International Journal of Pure and Applied Physics
ISSN 0973-1776 Volume 5, Number 1 (2009), pp. 41–47

http://www.ripublication.com/ijpap.htm

The Nuclear Field Produced by Relativistic Harmonic Oscillator Light Mesons

A.A. Rajabi

Physics Department, Shahrood University of Technology, P.O.Box 36155-316
Shahrood, Islamic Republic of Iran
E-mail: a.a.rajabi@shahroodut.ac.ir

Abstract

The strong nuclear interaction does not depend on the electric charge. It seemes natural to assume that the nuclear introduction could be associated with the presence of special nucleon charge g in nucleons, and a harmonic oscillator (H.O.) for confinement. Getting a correct treatment, we consider the exchange of spin zero particles, the spin zero particle like π- meson field with harmonic oscillator potential interacts with nucleon through the Kelein – Gordon Fok equation (K.G.F). In this model, we can find the known strong nuclear interaction. Hence harmonic oscillator potential is a confinement potential, causes each nucleon surrounded by a cloud of mesons.

Keywords: Harmonic Oscillator, π- mesons, Kelein – Gordon, Strong nuclear force, nucleon.

Introduction

In a traditional nuclear text one learns that a reasonable semi quantitative picture of the nucleon-nucleon (N-N) interactions provided by summing over exchange of light mesons $\pi, \rho, \omega, \eta, \sigma, \ldots$ between these nucleons, each having its own coupling and Yukawa potential but in this article we introduce the extra (H.O.) interacting potential. The range of usage of (H.O.) potential is so broad, e. g. from molecular vibrations to the behavior of quantized fields and another one on rotation –vibration spectra of diatomic molecules [1]. It is also worth to none a model of n- particle systems confined in a harmonic oscillator potential [2]. Harmonic oscillator (H.O.) potential may be used to describe spatial confinement of quantas, as for example the effects of the embedding of particle in nano- catties, or in fuller ones, liquid helium. A discussion of these issues and more reference to the literature may be found in [3, 4]. The Schrödinger equation for an electron in a uniform magnetic field confined by a harmonic oscillator potential was solved in 1928 by Fock [5] and 2 years later again by Darwin [6]. Though the non-relativistic harmonic oscillator seems to be well understood, there are many recent studies dealing with this subject. A relativistic generalization of (H.O.) is far from being trivial and is certainly not unique. In particular the model of generalization depends upon the definition of the relativistic harmonic oscillator (R.H.O.). Already in early (1930) Nikolsky [7] and Postepska [8] considered Dirac equation for an electron in the field of quadratic potential. The resulting eigenvalue problem reduces to a quadratic equation with no bound solutions. The (N.R.H.O.) discrete energy levels correspond in this oscillator resonances. Relativistic system which have an infinite number of bound states whose energy levels are all equally spaced have recently been discussed [9]. Such a system has been constructed using the inverse scattering method [9, 10]. Another approach, leading to the so called Dirac oscillator is based on a construction of the Dirac equation which is exactly solved and in the non-relativistic limit gives the

Schrödinger (H.O.) equation [11, 12, 13, 14 and 15]. During the next decades harmonium [16, 17, 18, and 19] and related systems [20, 21] were studied in a variety of contexts including very interesting and non-trivial generalization to case of three confined particles [22]. In this paper I am concerned with relativistic generalization (H.O.) and with influence as a nuclear mesons field. Then we can try to describe the nuclear interaction by analogy with the interaction of the electric field. The attempt was made to give up the interpretation of the (K.G.F.) equation because the spin of mesons like π is zero and hence we can use this equation as the equation for the wave function of this particle instead it was proposed to consider (H.O.) as a nuclear potential produced by nucleon and interchanging spin zero virtual mesons. Just as photons are quantum particles corresponding to the electromagnetic field (E.M.F.) they correspond to the nuclear field mesons. Hence mesons exchange and photon exchange are considered as the source of the strong nucleon interaction and the electromagnetic interaction, respectively. We can write the equation for the nuclear field (H.O.) potential

Relativistic Spin Zero particles in an External Field

The one-particle potential in the (K.G.F.) equation may transform in various ways under Lorentz transformation. There are forms in common use assume that particle moves in the scalar potential $U(x)$ and in an external field described by stationery model potential $A^{\nu} = [V(x), A]$. The zeroth component $A^{0} \equiv V_{0}(x)$ is referred to paper as the fourth component of the vector potential. The remaining three components of A^{ν} form a three-dimensional vector A which we shall refer to as vector potential. The particle is spinless, it is described by the (K.G.F.) equation. The scalar potential $U(x)$ is bracketed with the mass m, and the fourth component of the vector potential $V_{0}(x)$ goes with the energy E and the vector potential A goes with momentum ρ in the (K.G.F) equation as follows.

