

Hole Drift Mobility, Hall Coefficient and Coefficient of Transverse Magnetoresistance in Heavily Doped p-type Silicon

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Abstract

In this communication the normalized Fermi-energy, the normalized hole drift mobility, the normalized Hall coefficient and the normalized coefficient of transverse magneto resistance are numerically calculated for different doping concentrations of acceptor impurities at a temperature $T = 300^0\text{K}$ in a heavily doped p-type silicon. The analysis of the results show that the formation of valence band tails and its effect on transport properties become significant for acceptor doping concentration $\geq 10^{19}/\text{cm}^3$ and it is expected that results may be particularly of interesting above $\geq 10^{19}/\text{cm}^3$. The result of this article might be very important in characterizing semiconductor devices.

Introduction

Recently different workers have studied the effect of band tails due to heavy doping on band gap narrowing and some transport phenomena in heavily doped n-type silicon. Lee and Fossum^[1] have calculated the band gap narrowing in heavily doped n-type silicon. Poortmans et. al.^[2] have reported the band gap narrowing in heavily doped p-type silicon. Sharma^[3] have calculated the diffusion mobility ratio in heavily doped n-type silicon by taking in to account the effect of band tails. Y. Elfgad et.al^[4] have calculated some galvanomagnetic transport coefficients in heavily doped n-type silicon by taking into account the effect of band tails. T. Getinet^[5] has calculated hall coefficient and coefficient of transverse magnetoresistance of intermediately doped n-type silicon by taking in to account the effect of band tails with mixed scattering of electrons both by ionized ipurities and acoustic phonons.

In this paper, we numerically calculate some galvanomagnetic transport coefficients viz. hole drift mobility, Hall coefficient and coefficient of transverse magnetoresistance in heavily doped p-type silicon. To carry out the numerical

calculations we first derive explicit expressions of these transport coefficients by taking into account the effect of valence band tails, which extend above the valence band edge deep into the forbidden band gap. To derive the explicit expressions of the transport coefficients we used the following model: (1) complete ionization of acceptor impurities, (2) Fermi - Dirac statistics to describe the equilibrium distribution of holes in the valence band, (3) ionized impurities as a dominant scattering mechanism, and (4) the collision frequency due to ionized impurity scattering derived by Conwell-Weisskopf^[6]. Because we are interested only studying the effect of valence band tails on galvanomagnetic transport coefficients in heavily doped p-type silicon, we further assume a spherical constant energy surface in momentum space.

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Explicit expressions of the transport coefficients in heavily doped p-type silicon can be obtained if we know the concentration of holes in the valence band. The random distribution of impurity atoms in heavily doped p-type silicon gives rise to aperiodicity of the crystal. This crystal aperiodicity causes a fluctuation in the local electrostatic potential, which in turn results in a spatially dependent valence band edges and hence spatially dependent density of states. In our analysis we use the density of states function derived by Kane^[7], and Slotboom's^[8] approximation of the Kane's density of states function. In deriving the expressions of the transport coefficients we use the Boltzmann transport equation in the presence of the crossed d.c. electric and magnetic fields. We further use the collision frequency due to ionized acceptor impurities derived by Conwell-Weisskopf. To obtain the explicit expressions of the transport coefficients in heavily doped p-type silicon, we first derive the expressions by assuming parabolic density of states and then we modify the expressions when the density of states function is no longer parabolic using the concept suggested by Elfagd et. al. for heavily doped n-type silicon. For the sake of numerical calculations, we further normalize the hole concentration in the valence band and the transport coefficients. Thus the normalized hole concentration p_n , the normalized hole drift mobility μ_n , the normalized Hall coefficient r_n , and the normalized coefficient of transverse magnetoresistance b_n are derived to be

$$p_n = \frac{(m_p^*)^{\frac{3}{2}} 2^{\frac{5}{4}} \sigma^{\frac{3}{2}}}{10^{18} \pi^2 \hbar^3} \Phi_o, \quad (1)$$

