \( (m_{01} = m_{02} = m_{03} = m_{05}) = 1 = L_0 \)

Coeff. of \( (m_{10}) = p_{01} = L_1 \) Coeff. of \( (m_{20} = m_{24}) = p_{02} = L_2 \)

Coeff. of \( (m_{30}) = p_{03} = L_3 \) Coeff. of \( (m_{40}) = p_{02} p_{24} = L_4 \)

Coeff. of \( m_{50} = p_{05} = L_5 \) \[8.13-8.18\]

Thus the solution for the steady-state availability is given by

\[ A_0^*(\infty) = \lim_{t \to \infty} A_0^*(t) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2(0)}{D_2'(0)} = \frac{\mu_0 L_0 + \mu_2 L_2}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \] \[8.19\]
8. BUSY PERIOD ANALYSIS:

(a) Let \( W_i(t) \) \((i = 1, 2, 3, 4, 5)\) denote the probability that the repairman is busy initially with repair in regenerative state \( S_i \) and remain busy at epoch \( t \) without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments we have

\[
W_1(t) = \bar{S}_1(t), \quad W_2(t) = \bar{S}_2(t), \quad W_3(t) = \bar{S}_3(t)
\]  

[9.1-9.3]

Developing similar recursive relations as in availability, we have

\[
B_0(t) = \sum_{i=1,2,3,5} q_{i0}(t) c B_i(t) ; \quad B_1(t) = W_1(t) + q_{10}(t) c B_0(t) ;
\]

\[
B_2(t) = W_2(t) + \sum_{i=0,4} q_{i2}(t) c B_i(t) ; \quad B_3(t) = W_3(t) + q_{30}(t) c B_0(t) ;
\]

\[
B_4(t) = q_{40}(t) c B_0(t) ; \quad B_5(t) = q_{50}(t) c B_0(t);
\]

[9.4-9.9]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

\[
q_{6 \times 6}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*]' = [0, W_1^*, W_2^*, W_3^*, , 0, 0]' \]

[9.10]

Where \( q_{6 \times 6} \) is denoted by [8.9] and therefore \( D_2(s) \) is obtained as in the expression of availability.

Now \( N_3(s) = \)

\[
\begin{bmatrix}
0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & -q_{05}^* \\
W_1^* & 1 & 0 & 0 & 0 & 0 \\
W_2^* & 0 & 1 & 0 & -q_{24}^* & 0 \\
W_3^* & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}_{6 \times 6}
\]

Solving this Determinant, In the long run, we get the value of this determinant after putting \( s \rightarrow 0 \) is

\[
N_3(0) = (\mu_1 P_{01} + \mu_2 P_{02} + \mu_3 P_{03})
\]

\[
= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 = \sum_{i=1,2,3} \mu_i L_i
\]

[9.11]
Thus the fraction of time for which the repairman is busy with repair of the failed unit is given by:

$$B_0^r(\infty) = \lim_{t \to \infty} B_0^r(t) = \lim_{s \to 0} sB_0^r(s) = \frac{N_3(0)}{D_2^r(0)} = \sum_{i=1,2,3} \mu_i L_i$$ \[9.12\]

(b) Busy period of the Repairman in preventive maintenance in time \((0, t]\),

By probabilistic arguments we have

$$W_5(t) = \overline{B}(t)$$ \[9.13\]

Similarly developing similar recursive relations as in 9(a), we have

$$B_0(t) = \sum_{i=0,4} q_{40}(t) \overline{c} B_4(t) ; \quad B_1(t) = q_{30}(t) \overline{c} B_3(t) ;$$

$$B_2(t) = \sum_{i=0,2} q_{20}(t) \overline{c} B_2(t) ; \quad B_4(t) = q_{40}(t) \overline{c} B_0(t) ;$$

$$B_4(t) = q_{40}(t) \overline{c} B_0(t) ; \quad B_5(t) = W_5(t) + q_{50}(t) \overline{c} B_0(t) ;$$ \[9.14-9.19\]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{6 \times 6}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*] = [0,0,0,0,0,0,0]^t$$

Where \(q_{6 \times 6}\) is denoted by [8.9] and therefore \(D_2^r(s)\) is obtained as in the expression of availability.