The equation that will give the correct relativistic energy-momentum relation

$$E^2 = P^2c^2 + m^2c^4 \tag{1}$$

by letting

$$E \to E - V_q(x) \qquad P \to P - A \tag{2}$$

$$\frac{1}{c^2}\left[i\hbar\frac{\partial}{\partial t} - V\right]^2 \psi(x,t) = \left\{\left[\frac{\hbar}{i}\vec{\nabla} - A(x,t)\right]^2 + m^2\right\}\psi(x,t) \tag{3}$$

The relativistic K.G.F. equation goes over to the Schrödinger equation in the classical limit, $c \to \infty$. Since the zero point energies in non-relativistic theory and the theory of relativity differ by mc^2, it is convenient to introduce a transformation of the wave function ψ by means of the following relation

$$\psi(x,t) = \psi_k(x)e^{\frac{-i}{\hbar}(E+mc^2)t} \tag{4}$$

Substituting (4) into (3) and calculating the derivatives with respect to time, we obtain

$$2i\hbar mc^2\frac{\partial\psi'}{\partial t} - \hbar^2\frac{\partial^2\psi'}{\partial t^2} - 2V\left[mc^2\psi' + i\hbar\frac{\partial\psi'}{\partial t}\right] + V^2\psi' = c^2(P-A)^2\psi' \tag{5}$$

After dividing, both sides of the equation (5) by $2mc^2$ and substituting $\psi'(x,t) = \psi_k(x)e^{\frac{-iEt}{\hbar}}$ into the (K.G.F.) Eq. (5) we have

$$\left[\frac{(p-A)^2}{2m} - \frac{1}{2mc^2}(V-E)^2 + (V-E)\right]\psi_k(x) = 0 \tag{6}$$

where c is velocity of light, E is the energy relative to mc^2 and V is the fourth component of the vector potential, A that was introduced before. In the nonrelativistic limit the Klein - Gordon - Fok equation (5) transforms to the Schrödinger equation.

Two Different Sets of Harmonic Oscillator Potential

The Schrödinger equation describing a spinless particle confined by harmonic oscillator potential is usually written in the form

$$\left(\frac{p^2}{2m} + \frac{1}{2}kx^2\right)\psi = E\psi \tag{7}$$

Equation (7) corresponds to two different sets of potential:

I: The First one is as follow

$$a: V = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2, \quad A = 0 \tag{8}$$

But the second one is the super potential correspond to [23].

$$b: V = \frac{1}{2}m\omega^2x^2, \quad A = im\omega x \tag{9}$$

It may be tempting to expect that the potentials (a) and (b) are related by a gauge transformation. None of them corresponds to the realistic external electromagnetic field. But the option given by (b) is more important potential than (a) [23].

The potential (a) is more intuitive which is explicitly describes a Hook system, the second one which we will use in this problem has many formal advantages and allows for deeper understanding of not only harmonic oscillator itself, but also the structure of the quantum field theory for interesting problem of zero particle bound in (R.H.O.) potential , such as π bound to a nucleus, has the nice feature that it can be solved exactly Eigenfunction and Eigenvalue of π meson bound to nucleus by interacting (R.H.O.) potential (b). In the case described by (R.H.O.) potential (b) both, the Schrödinger and the (K.G.F.) equations from Eq (6) may be written as

$$\left[H-\frac{1}{2mc^2}(V-E)^2\right]\psi_k=(E-V)\psi_k \tag{10}$$

Where Hamiltonian H is like the Schrödinger equation for (H.O) problem as follows

$$H=\frac{1}{2m}(P-A)^2=\frac{1}{2m}(P-A)=\frac{1}{2m}(P-im\omega x)^{\dagger}(P-im\omega x)=\frac{P^2}{2m}+\frac{1}{2}m\omega^2x^2 \tag{11}$$

Therefore Eq (10) with $V=\frac{3}{2}\omega$ and H from Eq (11) may be written as follows.

