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Collisions of Positrons with Positroniums

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Abstract

A computational code based on an elaborate coupled static approach is developed for the treatment of the elastic and inelastic scattering of positrons by positroniums. The inelastic channel is chosen as a mixture of 2s- and 2p-states. The 2x2 reactance matrices as well as the corresponding partial cross sections corresponding to $0 \leq \ell \leq 6$ are determined for the first time at wide range of total energies below the second excitation threshold of the positronium, i.e. below 6.044 eV. The effect of adding the polarization potential to the first channel is explored. Partial and total cross sections are also calculated at high energies (between 10 and 500 eV) in order to compare present results of positronium formation with those obtained by other authors for inelastic collisions of positron with hydrogen atoms.

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Introduction

Positroniums are quasi-bound states each of which is composed of one electron and one anti-electron (or positron). Predictions of their existence were made immediately after Anderson's discovery of positrons [1] in the early thirties of the preceding century. Confirmation of their formation in Labs was announced later on by Deutsch et al. [2]. The experiments showed that both singlet and triplet (known as para- and ortho-positroniums, respectively) are produced with life times of the orders of 10^{-7} and 10^{-10} second. The formation of positroniums as a result of positron-atom and positron-molecule collisions at low energies was investigated over the years (for a review see Abdel-Raouf [3]. Also, the construction of both positive and negative ionic positroniums as well as molecular positroniums has gained much interest in the literature (Mills[4] , and Sarkar and Ghosh [5]). The last decade, however, has seen particular interest in the positroniums. This is attributed to two reasons; the first is the production of antihydrogens Amoretti et al. [6] is governed by the following reaction: $P_s + \bar{P} \rightarrow \bar{H} + e^-$, where P_s, $\bar{P}, \bar{H}$ and e^- , respectively, refer to the positronium, antiproton, anti-hydrogen and electron. (This process is enhanced in the presence of excited positroniums). The second reason is the formation of positronic molecules and positronium-atom chemical compounds Mitroy et al. [7]. On the other hand , formation of Positronium in the ground and first excited states as a result of positron-hydrogen inelastic scattering was studied by Nahar [8], Basu abd Ghosh [9], Khan et al. [10] and Saha and Roy[11]

The aim of the present work is to investigate the inelastic collisions of positrons with the positroniums using a coupled static formalism. Only two channels are opened, namely the elastic (1s) and the first excited (2s+2p) channels at total energies below the second excitation threshold, i.e. below 6.044 eV. The present work is devoted to the mathematical formalism of the problem and the construction of the computer code. Preliminary tests of the two channel programs are given. The effect of adding polarization to the first channel is tested. Details of the results will be presented somewhere else. The material of this paper contains other two sections. The next section involves the mathematical formalism of the problem. Section 3 contains preliminary results and discussion.

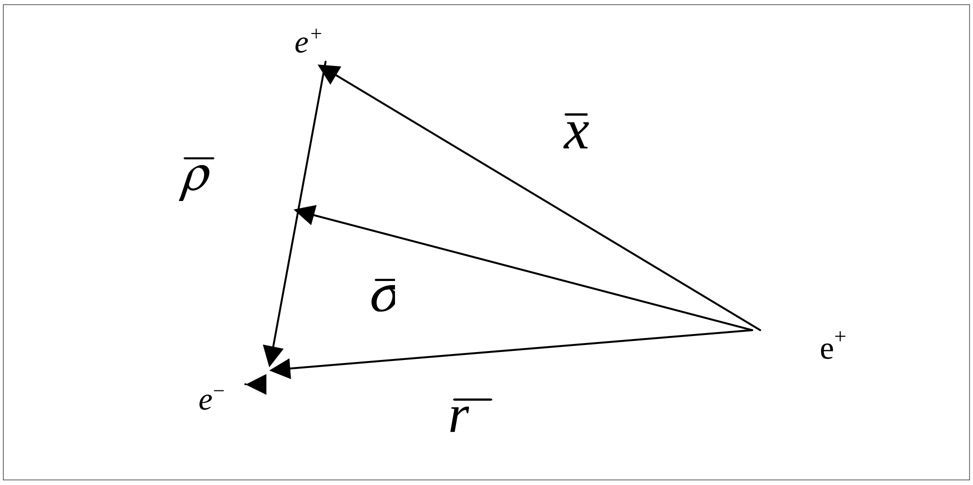

Figure 1

Theory

The two-channel problem under investigation can be sketched by

Ps (1s) + e^+ (Channel 1)

Ps (1s) + e^+ (e^-) = {

Ps*(2s+2p) + e^+ (Channel 2)

The total Hamiltonian of positron – positronium system can be written (in Rydberg units) as

$$\mathrm{H} = H_{Ps} - \frac{1}{2}\nabla^2_{\sigma} + \mathrm{V}^{(1)}_{\mathrm{int}} = -2\nabla^2_{\rho} - \frac{2}{\rho} - \frac{1}{2}\nabla^2_{\sigma} + \mathrm{V}^{(1)}_{\mathrm{int}} \qquad (1)$$

ρ is the internal distance of the positronium and σ is the distance between the centre of mass of Ps and the projectile e^+ (see Fig. 1). r is the distance between the incident positron and the positronium electron., x is the distance between the incident positron and the positronium positron. $V_{int}^{(1)}$, (see Fig 1), stands for the interaction potential between the projectile and the constituents of the positronium, i.e.

$$V_{int}^{(1)} = \frac{2}{x} - \frac{2}{r}, \tag{2}$$

where $\underline{r} = \underline{\sigma} + \frac{1}{2}\underline{\rho}, \quad \underline{x} = \underline{\sigma} - \frac{1}{2}\underline{\rho}$.

