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Drilling Rate at The Technical Limit

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Abstract

Recorded drilling times may show significant variations from one well to another even for the same total drilling depth, in the same field. Apart from the formation properties, drilling engineers' technical ability plays a significant role in required drilling time. Comparison of drilling performance with respect to previously drilled drilling records is a common technique applied to assess if there is any need for an improvement.

A graphical technique, known as 'learning curve analysis', is widely applied for performance evaluation. This approach has two major drawbacks. One, there may not be adequate number of drilled wells to make a healthy comparison. This is often the case in a newly developed field. Two, past drilling practices could be performed with bad engineering practices. In such a case, comparison of a given drilling performance with respect to bad engineering practices does not necessarily prove that the current practice represents a good performance. This is usually the case where previously drilled wells were carried out by inexperienced drillers and/or with old drilling technology.

In this paper, a different approach is introduced to assess drilling performance, and to alleviate the problems of learning curve analysis. The new approach suggests that the drilling rate is compared with a newly introduced parameter, called as *drilling rate technical limit*. It will be defined as the maximum achievable drilling rate without risking drilling safety. This method is superior to learning curve analysis because; one, it does not depend on the previous drilling records, two, it aims to drill a well at the fastest rate possible without jeopardizing the safety of drilling operation. It has a couple of disadvantages; one, the proposed method can only compare the drilling rates, two, it is laborious.

The new method calls for the assessment of the operational values that will maximize the drilling rate. Afterwards, it requires the quantification of drilling rate by one of the popular penetration rate models. This paper explains how to assess the most favorable values of key drilling parameters, so that the drilling rate can be maximized. This calls for the simultaneous consideration of drilling parameters' mathematical relationship with potential failures, as well as their mathematical relationship with the drilling rate

Introduction

The rate of drilling can be improved for a given field until it reaches its technical limit. This is the maximum achievable drilling rate (DR) without jeopardizing drilling safety. The drilling rate technical limit (DRTL) can only be achieved by carefully selecting all critical drilling parameters, which influences DR.

Several variables affect DR[1-4]. Some of these variables are formation properties, and nothing practical can be done to alter them favorably. Formation properties such as; pore pressure, compaction, in-situ stresses and mineral content are among the uncontrollable drilling variables. On the other hand, several drilling variables, when selected carefully, the rate of drilling improves significantly. Mud weight (MW), weight on bit (WOB), rotary speed or rotation per minute (RPM), bit type and hydraulic parameters, such as; flow rate (Q) and impact force (F_j) are among the controllable drilling variables.

Mathematical Relationships Between Drilling Rate And Controllable Drilling Variables

It has long been observed that the DR generally increases with increasing Q[5], WOB[6], RPM[6] and fractional bit tooth height. On the other hand, it decreases with increasing drilling fluid viscosity and MW[7-11]. Some of these variables may have significant effects on DR whereas others may have marginal effects.

Several authors have proposed mathematical relationships of DR with major drilling variables for rolling cutter bits[12-15]. Among them perhaps, the most complete mathematical drilling model being used is Bourgoyne and Young's model[21, 22].

$$DR = f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8 \tag{1}$$

In the above equation the functions f_1 through f_3 represent the effect of uncontrollable drilling variables on DR. Drilling performance cannot be improved practically by modifying them. Among these functions, the function f_1 represents the formation strength. The functions f_2 and f_3 model the effect of compaction on penetration rate. For example, the function f_2 accounts for the rock strength increase due to normal compaction with depth, and the function f_3 models the effect of undercompaction experienced in abnormally pressured formations[14].

The functions f_4 through f_8 represent the effect of controllable drilling variables on the DR. For example, the function f_4 models the effect of overbalance on the DR[21-22].

$$f_4 = e^{2.303\, a_4\, D\, (\rho_f - \rho)} \tag{2}$$

To graphically demonstrate the effect of MW on DR, Eq. 2 was substituted in Eq. 1, and DR in Eq. 1 was solved for the data given in Table 1. Note that, all the other controllable variables are kept constant while MW was changed.

Fig. 1 indicates that DR is 20 ft/hr at MW=12 ppg. If MW is increased to 13 ppg, the DR is cut down to 9 ft/hr. This is a reduction of more than 50%. Therefore, from the standpoint of DR, it is important to select MW as light as possible. Hence, it is crucial to determine the minimum MW, which will not lead to any other drilling problem.

In Eq. 1, the functions f_5 and f_6 model the effects of WOB and RPM on the DR[21-22].

$$f_5 = \left[\frac{\left(\frac{WOB}{d_b}\right) - \left(\frac{WOB}{d_b}\right)_t}{4 - \left(\frac{WOB}{d_b}\right)_t} \right]^{a5} \tag{3}$$

$$f_6 = \left(\frac{RPM}{60}\right)^{a6} \tag{4}$$

WOB vs. DR diagram (Fig. 2) can be produced similarly, by substituting Eq. 3 in Eq. 1, and solving for DR from the resulting equation. Again, the data in Table 1 is used to produce Fig. 2, however, in this particular case, while changing WOB, the following variables were kept constant: RPM, F_j, h, MW.

RPM vs. DR diagram (Fig. 3) is produced similarly, by substituting Eq. 4 in Eq.1, and by solving the resulting equation for DR. In this case, however, DR data is calculated at different values of RPM by keeping all other variables constant, such as; WOB, F_j, h, MW. As expected, Fig. 2 & 3 indicate that DR increases with increasing WOB and RPM. Therefore, it is important to determine the maximum allowable values of WOB and RPM for a given borehole condition and tubular configuration.

