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Pair Annihilation and Self Energy

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Abstract

Electrostatic self energy has always been a problem of perplexing magnitude. Matters become more complicated when we try to view the problem of pair annihilation in relation to self energy in the light of the classical laws. Where does the self energy go to when pair annihilation takes place? The field theories try to solve this problem intelligently with the help of the virtual photons. The interaction between the charged particle and the virtual photons creates self energy which is taken care of by the Feynman-Diagrams. But is this representation accurate enough to replace the classical self energy in all its totality? Is it at all possible to explain the self energy problem with the classical laws? Such matters have been investigated in this article.

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Introduction

Pair annihilation [1][2] is a commonly discussed topic in modern physics. A simple example is the case where an electron and a positron annihilate themselves to produce a pair of photons, the process being consistent with the principles of energy and momentum conservation. But in analyzing the process do we take into account the huge amount of static self energy [3] associated with the charges despite the fact that electrostatic energy is as real as any other form of energy?

Let us take a look at one of the remarks of Richard Feynman [4] in relation to self energy --------“What is wrong with an infinite energy? If energy cannot get out but must stay there for ever is there any difficulty with an infinite energy? Of course a quantity that works out to be infinite may be annoying but what really matters is whether there is any observable physical effect.” But in the event of pair annihilation is it reasonable to assume that self energy remains in tact despite the fact that the charges are no more there? And if self-energy really gets destroyed should it not reappear somewhere else, in conformity with the principle of energy conservation? Do we have a clear account of that?

Let us have a look at the above queries in the light of the quantum field theory. Pair annihilation is portrayed in this theory by the Feynman –diagrams [5]. Now the basic relativistic equation that relates to the energy of a particle (even in the realm of quantum field theory) is given below:

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

For a charged particle the above equation does not take into account the static field energy given below:

$$\text{Self-Energy} = \int [\frac{1}{2}\,\epsilon_0\, E^2 + \frac{1}{2\mu_0}\, B^2]dV \tag{2}$$

(All space)

Only if the value of the self-energy remains unaltered (even in the event of pair annihilation!) can the first equation do its job well in describing the energy changes that take place in processes involving charged particles. That is exactly what we intend to validate in the present article.

The modern field theory seeks to replace the action of the classical static field by the action of the virtual photons and hence the classical self energy by the energy gained by the interaction of the particle with the virtual photons.[The whole process is embodied in Dyson's Equation[6]] Interestingly the virtual photons are short-lived and should not be used to explain the interaction between two charges separated by a large distance. Let us have a look at the uncertainty relation:

$$\Delta E\Delta t\sim h/2\pi$$

If Δt is large, that is, the particles are separated by a large distance the uncertainty in energy becomes vanishingly small and the virtual photons should become real photons .This does not occur in practice. (Also real photons should satisfy the conservation of energy principle and in this case we have a theoretical contradiction).So the interaction between charged particles separated by a large distance cannot be ascribed to the action of virtual photons. We have to rely on the action of the classical electromagnetic field. Thus the classical static field is not fully accounted for by the action of the virtual photons .One thing has to be kept in mind. The static field cannot be given an oscillator expansion. The technique of the oscillator expansion can be given only to a radiation field (which finally leads to the creation and the annihilation operators).So the static field cannot be quantized into the action of the annihilation and the creation operators.(Even if we could do so the action of the virtual photons have to be dispensed with in that case) Now let us have a look at the relation:

$$E^2=p^2c^2 + m_0{}^2c^4$$

Here the correction induced on the bare mass takes care of the self energy. The interaction with the virtual photons increases the momentum which in turn accounts for the self energy. It gets incorporated into the mass of the particle (observed mass/corrected mass).But what about the self energy at a large distance from the particle/particles? It remains unaccounted for in the correction induced on the bare mass to produce the corrected mass. Some method has to be devised to remedy the situation. My article intends to do that. .