The solution of this problem is subjected to the solution of Schrödinger's equation:

$$H\,\psi = E\,\psi, \tag{3}$$

The total energy of the first channel is determined by:

$$E = E^{(1)} = E_{Ps} + k_1^2, \tag{4}$$

where k_1 is the momentum of the incident projectile relative to the centre of mass of the positronium. E_{PS}= -6.8 eV = -0.5 Ry. is the ground-state energy of the positronium.

The total Hamiltonian of the inelastic channel is obtained from (1) by replacing H_{Ps} with H_{Ps*}:

$$H = H_{Ps*} - \frac{1}{2}\nabla_{\sigma'}^2 + V_{int}^{(2)} = -2\nabla_{\rho'}^2 - \frac{2}{\rho'} - \frac{1}{2}\nabla_{\sigma'}^2 + V_{int}^{(2)} \tag{5}$$

And $$V_{int}^{(2)} = \frac{2}{x'} - \frac{2}{r'} \tag{6}$$

Also, the total energy of this channel is calculated by:

$$E = E^{(2)} = E_{Ps*} + k_2^2, \tag{7}$$

where k_2 is the momentum of the projectile relative to the centre-of-mass of the excited positronium. It is connected with the incident momentum through the relation

$$k_2^2 = E_{Ps} + k_1^2 - E_{Ps*} \tag{8}$$

where E_{PS*} = -0.125 Ry. is the energy of the excited positronium. Thus, the excited positronium channel is opened, if $E_{Ps} + k_1^2 - E_{Ps*} > 0.0$, i.e. $0.375 < k_1^2 < 0.44$ Ry. (The upper limit implies that the third channel is opened). The total wavefunction ψ of the system, (see eq. 3), is defined by

$$\left|\Psi\right\rangle = \left|\Phi_{Ps}(\rho)\right\rangle\left|\Psi_1(\sigma)\right\rangle + \left|\Phi_{Ps*}(\rho')\right\rangle\left|\Psi_2(\sigma')\right\rangle \tag{9}$$

$\Psi_1(\sigma)$ and $\Psi_2(\sigma')$ are the projectile scattering wavefunctions of the first and second channels, respectively. Φ_{Ps} and Φ_{Ps*} are the bound-state wavefunctions of the two channels. They possess the following exact forms:

$$\begin{aligned}\Phi_{Ps(1s)} &= \left(\frac{1}{\sqrt{8\Pi}}\right)\exp\left(-\frac{\rho}{2}\right),\\ \Phi_{Ps*(2s+2p)} &= \left(\frac{1}{8\sqrt{\Pi}}\right)\left(1-\frac{\rho'}{4}\right)\exp\left(-\frac{\rho'}{4}\right)+\left(\frac{1}{32\sqrt{\Pi}}\right)\rho'\cos\theta\exp\left(-\frac{\rho'}{4}\right),\end{aligned} \tag{10}$$

θ is the angle between ρ' and σ'.

The coupled-static approximation states that the solution of eq. (3) is subjected to the following conditions

$$<\Phi_{Ps}|H - E|\Psi> = 0 \tag{11a}$$

$$<\Phi_{Ps*}|H - E|\Psi> = 0 \tag{11b}$$

Substitution from (4), (7), (9) and (10) into eqs. (11a)and (11b), provides us with

$$(\nabla^2_\sigma + 2k_1^2)\left|\Psi_1\right\rangle = 2U_{st}^{(1)}(\sigma)\left|\Psi_1\right\rangle + 2\left\langle\Phi_{Ps}\left|H^{(2)} - E^{(2)}\right|\Phi_{Ps*}\Psi_2\right\rangle \tag{12a}$$

$$(\nabla^2_{\sigma'} + 2k_2^2)\left|\Psi_2\right\rangle = 2U_{st}^{(2)}(\sigma')\left|\Psi_2\right\rangle + 2\left\langle\Phi_{Ps*}\left|H^{(1)} - E^{(1)}\right|\Phi_{Ps}\Psi_1\right\rangle \tag{12b}$$

Using the partial wave expansion of the scattering wavefunctions in the form:

$$\begin{aligned}\Psi_1(\sigma) &= \frac{1}{\sigma}\sum_{\ell=0}^{\infty} i^\ell(2\ell+1)f_\ell(\sigma)Y_\ell^0(\hat{\sigma})\\ \Psi_2(\sigma') &= \frac{1}{\sigma'}\sum_{\ell=0}^{\infty} i^\ell(2\ell+1)g_\ell(\sigma')Y_\ell^0(\hat{\sigma}')\end{aligned} \tag{13}$$