$$\mu_n = \frac{2}{3} \left(\frac{\sqrt{2}\sigma}{k_B T} \right)^{\frac{5}{2}} \frac{\Phi_{\frac{5}{2}}}{\Phi_o}, \quad (2)$$

$$r_n = \frac{3k_B T}{2\sqrt{2}\sigma} \frac{\Phi_o \Phi_4}{\Phi_{\frac{5}{2}}^2}, \quad (3)$$

$$b_n = \left(\frac{\sqrt{2}\sigma}{k_B T} \right)^3 \left[\frac{\Phi_{\frac{11}{2}}}{\Phi_{\frac{5}{2}}} - \left(\frac{\Phi_4}{\Phi_{\frac{5}{5}}} \right)^2 \right], \quad (4)$$

where σ is the standard deviation of the Gaussian distribution of impurities considered by Kane and it is given by

$$\sigma = \left(\frac{p_o \lambda}{8\pi \epsilon_s^2} \right)^{\frac{1}{2}}, \quad (5)$$

The screening length λ is expressed as

$$\frac{1}{\lambda} = \left(\frac{e^2}{\epsilon_s} \frac{\partial p_o}{\partial (E_V - E_F)} \right)^{\frac{1}{2}}. \quad (6)$$

In these expressions p_o is the uniform hole density that prevails in the absence of ion coulomb interaction, ϵ_s is the dielectric constant of silicon, m_p^* is the hole density of states effective mass ($m_p^* = 0.81m_o$ at 300^0K) that accounts for the split off sub-bands as well as the two degenerate sub-bands with m_o being the free electron mass and

$$\Phi_o = \int_{-\infty}^{0.601} \frac{\{1.225 - 0.906(1 - \exp[2z])\} \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]}{2\pi^{\frac{1}{2}} \left(1 + \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]\right)} dz \quad (7)$$

$$+ \int_{0.601}^{\infty} z^{\frac{1}{2}} \frac{\left(1 - \frac{1}{16z^2}\right) \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]}{\left(1 + \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]\right)} dz,$$

$$\Phi_m = \int_{-\infty}^{0.601} |z|^m \frac{\{1.225 - 0.906(1 - \exp[2z])\} \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta - z^2\right]}{2\pi^{\frac{1}{2}} \left(1 + \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]\right)^2} dz \quad (8)$$

$$+ \int_{0.601}^{\infty} z^{(m+\frac{1}{2})} \frac{\left(1 - \frac{1}{16z^2}\right) \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]}{\left(1 + \exp\left[\frac{\sqrt{2}\sigma}{k_B T} z - \eta\right]\right)^2} dz,$$

where $\eta \equiv \frac{E_F}{k_B T}$ is the normalized (or dimensionless) Fermi-energy, z is defined in terms of the hole energy E as $z \equiv \frac{E}{\sqrt{2}\sigma}$, and in the integral expression Φ_m , $m = 5/2, 4, 11/2$.

Numerical calculation of the transport coefficients

To numerically calculate the transport coefficients, we first calculate the numerical value of the normalized (or dimensionless) Fermi-energy η for a given hole concentration from equation (1) by iteration. To do this, for a given normalized hole concentration we arbitrarily assume a value for η and then we compute RHS of equation (1) and compare the result with the LHS, i.e., with the normalized (or dimensionless) hole concentration. We then try some other value of η and then continue to change it until we get a numerical value for η , which makes equation (1) self-consistent. This value of η is the normalized value of Fermi-energy for the given hole concentration. After calculating η for various hole concentrations, we compute the definite integrals occurring in the expressions of the transport coefficients, i.e., equations (7) to (10). We then use these values of the definite integrals in equations (2), (3), and (4) to numerically calculate the normalized hole drift mobility, Hall coefficient and coefficient of transverse magnetoresistance, respectively, in heavily doped p-type silicon. Finally, we plotted the normalized Fermi-energy, hole drift mobility, Hall coefficient and coefficient of transverse magnetoresistance versus the normalized doping concentration of acceptor impurities.