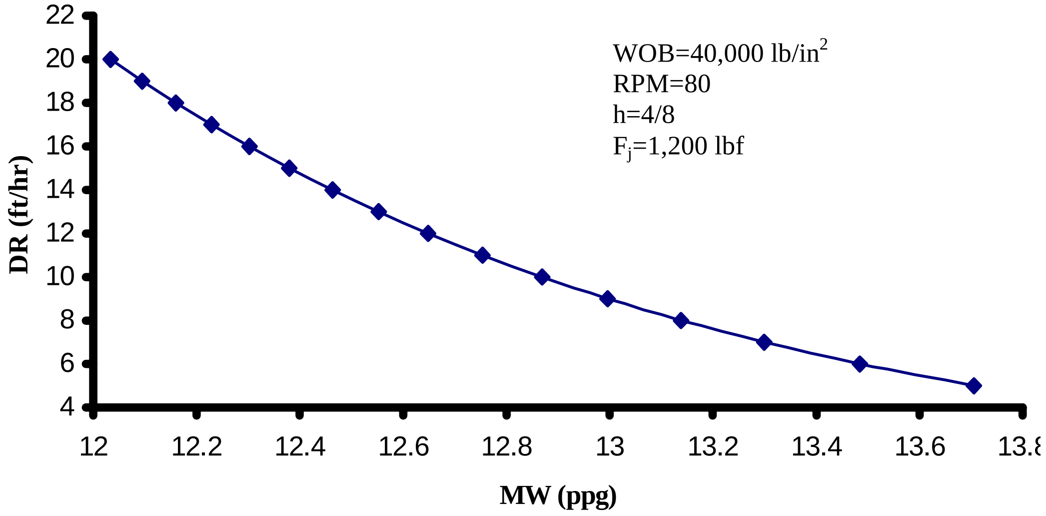

Figure 1: Effect of mud weight on drilling rate.

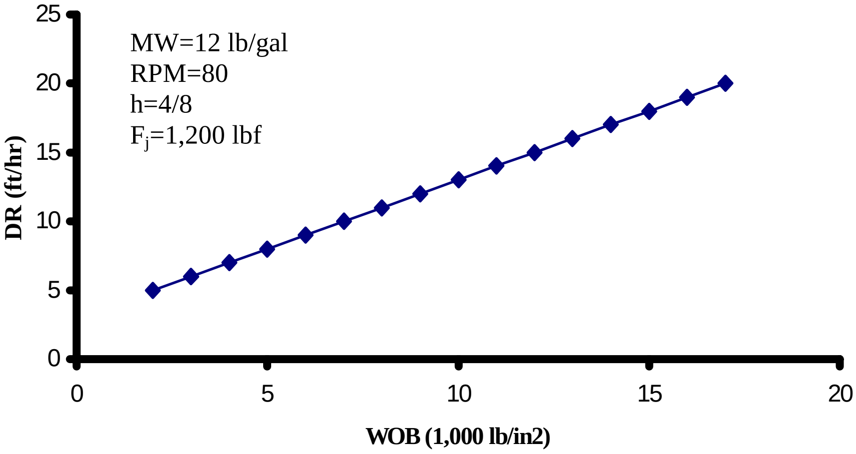

Figure 2: Effect of WOB on drilling rate.

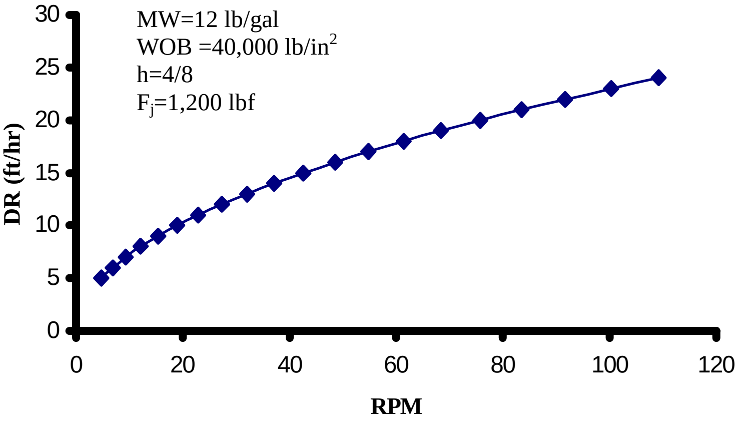

Figure 3: Effect of rotary table speed (RPM) on drilling rate.

In general, the DR increases with increased bit hydraulics and Q. However, once the bottom of the hole beneath the drill bit is efficiently cleaned off cuttings, a further increase in the Q (and/or F_j) is just a waste.

The function f_8 models the effect of bit hydraulics on DR[21-22].

$$f_8 = \left(\frac{F_j}{1{,}000} \right)^{a8} \tag{5}$$

The Q relates to F_j with the following equation:

$$Q = \left[\frac{2 * 57.66^2 * F_j^{\,2}}{j * m * MW} \right]^{\frac{1}{m+2}} \tag{6}$$

The DR for a given Q can be calculated as following. F_j is calculated from Eq. 6 for a given Q. Then, calculated F_j is substituted in Eq. 5 to determine function f_8, which in turn, is substituted in Eq. 1 to determine the DR.

Fig. 4 illustrates the relationship between the Q and the DR. As mentioned before, note that once an efficient cleaning is achieved at the bottom of the bit, a further increase in Q will not improve DR more. This is illustrated in Fig. 4 with two separate curves, which correspond to two separate 'recommended impact force per square inch of bottom hole area' values. These are 9 and 12 lbf/in^2 respectively. The lower curve indicates that the effective bottom hole cleaning can be achieved at 130 gal/min which corresponds to F_j=9 lbf/in^2, and the upper curve indicates that the effective bottom hole cleaning can be reached at about 150 gal/min, which corresponds to F_J=12 lbf/in^2. Note that, Eq. 6 can be used to determine the required Q for a given 'recommended impact force per square inch of bottom hole area.' This data is obtained either, in the laboratory with the laboratory tests, or at the field with drill off test.