We have

$$\left[\frac{d^2}{d\sigma^2} - \frac{\ell(\ell+1)}{\sigma^2} + 2k_1^2\right]f_\ell(\sigma) = 2U_{st}^{(1)}(\sigma)\, f_\ell(\sigma) + Q_1(\sigma) \tag{14a}$$

$$\left[\frac{d^2}{d\sigma'^2} - \frac{\ell(\ell+1)}{\sigma'^2} + 2k_2^2\right] g_\ell(\sigma') = 2U_{st}^{(2)}(\sigma')\ g_\ell(\sigma') + Q_2(\sigma') \quad (14b)$$

where

$$Q_1(\sigma) = \int_0^\infty K_{12}(\sigma,\sigma')\ g_\ell(\sigma')\ d\sigma' \quad (15a)$$

$$Q_2(\sigma') = \int_0^\infty K_{21}(\sigma',\sigma)\ f_\ell(\sigma)\ d\sigma \quad (15b)$$

The static potentials appearing at eqs.(12a) and (12b) are defined as:

$$\left.\begin{aligned} U_{st}^{(1)}(\sigma) &= <\Phi_{Ps}\left|V_{int}^{(1)}\right|\Phi_{Ps}> + V_{pol}(\sigma) \\ U_{st}^{(2)} &= <\Phi_{Ps*}\left|V_{int}^{(2)}\right|\Phi_{Ps*}>, \end{aligned}\right] \quad (16)$$

The fact that the target wavefunctions of the two channels are orthonormal implies that the first terms on the right hand sides of eqs. (16) vanish. The polarization potential $V_{pol}(\sigma)$ has the form (Stone [12]):

$$V_{Pol}(\sigma) = 2\ \beta(\sigma)\ V(\sigma)\ , \quad (17)$$

Where V(σ) is a potential of the form (see ref. [12])

$$V(\sigma) = (32/43)^{1/2}\{\ e^{-2\sigma}(\sigma^2 + 5\ \sigma + 9 + 9/\ \sigma) - 9\ (1 - e^{-2\sigma})/(2\ \sigma^2)\} \quad (18)$$

The calculations of the function β(σ) is accomplished as follows :

Let $\Delta E = E_{Ps}-E_{Pol}$, (where $E_{Pol} = -\ 21/\ 258$ Ry is the binding energy of the polarized positronium), and W= ΔE/ V (σ). Also, consider the function β such that

$$\beta(\sigma) = \frac{1}{2}(-W \pm \sqrt{W^2 + 4}) \quad (19)$$

The adiabatic energy of the positronium within the field of a unit positive charge is calculated (see the previous reference) by

$$E_{ad} = (1+ \beta^2)^{-1}(E_{Ps} + 2\ \beta\ V + E_{Ps*}\ \beta^2) \quad (20)$$

The correct value of β to be employed for calculating $V_{Pol}(\sigma)$ is the one of β which leads to smaller E_{ad} .

Eqs. (14a) and (14b) possess the general form

$$[\varepsilon - H_o(\sigma)]\zeta(\sigma) = \xi(\sigma) \quad (21)$$

ε - $H_0(\sigma)$ is one of the operators at the right hand side of eq. (14a) or (14b), ζ and ξ are $f_\ell(\sigma)$ or $g_\ell(\sigma)$. The solution of eq. (21) is given formally by the Lippmann-Schwinger equation:

$$\zeta = \zeta_0 + G_0 \xi \,, \tag{22}$$

where G_0 is the Green operator $(\varepsilon - H_0)^{-1}$ and ζ_0 is the solution of the homogeneous equation:

$$(\varepsilon - H_0)\ \zeta_0 = 0 \tag{23}$$

The partial-wave expansions of G_0 in the two channels enable us to express the formal solutions of (14a), 14b) as:

$$f_\ell^{(i)}(\sigma) = \left\{\delta_{i1} + \int_0^\infty \tilde{g}_\ell(k_1\sigma'')\left[2U_{st}^{(1)}(\sigma'')\ f_\ell^{(i)}(\sigma'') + Q_1^{(i)}(\sigma'')\right]d\sigma''\right\}_1 \tilde{f}_\ell(k_1\sigma)$$
$$+\left\{-\frac{1}{k_1}\int_0^\infty \tilde{f}_\ell(k_1\sigma'')\left[2U_{st}^{(1)}(\sigma'')\ f_\ell^{(i)}(\sigma'') + Q_1^{(i)}(\sigma'')\right]d\sigma''\right\}_2 \tilde{g}_\ell(k_1\sigma)\quad i = 1,2 \tag{24}$$