Discussion

Figures (1), (2), (3) and (4) show the plot of the normalized Fermi-energy, hole drift mobility, Hall coefficient and coefficient of transverse magnetoresistance versus the normalized doping concentration of acceptor impurities, respectively. Fig. (1) shows the dependence of the dimensionless Fermi-energy on dimensionless doping concentration. We see from this figure that the normalized Fermi-energy increases with increasing doping concentration. Fig. (2) shows the variation of the normalized hole drift mobility with doping concentration in heavily doped p-type silicon. It shows that the drift mobility increases slowly with increasing doping concentration of acceptor impurities for $p_n \leq 10$ and rapidly with increasing doping concentration for $p_n > 10$. Fig. (3) indicates the dependence of the normalized Hall coefficient on the normalized concentration of acceptor impurities in heavily doped p-type silicon. We note from this figure that the normalized Hall coefficient decreases monotonically with increasing normalized doping concentration. In contrast to this we see from fig. (4) that the normalized coefficient of transverse magnetoresistance increases with increasing doping concentration. As we can see from all graphs, the results of the analysis are very interesting around $p_n = 10$, i.e., when the doping concentration is

around $10^{19}/\text{cm}^3$. Around this doping concentration some phase change occurs in the system. Doping concentration around $10^{19}/\text{cm}^3$ may be thought as the point where the randomness of the impurity distribution, which give rise to the aperiodicity of the crystal which in turn causes the fluctuation of the electrostatic potential resulting in spatially dependent density of states function, starts to become significant. This means that the formation of the valence band tails and its effect on transport properties become significant for acceptor doping concentration $\geq 10^{19}/\text{cm}^3$ and it is expected that results may be particularly of interesting above $\geq 10^{19}/\text{cm}^3$. We could not compare our results with that of experiment for lack of experimental data on p-type silicon in which our theory is applicable. However it will be interesting to compare them with the experimental data whenever it becomes available.

The error incurred in the numerical calculation is one in 10^6 . The limitations in these calculations are (1) the logarithmically varying energy in the expression of the collision frequency has been assumed as a constant, and (2) only ionized impurity scattering as a dominant scattering mechanism has been considered, while of course one could also include acoustic phonon scattering in the analysis of the transport coefficients. These transport coefficients in heavily doped p-type silicon could also be calculated as functions of temperature. Calculations of these transport coefficients as a function of doping concentration and temperature simultaneously in heavily doped n-type silicon have been performed and the results will be reported soon. In all calculations I used $T = 300^0\text{K}$ and the dielectric constant of silicon $\epsilon_s = 11.8$.

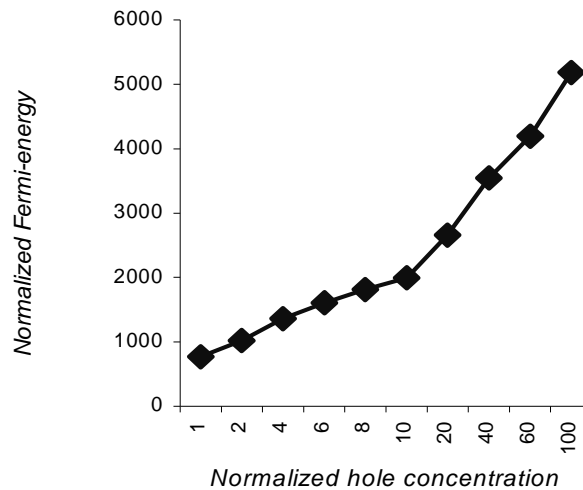


Figure 1: Dependence of the normalized Fermi-energy on the normalized doping concentration in heavily doped p-type silicon.