The basic aim of the article is to prove the classical self-energy indeed stays out there forever -------- even in the event of pair annihilation. The methodology adopted in the above context is as follows:

We shall treat the electron-positron pair approaching each other as a radiating dipole. The outwardly flowing radiation field will push out the static field energy. There will be no destruction of self energy as such. Only it will get pushed to an infinite distance. An interesting fact shall be deduced in relation to the above procedure. The decaying static field will produce an inwardly directed Poynting vector trying to make the self-energy move towards the annihilation point. The outwardly directed Poynting vector due to the expanding radiation field will overpower the inward energy flux resulting in a net outward flow of energy. There will be no loss of energy in the process.

Radiation from an arbitrary charge distribution (A Recapitulation)

Let us consider an arbitrary distribution of moving charges (obviously involving accelerations)

Confined to a small region of space as shown below:

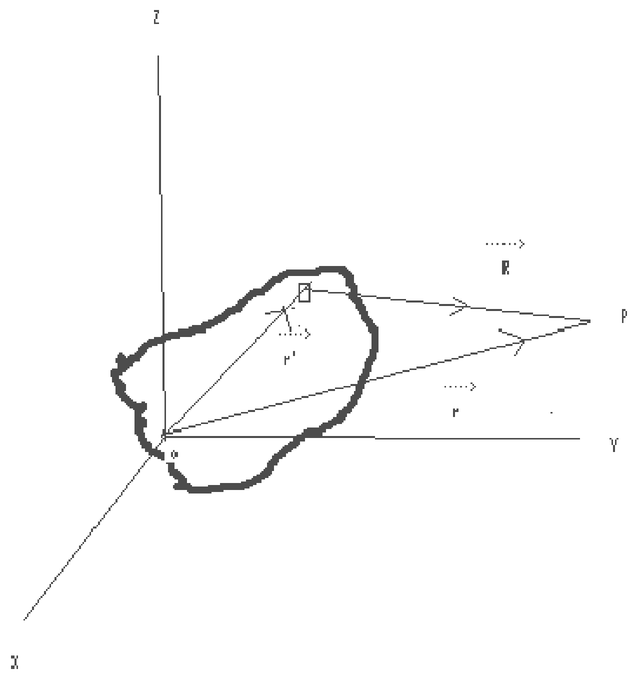

Figure 1

We have the following formulas [7](from III to VIII):

The scalar and the vector potentials are given by:

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r} + \hat{r}\,.\frac{\vec{p}(t_0)}{r^2} + \hat{r}\,.\frac{\dot{\vec{p}}(t_0)}{rc}\right] \tag{3}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\frac{\dot{\vec{p}}(t_0)}{r} \tag{4}$$

Q: Total Charge
t_0 :Retarded time wrt origin=$t - R/c$

$\vec{p}$: Dipole Moment=$\int \rho(\vec{r}'t)d\tau$,where $d\tau$ is the volume element

For a dipole like the electron-positron pair Q=0 .

The equations for E and B in spherical polar coordinates are given by:

$$\vec{E}(r,\theta,t) = \frac{\mu_0}{4\pi}\,\ddot{\vec{p}}\,(t_0)\left(\frac{sin\theta}{r}\right)\hat{\theta} \tag{5}$$

$$\vec{B}(r,\theta,t) = \frac{\mu_0}{4\pi c}\,\ddot{\vec{p}}\,(t_0)\left(\frac{sin\theta}{r}\right)\hat{\phi} \tag{6}$$

The Poynting vector takes the form :

$$\vec{S} = \frac{1}{\mu_0}[\vec{E}\times\vec{B}]$$
$$= \frac{\mu_0}{16\pi^2 c}\,\ddot{\vec{p}}\,(t_0)^2\frac{sin^2\theta}{r^2}\hat{r} \tag{7}$$

Total power radiated through a sphere of radius ‘r’ is given by:

$$P = \int S.d\vec{a} = \frac{1}{4\pi\epsilon_0}\frac{2\ddot{p}^2}{3c^3} \tag{8}$$

Now two important inferences may be made in relation to the above formulas:

(1) The electromagnetic field surrounding the dipole is created by the radiation flowing out of it.
(2) If the charges get annihilated the oscillatory field flows out to infinity preserving its ENERGY CONTENT.