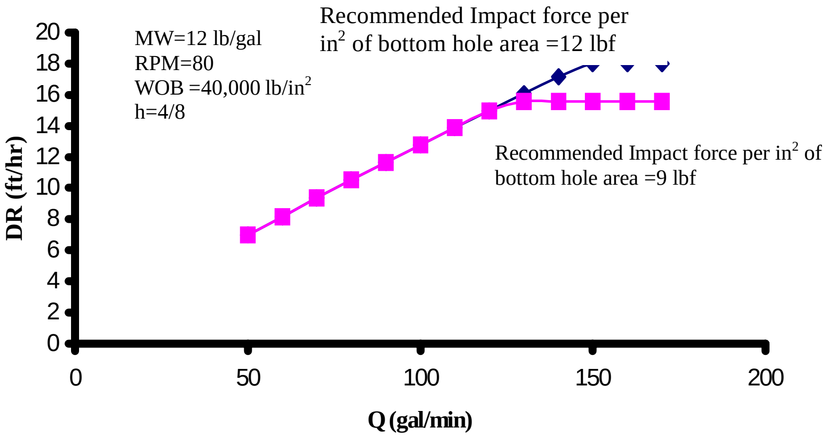

Figure 4: Effect of flow rate on drilling rate

The function f_7 models the effect of tooth wear (h) on the DR[21-22].

$$f_7 = e^{-a_7 h} \tag{7}$$

By substituting f_7 into Eq. 1 and using data in Table 1, DR was calculated and plotted as in Fig. 5 for h values ranging from h=0/8th (for a new bit tooth) to h=8/8th (fully worn out bit tooth). As expected and illustrated in Fig. 5, DR is inversely proportional with the value of h.

Drilling Rate Technical Limit

Generally speaking, if lighter MW, heavier WOB, faster RPM and higher Q are used, the expected outcome is faster DR. Fig. 7, generated by using Eq. 1 and the data in Table 1, demonstrates this fact graphically. Fig. 7 indicates that the DR of 22 ft per hr can be reached if controllable drilling parameters are selected as given under the upper must line. However, it does not mean that the maximum achievable DR is 22 ft per hr. It just demonstrates how DR improves as critical drilling parameters changes favorably. Obviously, if the MW is reduced below 12 ppg, the WOB is increased above 40K lbf, and the RPM is increased more than 100, the new DR line on the same diagram will fall above the upper most line. It is important to note that, once the Q reaches to 200 gal per min, the DR does not increase anymore with increasing Q, as this value corresponds to recommended impact force of 9 lb per square inch of bottom hole area which is needed for an effective cleaning of hole beneath the rock bit.

Now to assess the maximum achievable DR, which can also be called as the drilling rate technical limit (DRTL), following questions must be answered. These are:

1. What is the minimum acceptable MW?
2. What is the maximum acceptable WOB and RPM?
3. What is the maximum recommended Q?
4. What will be the drilling rate or DRTL, if drilling parameters are set as above values?

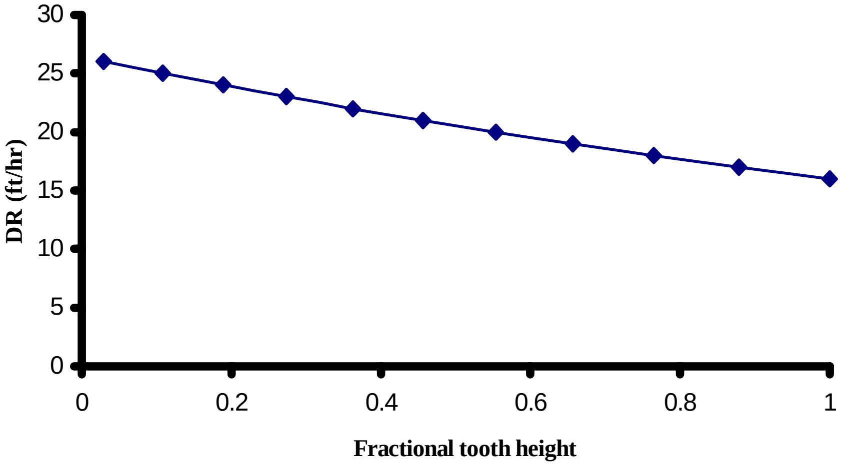

Figure 5: Effect of fractional tooth wear on drilling rate

Assessing Controllable Drilling Variables to Reach Drtl

Practicing engineers can improve their drilling efficiency significantly by carefully selecting controllable drilling variables. However, these variables cannot be selected solely based on DR without considering its consequence on the safety of drilling operation.

For a given drilling case, there exists an upper and a lower practical limit of each controllable drilling variable. These limits can be determined by selecting the controllable drilling variables at their most favorable values that will not cause any potential failure. This calls for the prediction of potential failure, as well as its mathematical relationship with the appropriate drilling variable.

The following drilling variables will be investigated; MW, WOB, RPM and Q

Table 1-A systematic approach in assessing the most favorable controllable drilling parameters and drilling rate technical limit for a given data.