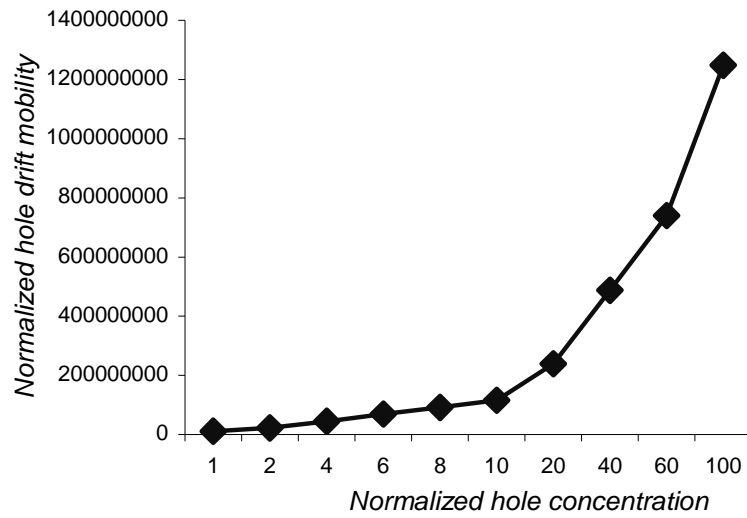


Figure 2: Variation of the drift mobility with doping concentration in heavily doped p-type silicon

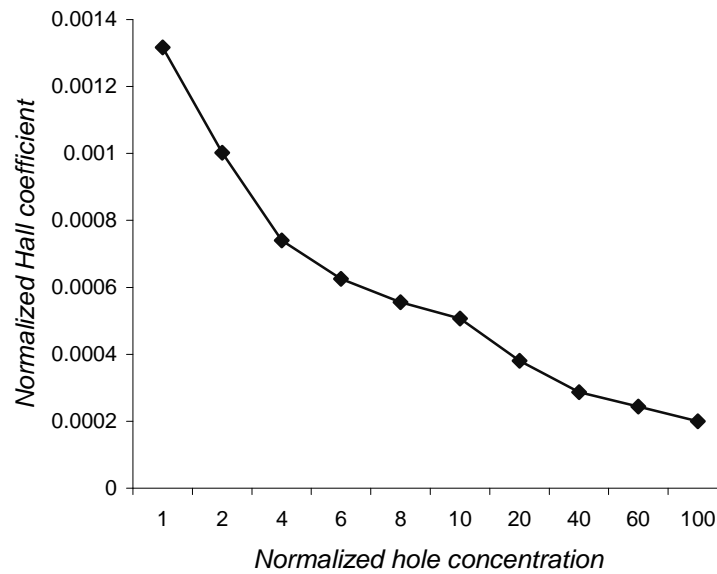


Figure 3: Variation of the normalized Hall coefficient with doping concentration in heavily doped p-type silicon.

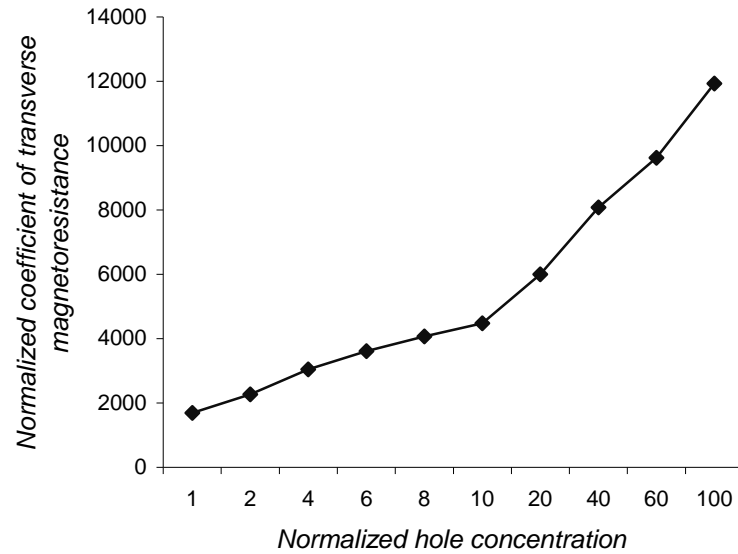


Figure 4: Dependence of the normalized coefficient of transverse magneto resistance on doping concentration in heavily doped p-type silicon.

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