and

$$g_\ell^{(i)}(\sigma') = \left\{\delta_{i2} + \int_0^\infty \tilde{g}_\ell(k_1\sigma'')\left[2U_{st}^{(2)}(\sigma'')\ g_\ell^{(i)}(\sigma'') + Q_2^{(i)}(\sigma'')\right]d\sigma''\right\}_3 \tilde{f}_\ell(k_2\sigma')$$
$$+\left\{-\frac{1}{k_2}\int_0^\infty \tilde{f}_\ell(k_1\sigma'')\left[2U_{st}^{(2)}(\sigma'')\ g_\ell^{(i)}(\sigma'') + Q_2^{(i)}(\sigma'')\right]d\sigma''\right\}_4 \tilde{g}_\ell(k_2\sigma')\quad i = 1,2 \tag{25}$$

where the delta functions δ_{ij}, i.j = 1,2 specify two independent forms of solutions for each of $f_\ell(\sigma)$ *and* $g_\ell(\sigma)$ in channels i = 1,2 according to the channel considered. Thus, if i = 1, the first element in the { } bracket of $f_\ell(\sigma)$, for example, will be 1 defining the first form of solution. For i = 2 this element will be zero defining the

second form. The functions $f_\ell(\sigma)$ or $g_\ell(\sigma')$, $\mu = k_1\,\sigma$ or $k_2\,\sigma'$ are related to the Bessel functions of the first and second channels, i.e. $j_\ell(\mu)$ and $y_\ell(\mu)$, respectively, by the relations

$$\tilde{f}_\ell(\mu) = \mu\ j_\ell(\mu) \text{ and } \tilde{g}_\ell(\mu) = -\mu\ y_\ell(\mu) \tag{26}$$

It is obvious from eqs. (24) and (25) that the solutions of the coupled integro-differential equations (14a) and (14b) can be only found iteratively and the iterative solutions of the order υ are calculated by:

$$f_\ell^{(i,\upsilon)}(\sigma)=\left\{\delta i1+\frac{1}{k_1}\int_0^{\Sigma}\tilde{g}_\ell(k_1\sigma'')\left[2U_{st}^{(1)}(\sigma'')\ f_\ell^{(i,\nu)}(\sigma'')+Q_1^{(i,\nu)}(\sigma'')\right]d\sigma''\right\}_1\tilde{f}_\ell(k_1\sigma)$$
$$+\left\{-\frac{1}{k_1}+\int_0^{\Sigma}\tilde{f}_\ell(k_1\sigma'')\left[2U_{st}^{(1)}(\sigma'')\ f_\ell^{(i,\nu)}(\sigma'')+Q_1^{(i,\nu)}(\sigma'')\right]d\sigma''\right\}_2\tilde{g}_\ell(k_1\sigma),\ i=1,2;\ \upsilon\geq 0 \tag{27}$$

$$g_\ell^{(i,\upsilon)}(\sigma')=\left\{\delta i2+\frac{1}{k_2}\int_0^{\Sigma}\tilde{g}_\ell(k_2\sigma'')\left[2U_{st}^{(2)}(\sigma'')\ g_\ell^{(i,\upsilon)}(\sigma'')+Q_2^{(i,\upsilon)}(\sigma'')\right]d\sigma''\right\}_3\tilde{f}_\ell(k_2\sigma')$$
$$+\left\{-\frac{1}{k_2}+\int_0^{\Sigma}\tilde{f}_\ell(k_2\sigma'')\left[2U_{st}^{(2)}(\sigma'')\ g_\ell^{(i,\upsilon)}(\sigma'')+Q_2^{(i,\upsilon)}(\sigma'')\right]d\sigma''\right\}_4\tilde{g}_\ell(k_2\sigma'),\ i=1,2;\ \upsilon\geq 0 \tag{28}$$

The zero-order iteration of $f_\ell(\sigma)$ is determined by

$$f_\ell^{(i,0)}(\sigma)=\left\{\delta i1+\frac{1}{k_1}\int_0^{\Sigma}2\tilde{g}_\ell(k_1\sigma'')\ U_{st}^{(1)}(\sigma'')f_\ell^{(i,0)}(\sigma'')\ d\sigma''\right\}_1\tilde{f}_\ell(k_1\sigma)$$
$$+\left\{-\frac{1}{k_1}\int_0^{\Sigma}2\tilde{f}_\ell(k_1\sigma'')U_{st}^{(1)}(\sigma'')f_\ell^{(i,0)}(\sigma'').d\sigma''\right\}_2\tilde{g}_\ell(k_1\sigma),\ i=1,2 \tag{29}$$

where Σ specifies the integration range (IR) away from the nucleus over which the integrals at eqs. (27)-(29) are calculated using Simpson's expansions. Eqs. (27), (28) and (29) can be abbreviated to :

$$\left.\begin{aligned} f_\ell^{(i,\nu)}(\sigma) &= a_1^{(i,\upsilon)}\tilde{f}_\ell(k_1\sigma)+b_1^{(i,\upsilon)}\tilde{g}_\ell(k_1\sigma),\ i=1,2,\\ g_\ell^{(i,\nu)}(\sigma') &= a_2^{(i,\upsilon)}\tilde{f}_\ell(k_2\sigma')+b_2^{(i,\upsilon)}\tilde{g}_\ell(k_2\sigma'),\ i=1,2,\end{aligned}\right] \tag{30}$$

where

$$a_1^{(i,\upsilon)}=\{\ \}_1,\ b_1^{(i,\upsilon)}=\{\ \}_2,\ a_2^{(i,\upsilon)}=\{\ \}_3,\ b_2^{(i,\upsilon)}=\{\ \}_4.$$