We shall examine the two postulates mentioned above before proceeding further.

Let us assume that the charges are initially at rest and then they are set into motion (involving accelerations, just as we have in the case of the positron-electron pair. Radiation spreads out from the distribution in an expanding sphere with the speed of light. In the interval from time t=t_1 to time t=t_2 the sphere expands from radius r_1=ct_1 to r_2=ct_2..[see figure (1) below] The to total amount of energy flowing out through the boundary of the smaller sphere(r=r_1) in the interval t_2-t_1 is given by(using equn (VIII)):

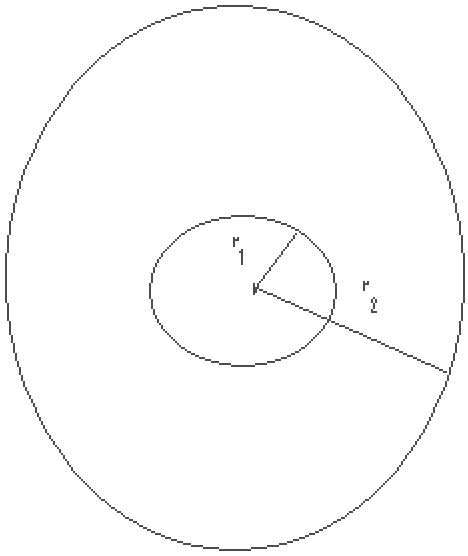

Figure 2

Energy Radiated= $\frac{1}{4\pi\epsilon_0}\frac{2}{3c^3}\int_{ct_1}^{ct_2}\ddot{p}^2\ dt$ (9)

The total amount of field energy in the annular shell(between r=ct_1 and r=ct_2) is given by :

Total energy in the annular shell $=\int_{Annularspace}[\frac{1}{2}\epsilon_0 E^2+\frac{1}{2\mu_0}B^2]dV$

$=\int_{ct_1}^{ct_2}\int_0^{2\pi}\int_0^{\pi} r^2\sin\theta[\frac{\epsilon_0}{2}E^2+\frac{1}{2\mu_0}B^2]d\theta d\phi\ dru$ (10)

Using the formulas (V) and (VI) and noting that r=ct, the above integral works out to

Energy contained in the annular shell= $\frac{1}{4\pi\epsilon_0}\frac{2}{3c^3}\int_{ct_1}^{ct_2}\ddot{p}^2\ dt$ (11)

The above result is in perfect agreement with equation (IX)

When annihilation takes place the charges stop radiating energy. But the existing oscillatory field continues to propagate outwards in the form of wave motion, always preserving its energy content

The Electron-Positron Dipole

The electron-positron pair behaves as an oscillating dipole to which we may apply the results of the previous section. The dipole moment is given by

$|(\vec{p})| = f(t)$

Assuming that the direction of $\vec{p}$ is fixed we carry out a Fourier decomposition of $f(t)$ on the time interval (0,2T).

We write

$$f(t) = a_0/2 + \sum_1^\infty [a_n \cos\omega t + b_n \sin\omega t] \quad (12)$$

Where,

$\omega = \frac{n\pi t}{T}$

$$a_0 = \frac{1}{T}\int_0^{2T} f(t)dt$$

$$a_n = \frac{1}{T}\int_0^{2T} f(t)\frac{\cos n\pi t}{T}dt$$

$$b_n = \frac{1}{T}\int_0^{2T} f(t)\frac{\sin n\pi t}{T}dt$$

Thus we have a huge number of radiating sinusoidal dipoles (elementary dipoles) which represents the action of the original one. To each of these dipoles we may apply the results of the previous section.