HOW TO OBTAIN THE MINIMUM APPLICABLE MUD WEIGHT			
Minimum MW needed to prevent formation kick			
Pore Pressure Gradient	g_p	12	ppg
True Vertical Depth	D	12,000	ft
$P_f = 0.052 MW x D$		7,488	psi
$MW = \frac{P_f + 150}{0.052\ D}$		12.24	ppg
Minimum MM needed to prevent borehole collapses			
Friction angle	ϕ	30	degree
Shear sonic interval transient time	Δt_s	425	micro sec/ft
Compress ional sonic interval transient time	Δt_c	286	micro sec/ft
Poisson's ratio $= \frac{\frac{1}{2}\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1}{\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1}$	ν	0.216	
Average bulk density of formations	ρ_b	2.4	gm/cm^3
Assuming data is not available then $\sigma_h = \frac{\nu}{1-\nu}(\sigma_z - P_f) + P_f$	σ_H	8,862	psi
Clay content	V_{CL}	0.15	fraction
$C_o = \frac{(1-\upsilon)\rho_b^2 \text{Cos}\,\phi}{5.355x10^{-11}(1-\text{Sin}\,\phi)}\left(\frac{1}{\Delta t_c^2 \Delta t_s^2} - \frac{1}{\Delta t_s^4}\right)[8V_{Cl} + 4.5(1-$		1760	
$MW = \frac{1}{D}\left[2\sigma_H - P_f - \frac{\sigma_z - P_f - C_o}{\left(\sqrt{\nu^2+1}+\nu\right)^2}\right]$		13	ppg
Since MW needed to prevent borehole collapse is greater than what is seeded to prevent formation fluid kick, then,	**MW**	**13**	ppg
HOW TO OBTAIN THE MAXIMUM APPLICABLE FLOW RATE (Q)			

Minimum Q Needed To Prevent Cuttings Bed Formation			
Hole Size	$H = d_b$	12.25	inch
Hole inclination	β	30	degree
Mud Weight	MW	13	ppg
Drill Pipe outside Diameter	OD_{DP}	5	inch
Specific weight of cuttings	SW	22	lbm/gal
Particle slip velocity in a vertical hole	V_1	41.5	fpm
$V_2 = 44\left[\left(\frac{SW - MW}{MW}\right)g^3\left(\frac{H - OD}{12}\right)^3\right]^{\frac{1}{6}}$		249	fpm
$Q = \frac{\pi}{4}\left(H^2 - OD^2\right)\left[\left(V_1 \,Cos\, \beta\right) + \left(V_2 \,Sin\, \beta\right)\right]$		650	gpm
Minimum Q needed to effectively clean the bottom hole area beneath the rock bit			
Power Law Constant	j	0.84	
Power Law Constant	m	1.492	$lb.sec^n$ $/100ft^2$
Recommended Impact force	IF_{max}	9	psi
$F_j = (\pi/4)\, H^2\, IF_{max}$		1,061	lbf
$Q = \left[\frac{6{,}649.35\, F_j^2}{j\ m\ MW}\right]^{\left(\frac{1}{m+2}\right)}$		302.4	gpm
Since Q needed for inclined hole cleaning (650 gpm) is more than what is needed to optimize bit hydraulics (302 gpm), then	**Q**	**650**	gpm
HOW TO OBTAIN THE MAXIMUM APPLICABLE WOB			
Minimum Cost WOB		40	1,000 lbf
Drill Collar Outside Diameter	OD_{DC}	8	inch
Drill Collar Outside Diameter	ID_{DC}	3	inch
$B_f = 1 - \frac{MW}{65.5}$		0.802	
$CBL = 1617\sqrt{\frac{B_f\left(OD_{DC}^2 - ID_{DC}^2\right)\left(OD_{DC}^4 - ID_{DC}^4\right)Sin(\beta)}{H - OD_{DC}}}$		23,334	lbf
Since Min Cost WOB>CBL of DCs , then WOB=CBL of DCs p.s. In this particular case, DPs are in tension and BHA is a slick assembly i.e, no support from stabilizers.	**WOB**	**23,334**	lbf

HOW TO OBTAIND THE MAXIMUM APPLICABLE ROP			
Minimum Cost RPM	N_{MC}	80	rpm
Estimation of DP torque created due to drill bit hang up			
Drill Pipe Outside Diameter	ID_{DP}	4.276	inch
Length of Drill Collars	L_{DC}	600	ft
Length of drill pipes = D-L_{DC}	L	11,400	ft
Force at DP (+ for Tension and - for Compression)	F	+500	lbf
$J_C = \frac{\pi}{32}\left(OD_C{}^4 - ID_C{}^4\right)$		1429	in^4
$I_C = J_C/2$		714	in^4
$J_P = \frac{\pi}{32}\left(OD_P{}^4 - ID_P{}^4\right)$		28.5	in^4
$I_P = J_P/2$		14.26	in^4
$\tau_r = \sqrt{833{,}333\ I_P\left(2{,}056{,}168\frac{I_P}{L^2} + F\right)}$		77,124	lbf-ft
$\tau = 0.795\ N_{HU}\ J_p\left[\frac{2\ J_C}{J_C + J_P}\right]$		3,559	lbf-ft
Estimation of DP torque due to rotational drag			
Voltage across motor	V	440	Volts
Amperes consumed	I	150	amps
Electrical efficiency of a big motor	eff	0.92	hp/hp
Mechanical efficiency of the rotary system	mff	0.95	
Fraction of the rotational drag of the drillstring that is created by BHA	η_{BHA}	0.75	
$\tau = 7.04\frac{V\ I\ eff\ \ mff}{RPM}\eta_{BHA}$		3,807	lbf-ft
Since τ_r> torque created due to the bit hang up and rotational drag at N_{MC}, then RPM=N_{MC}	RPM	80	rpm
HOW TO ESTIMATE THE DRILLING RATE TECHNICAL LIMIT			
Pore Pressure Gradient	g_p	12	ppg
a_2 through a_8	a_2	0.00007	
	a_3	0.000005	
	a_4	0.00003	
	a_5	1	
	a_6	0.5	
	a_7	0.5	
	a_8	0.5	
Threshold bit weight	$\left(\frac{WOB}{d_b}\right)_t$	0	1,000 psi