The coefficients at (30) are elements of the following matrices :

$$a^{\upsilon}=\begin{pmatrix}\sqrt{1/k_1}\,a_1^{(1,\upsilon)} & \sqrt{1/k_1}\,a_1^{(2,\upsilon)}\\ \sqrt{1/k_2}\,a_2^{(1,\upsilon)} & \sqrt{1/k_2}\,a_2^{(2,\upsilon)}\end{pmatrix},$$

and

$$b^{\upsilon}=\begin{pmatrix}\sqrt{1/k_1}\,b_1^{(1,\upsilon)} & \sqrt{1/k_1}\,b_1^{(2,\upsilon)}\\ \sqrt{1/k_2}\,b_2^{(1,\upsilon)} & \sqrt{1/k_2}\,b_2^{(2,\upsilon)}\end{pmatrix} \tag{31}$$

which are connected with the reactance matrix, , R^{υ} through the relation

$$R^{\upsilon}=b^{\upsilon}\left(a^{\upsilon}\right)^{-1} \tag{32}$$

On the other hand, the transition matrix, T^{υ} , is related to R^{υ} by:

$$T^{\upsilon}=R^{\upsilon}\left(I-\tilde{i}R^{\nu}\right)^{-1} \tag{33}$$

where I is a 2 x 2 unit matrix and $\tilde{i}=\sqrt{-1}$. The partial cross sections obtained in an iterative way are determined (in a_o^2) using the relations :

$$\sigma_{ij}^{(i,\nu)}=\frac{4\pi(2\ell+1)}{k_i^2}\left|\,T_{ij}^{\nu}\,\right|^2 \tag{34}$$

where $\left|T_{ij}^{\upsilon}\right|^2$ can be obtained from (33) through multiplication with its complex conjugate. This leads to the following explicit forms:

$$\begin{aligned}\left|T_{11}^{\upsilon}\right|^2 &= \frac{1}{D^2}\left\{\cdot\left[R_{11}\right]^2+\left[R_{12}^2-R_{11}R_{22}\right]^2\right\}\\ \left|T_{12}^{\upsilon}\right|^2 &= \frac{1}{D^2}\left[R_{12}\right]^2\end{aligned} \tag{35}$$

where

$$D^2=\left[\left(1+R_{12}^2\right)-\left(R_{11}R_{22}\right)\right]^2+\left[\left(R_{11}+R_{22}\right)\right]^2. \tag{36}$$

The total cross sections (in a_o^2) are expressed (in the υ th iteration) as :

$$\sigma_{ij}^{\upsilon}=\sum_{l=0}^{\infty}\sigma_{ij}^{(l,\upsilon)},\ \ i,j=1,2 \tag{37}$$

where $\boldsymbol{\sigma_{11}^{\upsilon}}$ is the total elastic cross section of the positrons scattered from the positronium, $\boldsymbol{\sigma_{12}^{\upsilon}}$ is the total excited positronium formation cross section.

Results and discussions

We started our investigation by testing the variation of the polarization potentials of the first channel V (σ) and $V_{pol,}$, eqs. (17) and (18), respectively, with the increase of σ. The values of σ were chosen such that $\sigma = \sigma' = \frac{1}{16}, \frac{2}{16}, \frac{3}{16}, ..., \frac{512}{16}$, where $h = \frac{1}{16}$ is the mesh size (or Simpson's step) employed for calculating the integrals in (27) - (29) using Simpson's rule. The results of these calculations are demonstrated in Figure 2. The figure shows that both potentials are attractive. In Figure (3) we illustrate the behavior of the quantity $-\sigma^4 V_{pol}$. It is obvious from the figure that this quantity converges to the value -36 which accounts for the polarizability of the positronium in the field of a unit charge. The results also show that the static potentials of the positron-positroniums scattering fall very rapidly .