$$\left[\frac{P^2}{2m}+\frac{1}{2}m\omega^2x^2\right]\psi_k(x)=\left[E(1-\frac{3\omega}{2mc^2}+\frac{E}{2mc^2})-\frac{2\omega}{2}(1-\frac{3\omega}{4mc^2})\right]\psi_k=\varepsilon\psi_k \tag{12}$$

The (K.G.F.) equation (10) is the same as the Schrödinger equations for three dimensional (H.O.). The energy of three dimensional (H.O.) may be written as $\varepsilon_{nl}=(2n+l+\frac{3}{2})\omega$. By a little calculation Eq (12) becomes

$$\left[P^2c^2+m^2c^4+m^2c^2\omega^2x^2\right]\psi_k(x)=(E+mc^2-\frac{3}{2}\omega^2)\psi_k \tag{13}$$

If we eliminate the effect of harmonic oscillator, i. e. if we put $\omega=0$ then equation (13) can be written as

$$\nabla^2\psi_k-\frac{(E+mc^2)^2-m^2c^4}{\hbar}\psi_k=0$$

$$\nabla^2\psi_k-\eta^2=0 \text{ where } \eta^2=\frac{(E+mc^2)^2-m^2c^4}{\hbar} \tag{14}$$

The equation (14) is a certain- analogue of the equation of electrostatic field.

Hence in the presence of a nucleon at the point $x=0$ it is natural to give equation (14) the form

$$\nabla^2\psi_k-\eta^2\psi=-4\pi g\delta(x) \tag{15}$$

Whence we shall find the solution satisfying the condition $\psi\to 0$ for $x\to\infty$ and by using the expansion of δ- function in a Fourier integral and Eq (15) after calculating sum abgeric and using the theory of residues for calculating integral we obtain the expression for the nuclear field potential from equation (14)

$$\psi=\frac{g}{x}e^{-kx} \quad \text{where} \quad \eta^2=\frac{(E+mc^2)^2-m^2c^4}{\hbar^2c^2} \tag{16}$$

which is the Yukawa potential.

This equation is certain analogue of the equation of the electrostatic field and goes over into the latter for $E=0$ or $\eta=0$ the potential ψ goes over into the potential of the electrostatic field $\psi=\frac{g}{x}$.

Π: In the case of (H.O.) potential corresponding to the Eq (13) with $\omega\neq 0$

$$\nabla^2\psi_k(x)-\frac{m^2\omega^2x^2}{\hbar^2}\psi_k(x)=\left[\frac{(E+mc^2-\frac{3}{2}\hbar\omega)^2-m^2c^4}{\hbar^2c^2}\right]\psi_k(x) \tag{17}$$

where ω comes from Eq.(9).

The behavior of the Eq. (17) near the origin of the nucleon ($x\to 0$) become as $\nabla^2\psi_k(x)=k^2\psi_k(x)$ where

$$\eta^2=\frac{(E+mc^2-\frac{3}{2}\hbar\omega)^2-m^2c^4}{\hbar^2c^2}\Rightarrow\psi_0=\frac{g}{x}e^{-kx} \tag{18}$$

And far away from the origin $x\to\infty$ we have the wave function as $\psi_\infty(x)\approx e^{-\frac{mwx^2}{2}}$

We can go over to the wave function $u(x)$

$$\psi(x)=\psi_0(x)\psi_\infty(x)u(x)=\frac{g}{x}e^{-kx-\frac{m\omega}{2}x^2}u(x) \tag{19}$$

After inserting $\psi(x)$ into equation (17) the solution of the differential equation for $u(x)$ will be held by suitable anzats [24, 25, and 26].

For $\omega=0$ the potential ψ goes over into the Yukawa potential $\psi(x)=\frac{g}{x}e^{-kx}$ and if $m=0$ and $\omega=0$ the potential $\psi_{(x)}$ goes over the potential of electrostatic field $\psi=\frac{g}{x}$. Indeed, the quantity g plays the same role as the charge e in the electrostatic potential, and can rightfully be called nucleon charge.