Annihilation of An Elementary Dipole

Let us assume that the elementary (sinusoidal) dipole lies vertically along the z-axis with its centre at the origin. Initially it is a static dipole with positive charge is in the northern hemisphere. Distant points do assume this situation in the event of an annihilation process . The radial and the cross radial components are given by:

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3} \quad (13)$$

$$E_\theta = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3} \quad (14)$$

When the information annihilation reaches a field -point the value of $\vec{E}$ decreases to zero value and hence the rate of change of the electric field produces a magnetic field in accordance with the fourth equation of Maxwell (in the absence of free currents)l:

$$\bigtriangledown \times \vec{B} = \mu_0 \epsilon_0 \frac{d\vec{E}}{dt} \tag{15}$$

The magnetic field so produced along with the existing part of the electric field produces an inwardly directed Poynting vector. This we shall prove now.

Let us consider sphere of radius R with the center at the origin.(see figure(2) below) In the northern hemisphere we draw a spherical triangle enclosed by the curves $\phi = k_1$, $\phi = k_2$ and $\theta = k_3$(the tip of the triangle obviously touches the axis on the north pole side).We apply Stoke's theorem to the above contour:

Stoke's Theorem:

$$\oint_C \vec{B}.d\vec{l} = \int_S \bigtriangledown \times \vec{B} d\vec{a} \tag{16}$$

By equation (XV) the above relation reduces to:

$$\oint_C \vec{B}.d\vec{l} = \mu_0 \epsilon_0 \int_S \frac{d\vec{E}}{dt} d\vec{a} \tag{17}$$

With the left hand side integral we choose an anticlockwise direction of integration (as indicated in the figure below) so that the normal to the surface for the right hand side integral is outwardly directed. (We shall apply this convention for both the northern and the southern hemispheres in our subsequent arguments)

[In the figure below the dipole is assumed to be at the origin and the axis along the Z-axis]

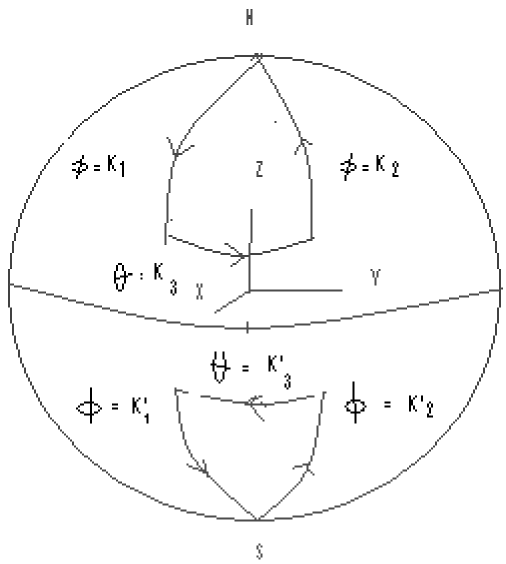

Figure 3

Now $\vec{E}_r$ is outwardly pointing in the northern hemisphere (see formula (XIII)) and its magnitude decreases with time. Hence $\frac{d\vec{E}}{dt}$ is a negative and the right hand side of the integral is negative. Therefore the left hand side is also negative. Now the value of this integral (direction if integration is anticlockwise) cancels along the curves $\phi = k_1$, and $\phi = k_2$, due to azimuthal symmetry. Only the contribution from the curve $\theta = k_3$ remains and this has to be negative according to our previous arguments. Since the value of $\vec{B}$ should be constant along the above curve (again by azimuthal symmetry) we may assert that the direction of $\vec{B}$ is along the direction of decreasing ϕ.Now $\vec{E}_\theta$ is in the direction of increasing θ(see formula XIV).The two vectors $\vec{E}_\theta$ and $\vec{B}$ produce a Poynting vector pointing towards the origin.