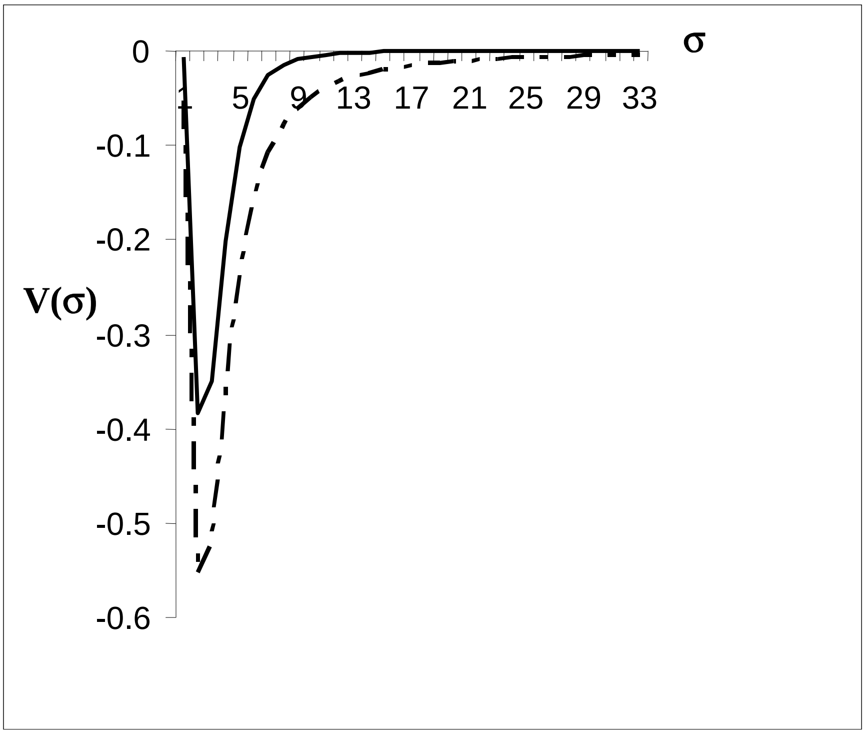

Figure 2: Variation of $V_{12}(\sigma)$ (__ . __) and $V_{pol}(\sigma)$ (____) with σ.

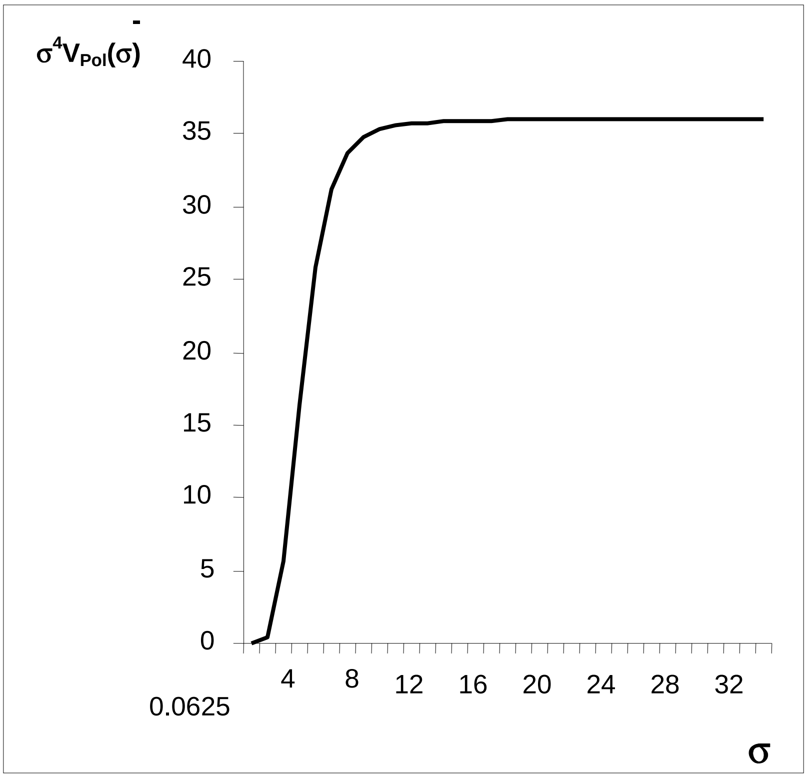

Figure 3: Asymptotic behavior of – σ^4 Vpol(σ).

The calculation of the cross sections of e^+ - Ps scattering has been preceded by investigation the variation of the elements of R^{υ} with the increase of the integration range (IR) and the number of iterations(see Table 1). (IR is related to the number of mesh points n and the mesh h (=1/16) by IR= nh. at n = 512, IR =32 a.u). We found that the results are sensitive to n and the number of iterations υ. Preliminary calculations show that more than 256 points are needed to obtain reasonable convergence and stable values for the elements of the reactance matrix and the partial cross sections. It is intended to use the final values of n and υ to calculate the partial cross sections corresponding to seven partial waves ranging from ℓ= 0 to ℓ= 6 at two regions of incident energy. The first lying in the range $0.375 < k_1^2 < 0.44$ Ry. The second region starts at 10.0 eV and ends at 500 eV which is suitable for comparison with the results mentioned in the literature for positron-hydrogen scattering .

The partial and total elastic cross sections of positron-positronium scattering σ_{11} are tabulated in Table 2 for incident energy lying in the range $0.375 < k_1^2 < 0.44$ Ry., This Table emphasizes that the total elastic cross section decreases smoothly with the increase of the incident energies. Table 3 contains the partial and total excited positronium cross-sections of positron-positronium scattering for the considered range of energies (see also Figures 4 and 5). The seven partial waves employed are quite satisfactory for calculating the σ_{12} to a high degree of accuracy within the framework of the coupled-static approximation. We also notice that the total excited positronium formation cross section σ_{12} have maximum value between the incident energies .395 Ry. and .405 Ry.