Fractional bit tooth height	h	8/8	
Formation drillability	f_1	55.6	fph
$f_2 = e^{2.303 a_2 (10{,}000 - D)}$		0.724	
$f_3 = e^{2.303 a_3 D^{0.69} (g_p - 9)}$		1.023	
$f_4 = e^{2.303 a_4 D (g_p - MW)}$		0.436	
$f_5 = \left[\frac{\left(\frac{WOB}{d_b}\right) - \left(\frac{WOB}{d_b}\right)_t}{4 - \left(\frac{WOB}{d_b}\right)_t} \right]^{a_5}$		0.476	
$f_6 = \left(\frac{RPM}{60}\right)^{a_6}$		1.115	
$f_7 = e^{-a_7 h}$		0.607	
$F_j(Q) = \frac{\sqrt{\frac{j\ m\ MW\ Q^{m+2}}{2x57.66^2}}}{\frac{\pi}{4} d_b^2}$		34.24	lbf/in^2
Since $F_j(Q) > IF_{max}$, $F_j(Q) = IF_{max} \pi d_b^2 / 4$		1061	lbf
$f_8 = \left(\frac{F_j(Q)}{1{,}000}\right)^{a_8}$		1.03	
$DR = f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8$	**DRTL**	**6.09**	fph

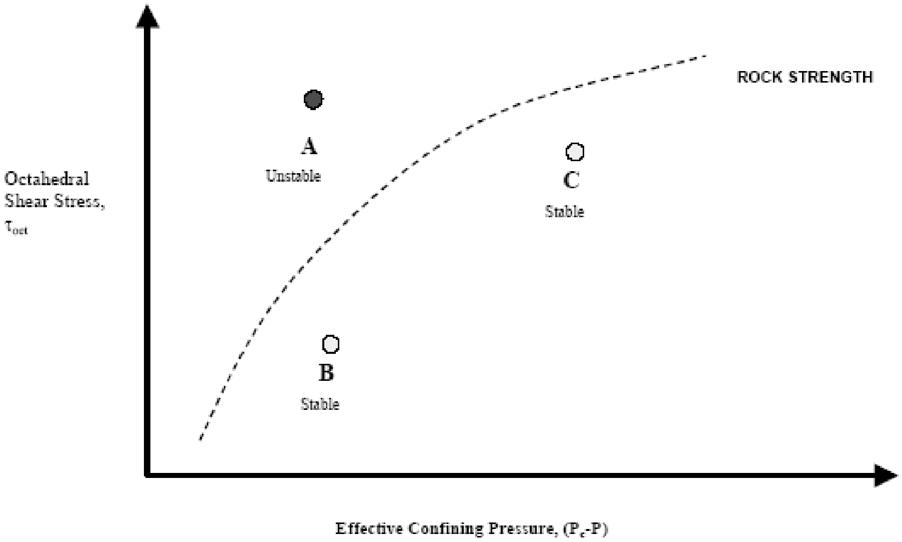

Figure 6: Experimentally determined rock failure envelop (15)

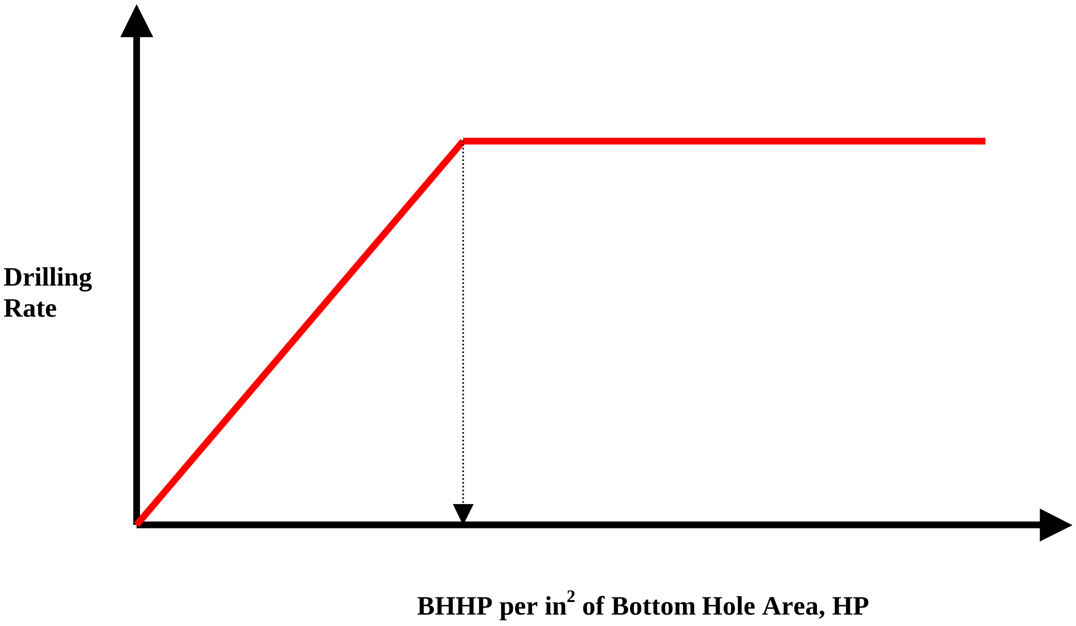

Figure 7: Laboratory and field drilling tests show that drilling rate rises with increased bit hydraulics to a maximum value and thereafter fails to cause a further rise.