Now \(N_4(s) = \begin{bmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & -q_{05}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -q_{24}^* & 0 & 0 \\ 0^* & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ W_5^* & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 6}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting \(s \to 0\) is

$$N_4(0) = \mu_5 p_{05} = \mu_5 L_5$$ \[9.20\]
Thus the fraction of time for which the system is under preventive maintenance is given by:

\[ B_0^\ast(\infty) = \lim_{t \to \infty} B_0^\ast(t) = \lim_{s \to 0} sB_0^\ast(s) = \frac{N_4(0)}{D_2'(0)} = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \]  

[9.21]

(c) Busy period of the Repairman in Shut Down repair in time \((0, t]\),

By probabilistic arguments we have

\[ W_4(t) = \bar{G}_4(t) \]  

[9.22]

Similarly developing similar recursive relations as in 9(b), we have

\[ B_0(t) = \sum_{i=1,2,3,4,5} q_{i0}(t) \begin{bmatrix} c \\ B_i(t) \end{bmatrix} ; \quad B_1(t) = q_{10}(t) \begin{bmatrix} c \\ B_0(t) \end{bmatrix} ; \]

\[ B_2(t) = \sum_{i=0,4} q_{2i}(t) \begin{bmatrix} c \\ B_i(t) \end{bmatrix} ; \quad B_3(t) = q_{30}(t) \begin{bmatrix} c \\ B_0(t) \end{bmatrix} ; \]

\[ B_4(t) = W_4(t) + q_{40}(t) \begin{bmatrix} c \\ B_0(t) \end{bmatrix} ; \quad B_5(t) = q_{50}(t) \begin{bmatrix} c \\ B_0(t) \end{bmatrix} ; \]

[9.23-9.28]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

\[ q_{6 \times 6}[B_0^\ast, B_1^\ast, B_2^\ast, B_3^\ast, B_4^\ast, B_5^\ast] = [0,0,0,0,W_4^\ast,0]^\prime \]

Where \(q_{6 \times 6}\) is denoted by [8.9] and therefore \(D_2'(s)\) is obtained as in the expression of availability.

Now \(N_5(s) = \begin{bmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & -q_{05}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 \\ 0' & 0 & 0 & 1 & 0 & 0 \\ W_4^* & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 6} \)

In the long run, we get the value of this determinant after putting \(s \to 0\) is

\[ N_5(0) = \mu_4 p_{02} p_{24} = \mu_4 L_4 \]  

[9.29]

Thus the fraction of time for which the system is under shut down is given by:

\[ B_0^\ast(\infty) = \lim_{t \to \infty} B_0^\ast(t) = \lim_{s \to 0} sB_0^\ast(s) = \frac{N_4(0)}{D_2'(0)} = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \]  

[9.30]
PARTICULAR CASES:
When all repair time distributions are n-phase Erlangian distributions i.e.

\[ g_i(t) = \frac{n!}{j!(nrt_j)^j} e^{-nrt_i} \]

And Survival function
\[ A_i(t) = e^{-\eta t_i}, \quad B_i(t) = e^{-\eta t_i} \]

For n=1 \[ g_i(t) = r_i e^{-\eta t_i}, \quad G_i(t) = e^{-\eta t_i} \]

Also
\[ p_{01} = \frac{\alpha}{x_1 + \theta}, \quad p_{02} = \frac{\beta}{x_1 + \theta}, \quad p_{03} = \frac{\gamma}{x_1 + \theta}, \quad p_{05} = \frac{\theta}{x_1 + \theta} \]

\[ p_{10} = 1, \quad p_{20} = \frac{r_2}{x_1 + r_2}, \quad p_{24} = \frac{x_i}{x_1 + r_2}, \]

\[ p_{30} = p_{40} = p_{50} = 1, \quad \mu_0 = \frac{1}{x_1 + \theta}, \quad \mu_1 = \frac{1}{r_i}, \]

\[ \mu_2 = \frac{1}{x_1 + r_2}, \quad \mu_3 = \frac{1}{r_3}, \quad \mu_4 = \frac{1}{r_4}, \quad \mu_5 = \frac{1}{\eta} \]