Equating the energy of the three-dimensional (H.O.) equation (12) with energy of equation (11) leads

$$E_{nl}=(2n+l+\frac{3}{2})\hbar\omega=\frac{1}{2mc^2}\left[(E_{nl}+mc^2-\frac{3}{2}\omega)^2-m^2c^4\right] \tag{20}$$

Hence the π - meson spectrum with (H.O.) potential field are, (let $\hbar=1$)

$$E_{nl}^{+}=-mc^2+\frac{3}{2}\omega+\left[m^2c^4+2mc^2(2n+l+\frac{3}{2})\omega\right]^{\frac{1}{2}} \tag{21}$$

$$E_{nl}^{-}=-mc^2+\frac{3}{2}\omega-\left[m^2c^4+2mc^2(2n+l+\frac{3}{2})\omega\right]^{\frac{1}{2}} \tag{22}$$

Hence, the π-meson spectrum is compared of two discrete energy level separated by the

$$(\Delta E_{nl})^2 = 8mc^2(2n + l + \frac{3}{2})\omega \tag{23}$$

The distance between two levels.

$$(\Delta E)^2 = (\Delta E_{n+1} - \Delta E_n)^2 = 8mc^2\omega \tag{24}$$

This is the square distance between two succeive levels, for $\omega = 0$ we have $\Delta E_{nl} = 0$. There no separation between energy level.

Conclusion

In this paper the relativistic generalization of the harmonic oscillator are solved by changing the Klien Gorden equation into Schrödinger equation. In particular of the model of two spinless particles interacting by an instantaneous coulomb potential (Yukawa) and confined by the harmonic oscillator potential. We have shown the presence of a special nucleon charge g in nucleons and the confinement of spin zero particles such as π- meson change. Hence each nucleon should be considered as a particle surrounded by a cloud of π- meson. The validity of such a picture is hidden in the confinement potential. Also the picture of π- meson interaction of nucleon turns out to be much more complex than that of this simple method but this model has been developed the concept of nuclear force field.

Acknowledgments

This work was supported by the shahrood university research grants.

Reference

[1] Rowe DJ, Bahri C (1988) J Phys A: Math Gen 31:4947-4961
[2] Grudzinski K, Wybourne BG (1996) Rep Math Phys 38: 251-266.
[3] Jakolski W (1996) Phys Rep 271:1-66.
[4] Bielinska- Waz D, Karwowski J, Diercksen GHF (2001) J Phys B: At Mol Opt Phys 34: 1987-2000
[5] Fock V (1928) Z Phys 47: 446-448
[6] Darwin cG (1930) Proc Cambridge Philos Soc 27: 86-90
[7] Nikolsky K (1930) Z Phys 62: 677-681
[8] Postepska I (1935) Acta Phys polon 4: 269-280
[9] Toyama FM, Nogami Y (1999) Phys Rev A 59: 1056-1062
[10] Nogami Y, Toyama FM (1993) Phys Rev A 47: 1708-1714
[11] Cook PA (1971) Lettere al Nuovo Cimento 1 : 419-426
[12] Moreno M. Zentella A (1989) J Phys A : Math Gen 22: L821-L825
[13] Moshinsky M, Szczepaniak A (1989) J Phus A : Math Gen 22 : L 817-L820
[14] Mir –kasimovRM (1991) J Phys A : Math Gen 24:4283-4302

[15] Moshinsky M, delSol Mesa A (1996) J Phus A: Math Gen 29:4217-4236
[16] Taut M (1993) Phys Rev A 48:3561-3566
[17] Karwowski J(2003) Harmonic oscillators revisited: relativistic formulations, confined particles, and several other aspects. In: Lulek T, Lulek B. Wal A (eds) Symmerty and Structural Properties of Condensed Matter. World Scientific, Singapore, pp 352-365.
[18] Karwowski J, Cyrnek L (2003) Comp Methods Sci Technol 9:67-78
[19] Karwowski J, Cyrnek L (2004) Ann Phys (Leipzig) 13: 181-193
[20] Samanta A, Ghosh Sk (1990) Phys Rev A 42:1178-1183.
[21] Ghosh Sk Samanta A (1991) J Chem Phys 94: 517-522.
[22] Taut M, Pernal K, Cioslowski J, Staemmter V (2003) J Chem Phys 118:4861-4871.
[23] Cooper F, Khare A, Sukhatme U (1995), Physics Rep., 251: 267-385.
[24] A. A. Rajabi, (2006) Few-Body Systems 40, PP. 21-33.
[25] A. A. Rajabi, (2008) International Journal of Modern Physics E, Vol. 17, No. 4, 1-6.
[26] A. A. Rajabi, (2005) Few-Body System, Vol. 37, pp. 197-213.