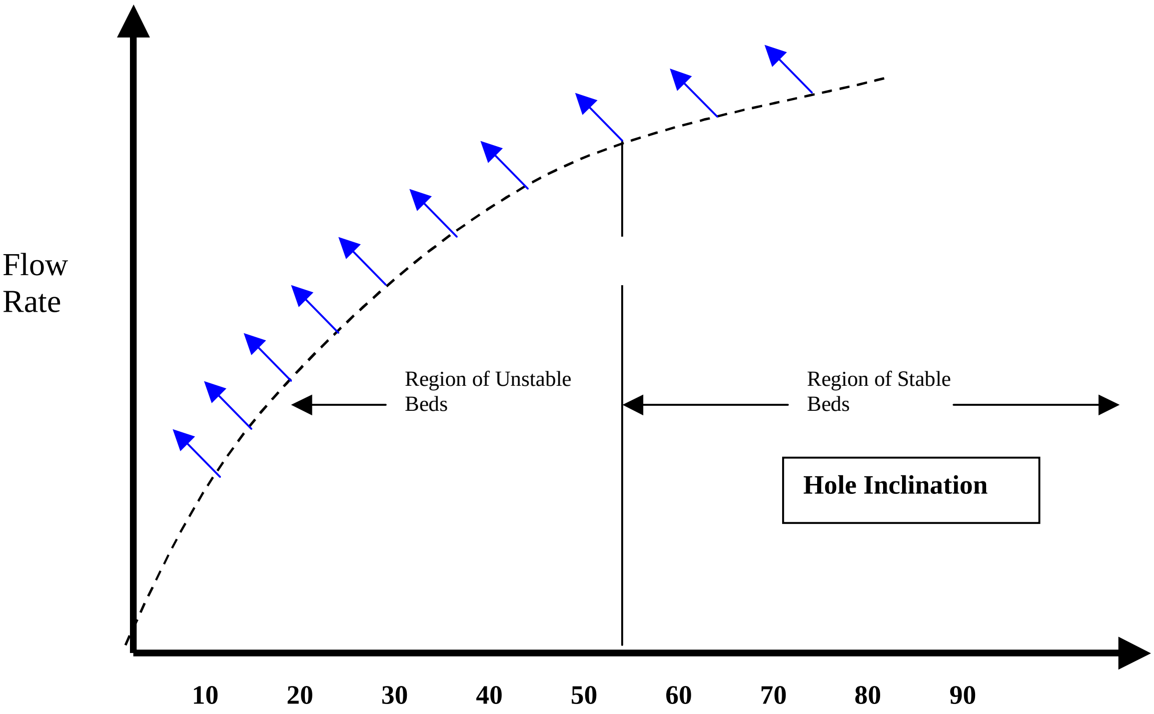

Figure 8: Full transport annular flow rate diagram.

Mud Weight

It has been stated that the drilling fluid is probably the most important variable to be considered in drilling optimization and hydraulics is the second[16]. Drilling fluid density or MW has a considerable effect on the DR. It is one of the variables inversely proportional to the DR. It has been observed that the DR generally increases with decreasing equivalent circulating density (ECD). One way to decrease ECD is to reduce the MW, and the other is to reduce its viscosity.

So, what is the minimum MW so that drilling operation is no in danger? The answer to this question can be obtained by investigating the possible problems that can be anticipated due to insufficient MW. The following two drilling problems may arise because of insufficient MW:

1. Formation fluids may flow into the borehole (kick),
2. The borehole may collapse (formation instability).

These two problems usually appear at two different MWs. Therefore, the lowest acceptable drilling fluid density is going to be the higher one of the two values. This ensures that both problems do not show up during drilling.

While drilling a permeable formation, formation fluids may flow into the borehole if the drilling mud hydrostatic pressure falls below the formation fluid pressure. In such a case, the lower limit of drilling fluid density is selected in a way that the hydrostatic pressure of the mud column is slightly higher that the formation fluid pressure (about 150 psi). Therefore, the lower limit of formation fluid density, MW, is determined simply by:

$$MW = \frac{P_f + 150}{0.052\ D} \tag{8}$$

Formation breakdown or formation compressive failure is a type of the borehole instability, which emerges when insufficient mud weight is in use during the drilling of sensitive formations[17-18]. To determine whether compressive failure will occur at borehole wall for a given mud weight, the stress state defined by two variables - octahedral shear stress, τ_{oct} and effective confining pressure, $(P_c\text{-}P_f)$ - is compared with an experimentally determined rock failure envelope[19], such as that shown in Fig. 6. If the stress state at the borehole wall falls below the rock strength curve, as at points A and C in Fig. 6, it is assumed that compressive failure will not occur, otherwise, the borehole wall will crumble and collapse.

For the assumed condition of no flow, vertical well and normally stressed formation the stresses at the borehole wall are given in polar coordinates by;

$$\sigma_z = \int_0^z \rho_b dz \tag{9}$$

$$\sigma_r = P_m = MW\ D \tag{10}$$

$$\sigma_\theta = 2\sigma_H - P_m = 2\sigma_z\left(\frac{\nu}{1-\nu}\right) - MW\,D \tag{11}$$

In terms of σ_r, σ_θ, σ_z, the effective confining pressure and the octahedral shear stress are given by:

$$P_c - P_f = \frac{\sigma_\theta + \sigma_z + \sigma_r}{3} - P_f \tag{12}$$

$$\tau_{oct} = \sqrt{\frac{(\sigma_\theta - \sigma_r)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2}{6}} \tag{13}$$

Therefore, once an experimentally produced rock failure envelope is obtained, for a given effective confining pressure, the minimum mud density can be determined as following:

1. $(P_c\text{-}P_f)$ is calculated from Eq. 12.
2. τ_{oct} is determined from Fig. 6.
3. Finally, MW is solved from Eq. 13 after substituting MW D for σ_r and $2\sigma_z\left(\frac{\nu}{1-\nu}\right) - \text{MW D}$ for σ_θ

In cases where no laboratory data is available for determining a rock failure envelope, the following empirical correlations can be used to determine whether the rock fails or not [20,21].