MTSF = \[ \frac{\mu_0 + \mu_2 P_{02}}{1 - p_{02} P_{20}}, \quad A_i(\infty) = \frac{\mu_0 L_0 + \mu_2 L_2}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}, \]

\[ B_0^{1*}(\infty) = \frac{\sum_{i=0,1,2,3,4,5} \mu_i L_i}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}, \quad B_0^{2*}(\infty) = \frac{\mu_5 L_5}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}, \quad B_0^{3*}(\infty) = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \]

10. PROFIT ANALYSIS

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t).
Therefore, \( G(t) = \) Expected total revenue earned by the system in \((0,t]\) - Expected repair cost of the failed units

- Expected repair cost of the repairman in preventive maintenance
- Expected repair cost of the repairman in shut down

\[
\begin{align*}
G(t) &= C_1 \mu_{up}(t) - C_2 \mu_{b1}(t) - C_3 \mu_{b2}(t) - C_4 \mu_{b3}(t) \\
&= C_1 A_0 - C_2 B^1_0 - C_3 B^2_0 - C_4 B^3_0
\end{align*}
\]

[11.1]

Where

\[
\begin{align*}
\mu_{up}(t) &= \int_0^t A_0(t) \, dt \\
\mu_{b1}(t) &= \int_0^t B^1_0(t) \, dt \\
\mu_{b2}(t) &= \int_0^t B^2_0(t) \, dt \\
\mu_{b3}(t) &= \int_0^t B^3_0(t) \, dt
\end{align*}
\]

[11.2-11.5]

\( C_1 \) is the revenue per unit time and \( C_2, C_3, C_4 \) are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

\[C_1\] is the revenue per unit time and \( C_2, C_3, C_4 \) are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

\[C_1\]

Table 1: Variation in MTSF vis-a-vis failure rate of associate unit for varying values of \( \alpha \) & \( \gamma \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \theta, \eta )</th>
<th>( r_1, r_2 )</th>
<th>( r_3, r_4 )</th>
<th>MTSF</th>
<th>MTSF1</th>
<th>MTSF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.001</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>113.86298</td>
<td>9.950872</td>
<td>38.955257</td>
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<tr>
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<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>49.337563</td>
<td>5.014274</td>
<td>30.163548</td>
</tr>
<tr>
<td>0.03</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>32.614502</td>
<td>3.35257</td>
<td>22.027822</td>
</tr>
<tr>
<td>0.04</td>
<td>0.004</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>24.516527</td>
<td>2.518226</td>
<td>18.528139</td>
</tr>
</tbody>
</table>

Table 2: Variation in Availability vis-a-vis failure rate of associate unit for varying values of \( \alpha \) & \( \gamma \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \theta, \eta )</th>
<th>( r_1, r_2 )</th>
<th>( r_3, r_4 )</th>
<th>Availability</th>
<th>Availability1</th>
<th>Availability2</th>
</tr>
</thead>
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<td>0.01</td>
<td>9.204318</td>
<td>1.041542</td>
<td>0.26163</td>
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<tr>
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<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>5.172541</td>
<td>0.028335</td>
<td>0.251371</td>
</tr>
<tr>
<td>0.04</td>
<td>0.004</td>
<td>0.002</td>
<td>0.001</td>
<td>0.01</td>
<td>4.147388</td>
<td>0.021387</td>
<td>0.228383</td>
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</tbody>
</table>
11. **DISCUSSION:**

It is seen from the graphs that value of MTSF decreases with increase in the failure rate of associate unit. The same can be predicted in the case of Availability. It is also seen that with the application of preventive maintenance technique Availability increases to some extent.
REFERENCES