International Journal of Pure and Applied Physics
ISSN 0973-1776 Volume 4 Number 1 (2008) pp. 65–70

http://www.ripublication.com/ijpap.htm

Analytical expression for p-^{3}He total reaction cross section

M.A. Alvi

*Department of Physics, Science Faculty,
King Abdulaziz University, Jeddah-21589, Saudi Arabia.
E-mail: alveema@hotmail.com*

Abstract

Applying Coulomb correction factor to the Glauber multiple scattering model an analytical expression has been derived for microscopic study of proton-^{3}He total reaction cross section. Using independent single particle model for the density of ^{3}He, reasonably good account of the data has been achieved.

PACS: 25.40.Cm; 25.40.Dn; 24.10.Ht; 25.70Bc

Keywords: Coulomb modified Glauber model; Microscopic analysis; Independent particle model density

Introduction

The total reaction cross section, σ_R, gives the probability that a projectile will undergo a non-elastic process when passing through a nuclear medium. It is, therefore, of fundamental importance to our understanding of induced nuclear reactions as well as important for application in several fields such as medicines, chemistry, biology, detector simulation in particle and high energy physics, astrophysics etc. [1-3].

Theoretical studies of proton σ_R have generally been made using either the optical model potential as derived by fitting the elastic scattering differential cross section data [4-8] or the Glauber multiple scattering model [9-13]. In the latter case it is found that the Glauber model though based on the high energy approximation, works fairly well down to energy about 20 *MeV/nucleon* provided that the model is suitably modified to account the deviation of projectile trajectory due to Coulomb field [10]. Strictly speaking, in almost all the application of the Glauber model to study the

σ_R data, the elastic S- matrix element S_{el} is evaluated using the so-called optical limit approximation (*OLA*), where one only needs to evaluate the first term of the complete Glauber S_{el}. Further, one of the assumptions of this approximation is that the number of nucleons in the target nucleus is sufficiently large. Hence, despite of the fair success of *OLA* in reproducing the proton σ_R data, it should not be applied for studying the total reaction cross section for light target nuclei.

We present in this work a microscopic study of *p*- ^3He reaction cross section in the energy range about 20 to 50 *MeV* [14] by calculating all the terms of Glauber S_{el}. An analytical expression for σ_R is derived where for the simplicity the density of ^3He is constructed from uncorrelated wave functions. The reaction cross sections calculated by this closed expression are found to be much closer to experimental values than those by the *OLA*.

Theoretical Formulation

The Glauber theory provides us with an excellent framework to describe the high energy reaction in various fields of physics. The proton-nucleus reaction cross section is specified by the elastic S-matrix element S_{el} defined by:

$$S_{el}(b) = \left\langle \Psi_O \left| \prod_{j=1}^{A} \left[1 - \Gamma_{NN}(\boldsymbol{b} - \boldsymbol{s}_j)\right] \right| \Psi_O \right\rangle, \tag{1}$$

where $\boldsymbol{b}$ is the impact parameter, Ψ_O is the target ground state wave function, $\boldsymbol{s_j}$ is the two dimensional coordinate of j-th nucleon of the target relative to its center of

(*c.m.*), which lies on the plane perpendicular to the incident momentum of the projectile. The *NN* profile function $\Gamma_{NN}(\boldsymbol{b})$, which is generally parameterized as [10]:

$$\Gamma_{NN}(b) = \frac{\sigma(1 - i\gamma)}{4\pi\beta^2} e^{-b^2/2\beta^2} , \tag{2}$$

where σ is the *NN* total cross section, γ is the ratio of the real to the imaginary parts of the forward scattering amplitude and β^2 is the slope parameter. The expression for total reaction cross section is obtained by subtracting the elastic cross section from the total cross section:

$$\sigma_R = 2\pi \int_0^\infty bdb \left[1 - \left|S_{el}(b)\right|^2\right]. \tag{3}$$