The uniaxial compressive strength is calculated from bulk density, shear and compressional sonic velocities and gamma ray data.

$$C_o = \frac{(1-\upsilon)\,\rho_b^2 \,\mathrm{Cos}\,\phi}{5.355\mathrm{x}10^{-11}(1-\mathrm{Sin}\,\phi)}\left(\frac{1}{\Delta t_c^2\,\Delta t_s^2} - \frac{1}{\Delta t_s^4}\right)\left[8\,V_{Cl} + 4.5\,(1-V_{Cl})\right] \tag{14}$$

Poisson's ratio, ν, can be empirically determined from shear and compressional sonic velocities as in the following equation:

$$\nu = \frac{\frac{1}{2}\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1}{\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1} \tag{15}$$

Once the value of C_o is determined empirically, then it can be introduced into Coulomb's criterion, to determine whether the rock fails or not under present borehole stress, it can be stated as follows:

$$C_o = (\sigma_{max} - P_f) - \left(\sqrt{\nu^2 + 1} + \nu\right)^2 (\sigma_{min} - P_f) \tag{16}$$

For a normally stressed, tectonically inactive formation where maximum and minimum horizontal stresses are equal, it is reasonable to assume that $\sigma_{max}=\sigma_z$ and $\sigma_{min}=\sigma_\theta$. Hence, by combining Eq. 9 through Eq. 16, the lowest MW that satisfies the mechanical borehole stability criteria can be determined from the following relationship:

$$\mathrm{MW} = \frac{1}{D}\left[2\sigma_H - P_f - \frac{\sigma_z - P_f - C_o}{\left(\sqrt{\nu^2 + 1} + \nu\right)^2}\right] \tag{17}$$

Weight on Bit

One of the major variables, which significantly affects DR, is the WOB. Given that there is efficient bottom hole cleaning beneath the bit teeth, generally, the DR increases with increasing WOB. However, as in the case of many controllable drilling variables, there is an upper limit of WOB that must not be exceeded. The upper limit of WOB is decided after following two critical loads are determined:

1. The WOB at which the bit drills at minimum cost
2. Critical Buckling Load (CBL) of drill string.

The first one of the above two ensures that operating cost of drill bit is at its minimum value. Notice that the minimum cost WOB does not ensure that the DR is at its maximum. This is considered to be one of the fundamental requirements of cost effective drilling. Therefore, if the DR is to be maximized as a target, then minimizing cost per foot criteria can be sacrificed.

If sufficient data is available to produce a table of bit operating cost as a function of WOB and RPM, then graphical technique can be used to determine the minimum cost WOB and RPM[22]. However, in the absence of such a table, several analytical methods can be used. There are two popular analytical models that can be used to produce a table of 'cost per foot' for a range of practical WOB and RPM[12-15]. They are both used to model DR for tri-cone roller cone bits. Once the cost per footage table is constructed, the minimum cost per footage and corresponding values of WOB and RPM can easily be identified.

Minimum cost WOB does not guarantee that under such WOB the existing drill string is going to be mechanically stable. Therefore, once the minimum cost WOB is determined, the stability of those pipes must be investigated. If the minimum cost WOB needs more compression than the critical buckling load (CBL) of drill collars, than the maximum value of WOB has to be reduced to CBL of the drill collars.

During the drilling of horizontal and extended reach wells, sometimes, drill pipes have to be put in compression to achieve the required WOB. In this case, the CBL of drill pipes must be determined and added to the longitudinal component of drillcollar weight to determine the maximum applicable WOB from the standpoint of drill string stability.

The following equation can be used to estimate the CBL of pipes in inclined and straight holes[23-27].

$$\mathrm{CBL} = 1{,}617\sqrt{\frac{\mathrm{B_f}\,(\mathrm{OD}^2 - \mathrm{ID}^2)(\mathrm{OD}^4 - \mathrm{ID}^4)\,\mathrm{Sin}\,ß}{(\mathrm{H} - \mathrm{OD})}} \qquad (18)$$

It is worth to mentioning that the CBL of pipes can be increased substantially, if stabilizers are attached to pipes. In such cases, not only the number of stabilizers, but also the location of stabilizers within the string determines the value of CBL[32].

Rotary Speed

Rotary Speed or RPM is among the controllable variables that significantly affect the DR. Assuming that bottom hole cleaning is adequate, DR generally increases with increased RPM. However, in most practical applications, the optimum RPM is selected so that per footage cost of drilling is at a minimum. This is called minimum cost rotary speed (N_{MC}). This number and minimum cost WOB can be determined simultaneously[12-15]. However, before applying minimum cost RPM, the torsional strength of pipes must be investigated. If minimum cost RPM will put drill pipes in helical buckling mode before it reaches to drill bit, than the maximum safe RPM must be determined by considering the torsional strength of drill pipe.

Drill pipes will torsionally buckle if torsional loads exceed the minimum torque required to buckle them. The buckling strength of a drill pipe against torsional load depends on how much tension or compression is placed on it. The following formula can be used to determine the torsional resistance of a tubular under a given tension or compression[22].

$$\tau_r = \sqrt{833{,}333\, I_P \left(2{,}056{,}168 \frac{I_P}{L_P^{\,2}} + F \right)} \tag{19}$$

The drill bit hang-up and Bottom Hole Assembly (BHA) rotational drag are considered to be the two factors, which cause drill pipe helical buckling. The following BHA torsional model predicts the transmission of torsional loads created by the bit through the drill collars and then into the drill pipe[28].

$$\tau = 0.795\ N_{HU}\ J_p \left[\frac{2\ J_c}{J_c + J_p} \right] \tag{20}$$

By substituting τ_r from Eq.19 into τ in Eq. 20, one can solve for the maximum value of rotary speed (N_{HU}) that can be applied to avoid helical buckling of drill pipes in case the bit hangs-up.

Torsional buckling of the drill pipes may also be anticipated if the input torque by the rotary table is excessive. The following equation uses watts consumed by an electric rotary drive to estimate the drill pipe torque by the rotary system[24].

$$\tau = 7.04 \frac{V\ I\ eff\ mff}{N_{RD}} \eta_{BHA} \tag{21}$$

Similarly, by substituting τ_r from Eq. 19 into τ in Eq. 21, one can solve for the maximum value of rotary speed (N_{RD}) to avoid drill pipe helical buckling.

Finally, among the three rotary speeds, N_{MC}, N_{HU} and N_{RD}, the smallest one is selected as the maximum applicable RPM.