Now on to the southern hemisphere: .We draw a spherical triangle enclosed by the curves $\phi = k'_1$, $\phi = k'_2$ and $\theta = k'_3$(the tip of the triangle obviously touches the axis on the south pole side).W apply Stoke's theorem to the above contour:

Now $\vec{E}_r$ is inwardly pointing in the southern hemisphere(see formula (XIII)) and its magnitude decreases with time. Hence $\frac{d\vec{E}}{dt}$ is positive and the right hand side of the integral is positive. Therefore the left hand side is also positive. Now the value of this integral (direction of integration is anticlockwise) cancels along the curves $\phi = k'_1$, and $\phi = k'_2$, due to azimuthal symmetry. Only the contribution from the curve $\theta = k'_3$ remains and this has to be positive according to our previous arguments. Since the value of $\vec{B}$ should be constant along the above curve (again by azimuthal symmetry) we may assert that the direction of $\vec{B}$ is along the direction of decreasing ϕ.Now $\vec{E}_r$ is in the direction of increasing θ(see formula(XIV)).The two vectors $\vec{E}_r$ and $\vec{B}$ produce a Poynting vector pointing towards the origin.

Thus the net effect of the destruction of an elementary sinusoidal dipole is the is the interaction between outwardly moving radiant energy and inflowing field energy of the so long static field.

Integration of The Elementary Dipoles

The original electron-positron dipole may be represented by the numerous elementary dipoles.

If the charges are initially assumed to be at rest we had a static field extending up to infinity. When the charges get into an accelerated motion we have a radiation which creates an oscillatory electromagnetic field surrounding the dipole beyond which we have a static field which assumes the initial STATIC CONFIGURATION of the dipole. The process of growth of the static field may be given the following interpretation---------growth of the oscillatory field coupled with the decay of the static field. The growing oscillatory field produces an outwardly pointing Poynting vector while the decaying static field produces an inwardly directed Poynting vector. Obviously the outwardly pointing vector has to be stronger. This is borne out by the experimental fact that an oscillating dipole(or a set of randomly moving charges) indeed radiates energy! When annihilation takes place further radiation stops but the existing oscillatory field continues to move forward by the action of the Poynting vector, pushing out the static field. Indeed there is no destruction of energy and the validity of equation (I) is substantially established. And that resolves the problem.

The Quantum Perspective

The treatment of this article is based primarily on classical lines but it would be quite interesting to make a passing reference to quantum theory here. We have the equation of continuity [8]:

$J^{\mu}{}_{,\mu} = 0$ (18)

Where,

$\rho = \psi^{\dagger}\psi$

$J^{0} = c\rho = c\psi^{\dagger}\psi$

$J^{k} = c\psi^{\dagger}\alpha^{k}\psi$, k=1, 2, 3

[$j, \rho, \alpha \quad \psi \quad and \quad \psi^{\dagger}$ have their usual meanings]

Equivalently we may write,

$\frac{d\rho}{dt} + Div\vec{J} = 0$ (19)

Using the divergence theorem to transform the above equation we may write

$$\int_{V} \rho dV = \int_{S} \vec{J}.d\vec{a} \quad (20)$$

The above relations are consistent with the Dirac equation. It is clear that in the event of annihilation the quantity $\rho = \psi^{\dagger}\psi$ is not getting destroyed but it is flowing out of the volume where the charges are contained. It stays out there in space. So the electric (or magnetic) field associated with the psi function should not get destroyed— it must stay out there for ever in some form. It is the conservation of the quantity $\rho = \psi^{\dagger}\psi$ that leads to the success of equation (I). Since the psi function is not getting destroyed, the field associated with stays out somewhere in space and this idea falls in line with the classical analysis.

Conclusion

It is clear from the above considerations that self energy (static field energy) stays out there or ever, even in the event of the annihilation processes. This accounts for the success of our equations describing the annihilation events despite the fact that we tend to ignore completely the association of static self energy in these processes. Basically our logic has been based on classical principles though we have made some references to the quantum field theory.

Acknowledgements

Finally I express my indebtedness to all the authors whose works have inspired me to maintain an active interest in the subject of physics.

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