Table 1: Variation of the elements of the p-wave elastic cross section R-matrix with the number of iteration (υ) at k_1^2 =0.38 eV and integration range (IR) = 32 a_o

υ	R11	R12	R22
2	52.693492	-0.5438852	0.0447479
5	49.589008	0.6383906	0.0503000
10	49.587952	0.6385514	0.0503001
15	49.587958	0.6385516	0.0503003
20	49.587958	0.6385516	0.0503003
25	49.587958	0.6385516	0.0503003
30	49.587958	0.6385516	0.0503003
40	49.587958	0.6385516	0.0503003

Table 2: Present Partial and total elastic cross-sections $\sigma_{11}(\pi\ a_o^2)$ of positron-positronium scattering.

K^2 Ry.	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	TOTAL
0.38	9.38032	31.55570	26.96221	6.50492	2.06782	1.50746	0.94298	78.92142
0.385	9.08945	30.99381	26.82892	6.57588	2.04400	1.39456	0.97500	77.90162
0.39	8.93728	30.21846	26.68834	6.64343	2.03290	1.29685	0.98175	76.79900
0.495	8.84000	29.58452	26.54310	6.71582	2.03168	1.21366	0.96875	75.89753
0.4	8.77234	29.06665	26.39269	6.79326	2.03790	1.14395	0.94137	75.14817
0.405	8.72301	28.58767	26.23277	6.87213	2.04959	1.08649	0.90439	74.45605
0.41	8.68498	28.12001	26.05986	6.94824	2.06519	1.03999	0.86179	73.78005
0.415	8.65348	27.65839	25.87456	7.01858	2.08354	1.00314	0.81676	73.10846
0.42	8.62530	27.20313	25.68163	7.08196	2.10373	0.97469	0.77173	72.44217
0.425	8.59833	26.75472	25.48790	7.13869	2.12504	0.95343	0.72843	71.78654
0.43	8.57118	26.31286	25.29977	7.18998	2.14687	0.93824	0.68806	71.14694
0.435	8.54290	25.87684	25.12148	7.23730	2.16869	0.92805	0.65133	70.52659

Table 3: Present partial and total excited positronium cross-sections $\sigma_{12}(\pi\ a_o^2)$ of positron-positronium scattering.

K^2 Ry.	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=5$	$\ell=6$	TOTAL
0.38	0.55587	0.00521	0.00328	0.05500	0.00002	0.00015	0.00000	0.61953
0.385	0.62689	0.07865	0.00069	0.18525	0.00003	0.00231	0.00003	0.89385
0.39	0.63498	0.26578	0.00003	0.28847	0.00117	0.00944	0.00032	1.20020
0.495	0.61949	0.38823	0.00027	0.31677	0.00604	0.02192	0.00158	1.35430
0.4	0.59186	0.45704	0.00333	0.27799	0.01581	0.03680	0.00500	1.38784
0.405	0.55707	0.51030	0.01463	0.20382	0.02848	0.04963	0.01188	1.37582
0.41	0.51824	0.56170	0.03805	0.12561	0.03967	0.05681	0.02298	1.36305
0.415	0.47751	0.61381	0.07264	0.06319	0.04510	0.05704	0.03800	1.36729
0.42	0.43635	0.66645	0.11288	0.02360	0.04266	0.05129	0.05551	1.38873
0.425	0.39571	0.71933	0.15125	0.00466	0.03329	0.04194	0.07318	1.41936
0.43	0.35624	0.77257	0.18110	0.00000	0.02034	0.03166	0.08845	1.45037
0.435	0.31839	0.82641	0.19855	0.00284	0.00826	0.02250	0.09913	1.47607

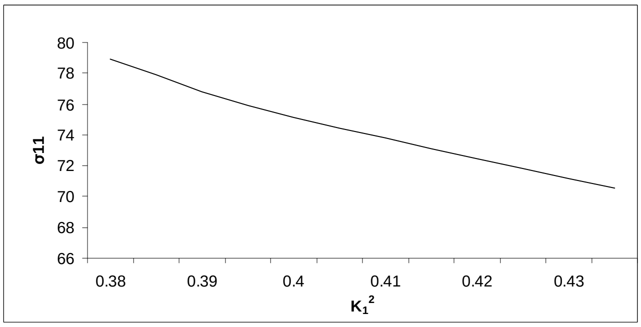

Figure 4: Present Total Elastic Positronium Cross Sections σ_{11} ($\pi\ a_o^2$) of Positron-Positronium Scattering.

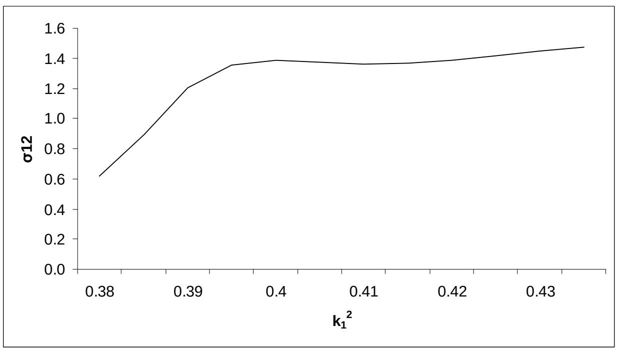

Figure 5: Present Total Exited-Positronium(2s+2p) Cross Sections σ_{12} ($\pi\ a_o^2$) of Positron-Positronium Scattering by the Coupled-Static Approximation.