Circulation Rate

Laboratory and field drilling tests show that the DR rises with increased bit hydraulics to a maximum value and thereafter fails to cause a further rise[29]. This phenomenon is interpreted to mean that once the bottom of the hole is cleaned that further efforts at cleaning are a waste of bit hydraulics (Fig. 7). Therefore from the standpoint of bit hydraulics, the Q can be increased until drilling fluid fully cleans the cuttings beneath the bit. However, this rate may not be adequate to circulate cuttings out of the hole. Higher Q is often needed to prevent cuttings bed formation in inclined and horizontal wells.

The following formula can be used to find the Q, which maximizes bit hydraulic horsepower (BHHP) with the constraint of a selected Bit hydraulic horsepower per square inch of bottom hole area[30].

$$Q = \left[\frac{1714\ \text{BHHP}}{j\ m}\right]^{\left(\frac{1}{m+1}\right)} \tag{22}$$

Similarly, the following formula can be used to find the Q, which maximizes jet impact force (F_j) with the constraint of a selected impact force per square inch of bottom hole area[30].

$$Q = \left[\frac{6{,}649.35\, F_j^{\,2}}{j\ m\ MW}\right]^{\left(\frac{1}{m+2}\right)} \tag{23}$$

If the Q needed to optimize bit hydraulics is inadequate for efficient borehole cleaning then full transport Q has to be selected as the minimum Q. The following formula was proposed to determine the full transport Q in directional wells[31].

$$Q \ = \frac{\pi}{4}\left(H^2 - OD^2\right)\left[\left(V_1 \,\text{Cos}\ \beta\right) + \left(V_2 \,\text{Sin}\, \beta\right)\right] \tag{24}$$

$$V_2 = 44\left[\left(\frac{SW - MW}{MW}\right) g^3 \left(\frac{H - OD}{12}\right)^3\right]^{\frac{1}{6}} \tag{25}$$

Eqn.'s 24 and 25 are used to construct full transport annular Q diagram (Fig. 8). The required Q is selected by entering the chart with hole inclination (β).

Conclusions

DR strongly depends on several controllable drilling variables. Appropriate selection of these variables can significantly improve the DR. However, there is an upper limit of DR, which cannot be exceeded without risking the safety of drilling operation. This rate is called the drilling rate technical limit and can be approached if:

- MW is selected as the bigger of following two values; a) minimum MW needed to prevent formation fluid kick, and b) minimum MW needed to prevent borehole collapse.
- WOB is selected as the smaller of following two values; a) optimum WOB for minimum cost per footage, and b) minimum critical buckling load of drillstring portion that will be put in compression.
- RPM is selected as the smallest of following three values; a) maximum allowable RPM for rotational drag, b) maximum allowable RPM for bit hang up, c) optimum RPM for minimum cost per footage.
- Q is selected as the bigger of following two values; a) minimum flow rate needed to prevent cuttings bed formation in inclined holes and vertical holes, b) the flow rate need for an effective cleaning beneath the bit tooth.

Once controllable drilling parameters are determined as above, and the DR is calculated by one of the drilling rate models, drilling engineers will be able to evaluate their drilling performance with a better tool, rather than using classic learn curve analysis technique which is believed to have major drawbacks

Nomenclature

a_1-a_8	Coefficients
B_f	Buoyancy factor
BHHP	Bit hydraulic horse power
C_o	Uniaxial compressive strength
CBL	Critical buckling load (lbf)
D	Formation depth (ft)
eff	Electrical efficiency of motor
f_1	Formation drillability (ft/hr)
f_2-f_8	Functions
F	Tension (+) or Compression (-)
F_j	Jet Impact Force (lbf)
g	Acceleration of Gravity (ft/s^2)
g_p	Pore pressure gradient (lbf/gal)
H	Hole size (in)
h	Fractional tooth height
I	Amperes consumed
I_c	Cross-sectional moment of inertia of drill collars (in^4)
I_p	Cross-sectional moment of inertia of drill pipes (in^4)
IF_{max}	Recommended impact force per square inch of bottom hole area (lbf/in^2)
ID	Inside diameter (in)
j & m	Power law constants
J_c	Polar moment of inertia of drill collars (in^4)
J_p	Polar moment of inertia of drill pipes (in^4)
L_P	Length of pipes (ft)
P_f	Formation fluid pressure (psi)
P_m	Hydrostatic pressure (psi)
MW	Drilling fluid density (lbf/gal)
mff	Mechanical efficiency of rotary system
N_{MC}	Minimum cost rotary speed (rev/min)
N_{HU}	Critical rotary speed to avoid helical bucking due to bit hang up (rev/min)
N_{RD}	Critical rotary speed to avoid helical bucking due to rotational drag (rev/min)
OD	Outside diameter (inch)
Q	Flow rate (gal/min)
RPM	Rotary table speed (rev/min)
SW	Specific weight of solids
V_{Cl}	Percent clay volume
V_1	Particle slip velocity in a vertical hole
V_2	Critical transport velocity for large solids in a horizontal annulus
WOB	Weight on bit (1000 lbf)

$\left(\frac{WOB}{d_b}\right)_t$	Threshold bit weight per inch of bit diameter at which the bit begins to drill, 1,000 lbf/in
β	Hole inclination
Δt_s,	Shear sonic velocity
Δt_c	Compressional sonic velocity
ϕ	Friction angle (≅30o for most rocks)
η_{BHA}	Rotational drag due to BHA
ν	Poisson’s ratio
ρ_b	Formation bulk density
ρ_f	Formation fluid gradient
σ_H	Horizontal in-situ stress
σ_r	Radial borehole stress
σ_z	Vertical in-situ stress
σ_θ	Tangential (Hoop) borehole stress
τ	Torque transmitted
τ_r	Torsional buckling strength

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