The total non-exited and excited positronium formation cross sections (measured in $\pi\ a_o^2$ units) of the present work are tabulated in Tables 4 and 5 for energy range starts at 10.0 eV and ends at 500 eV which is suitable for comparison with the results given in the literature for e+-H . From Table 4, we notice that our total cross sections for the considered range of energy behave similar to those presented in Table 2. The values of the excited positronium cross sections are given in Table 5.

In Tables 6, our results of the total cross sections of positronium formation in excited (2s+2p)-state for e+-Positronium are compared with the available results of e+-Hydrogen of Nahar [8] , Saha and Roy [11], Khan and Mazumdar [10] and Basu and Ghosh [9] (in units of πa_o^2) at incident positron energies 20-500 eV. We notice that, our values are larger in size than the compared results We hope that the present work would attract both theorists and experimentalists to investigate the present scattering process in detail in future

Table 4: Partial and total elastic cross-sections $\sigma_{11}(\pi\ a_o^2)$ of positron-positronium scattering at different incident energies.

E eV.	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=5$	$\ell=6$	TOTAL
10	3.74438	13.87922	16.89003	8.32559	3.03350	1.20890	0.55890	47.64052
20	1.75027	5.75369	7.65815	6.14589	3.46584	1.70946	0.84556	27.32887
30	0.91556	3.00298	4.31468	4.14910	2.97170	1.77848	0.99011	18.12260
40	0.54543	1.81874	2.76425	2.93484	2.41696	1.66537	1.03714	13.18273
50	0.35925	1.21295	1.91930	2.17433	1.95937	1.49313	1.01947	10.13779
60	0.25399	0.86376	1.40770	1.67070	1.60380	1.31592	0.96660	8.08246
75	0.18899	0.64491	1.07440	1.32097	1.32923	1.15380	0.89828	6.61057
100	0.16549	0.56506	0.94986	1.18500	1.21588	1.08027	0.86218	6.02374
200	0.14613	0.49896	0.84546	1.06844	1.11561	1.01190	0.82604	5.51253
300	0.11637	0.39701	0.68148	0.88054	0.94709	0.89021	0.75550	4.66820
400	0.09489	0.32305	0.56020	0.73708	0.81244	0.78642	0.68952	4.00360
500	0.02454	0.08108	0.14540	0.20778	0.25779	0.28867	0.29856	1.30383

Table 5: Present partial and total excited positronium cross-sections $\sigma_{12}(\pi\ a_o^2)$ of positron-positronium scattering at different incident energies.

E eV.	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	$\ell=5$	$\ell=6$	TOTAL
10	0.72055	0.56152	0.01248	0.22863	0.40582	0.29647	0.17385	2.39932
20	0.02781	0.00104	0.07305	0.27417	0.47062	0.53158	0.49733	1.87561
30	0.00076	0.03604	0.10567	0.20427	0.29930	0.34738	0.35226	1.34569
40	0.00527	0.04122	0.08731	0.14100	0.18835	0.21847	0.22507	0.90670
50	0.00622	0.03378	0.06460	0.09654	0.12370	0.14183	0.14787	0.61456
60	0.00550	0.02551	0.04660	0.06726	0.08424	0.09605	0.10063	0.42578
75	0.00445	0.01886	0.03378	0.04778	0.05934	0.06728	0.07099	0.30248
100	0.00396	0.01621	0.02891	0.04056	0.05037	0.05690	0.06038	0.25730
200	0.00350	0.01399	0.02479	0.03467	0.04296	0.04854	0.05162	0.22007
300	0.00272	0.01051	0.01841	0.02577	0.03171	0.03607	0.03830	0.16349
400	0.00213	0.00797	0.01395	0.01942	0.02401	0.02724	0.02920	0.12391
500	0.00029	0.00096	0.00166	0.00236	0.00297	0.00350	0.00387	0.01561

Table 6: Comparison of the total Cross Sections $\sigma_{12}(\pi\ a_o^2)$ for the formation of positronium in the (2s+2p)-state from a hydrogen atom and from a positronium (present work).

E Ry.	PRESENT e+-Ps	Ref.[8] e^+-H	Ref.[11] e+-H	Ref[10] e+-H	Ref.[9] e+-H
20	1.87561	0.41858	0.35523	0.346	
30	1.34569	0.28361	0.27216	0.24287	
40	0.90670	0.16425			
50	0.61456	0.09422	0.09963	0.07671	0.20149
100	0.12391	0.00939	0.01089	0.0105	0.01458
200	0.01561	0.00049	0.00061		0.00057
300	0.00397	0.00007			0.00008
500	0.00063	0.000006	0.000007		0.000006

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