For simplicity, treating the target nucleons as identical, the expression for S_{el} for *p*-^3He system may be written as:

$$S_{el}(b) = \int d_{\ 1}\, d_{\ 2}\, d_{\ 3}\, \rho^{(3)}(\ _1,\ _2,\ _3)[1 - \Gamma_{NN}(\boldsymbol{b} - \boldsymbol{s_1}) - \Gamma_{NN}(\boldsymbol{b} - \boldsymbol{s_2}) - \Gamma_{NN}(\boldsymbol{b} - \boldsymbol{s_3})$$

$$+\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_1})\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_2})+\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_1})\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_3})+\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_2})\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_3})$$

$$-\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_1})\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_2})\Gamma_{NN}(\boldsymbol{b}-\boldsymbol{s_3})], \tag{4}$$

where the three-body density is given by:

$$\rho^{(3)}(\ _1,\ _2,\ _3) = \left|\Psi_0(\ _1,\ _2,\ _3)\right|^2 \delta\left(\frac{\ _1+\ _2+\ _3}{3}\right). \tag{5}$$

In the above expression $\ _1$, $\ _2$ and $\ _3$ denote the position vectors of nucleons from the *c.m.* of the ^3He nucleus and the δ-function is the *c.m.* constraints. Using the independent particle model Gaussian density for ^3He, the expression for the intrinsic three-body density may be written as:

$$\rho^{(3)}(\ _1,\ _2,\ _3) = \left(\frac{\alpha_m^2}{\pi\sqrt{3}}\right)^3 e^{-\alpha_m^2(\xi_1^2+\xi_2^2+\xi_3^2)}\ \delta\left(\frac{\ _1+\ _2+\ _3}{3}\right), \tag{6}$$

where α_m^2 is the size parameter of the independent particle model Gaussian density. The intrinsic one-body density can easily be derived as:

$$\rho^{(1)}(\xi_1) = \left(\frac{\alpha_0^2}{\pi}\right)^{3/2} e^{-\alpha_0^2\xi_1^2} \tag{7}$$

with $\alpha_0^2 = \frac{3}{2}\alpha_m^2$. The root mean square (*rms*) radius of this density is given by:

$$\left(r^2\right)^{1/2} = \sqrt{1.5/\alpha_0^2}\ . \tag{8}$$

Substituting eqs. (2) and (6) in eq. (4) we get expression for S_{el} as:

$$S_{el}(b) = 1-3c_1 e^{-p_1b^2}+3c_2 e^{-p_2b^2}-c_3 e^{-p_3b^2}, \tag{9}$$

where

$c_1 = 3a/(1+3\alpha_m^2\beta^2)$, $p_1 = 1.5\alpha_m^2/(1+3\alpha_m^2\beta^2)$

$c_2 = 12a^2/(1+6\alpha_m^2\beta^2)(1+2\alpha_m^2\beta^2)$, $p_2 = \alpha_m^2/(1+6\alpha_m^2\beta^2)$

$c_3 = \{2/(1+2\alpha_m^2\beta^2)\}^2\ a^3/(\alpha_m^2\beta^2)$, $p_3 = 1.5/\beta^2$

with $a = \alpha_m^2\,\sigma(1-i\gamma)/4\pi$.

The closed expression for total reaction cross section can now be written as:

$$\sigma_R = \pi[\frac{3(c_1+c_1^*)}{p_1}-\frac{3(c_2+c_2^*)}{p_2}+\frac{c_3+c_3^*}{p_3}+\frac{9(c_1^*c_2+c_1c_2^*)}{(p_1+p_2)}-\frac{3(c_1c_3^*+c_1^*c_3)}{(p_1+p_3)}$$

$$+\frac{3(c_2 c_3^* + c_2^* c_3)}{(p_2+p_3)} - \frac{9c_1^2}{2p_1} - \frac{9c_2^2}{2p_2} - \frac{c_3^2}{2p_3}]. \quad (10)$$

One of the basic assumptions of the Glauber model is that the projectile follows a straight line trajectory during a collision with the target nucleus. This is not a good approximation at lower energies. To account the correction for the trajectory deviation due to Coulomb field, we following Charagi and Gupta [10] multiply the right hand side of eq. (10) with the Coulomb correction factor $\left(1-\frac{1.44Z}{E_{cm}R}\right)$, where E_{cm} is the *c.m.* energy in *MeV* and *R* (defined by $\left|S_{el}(R)\right|^2 = \frac{1}{2}$) is the interaction radius in *fm*. In this work *R* is taken to be equal to the charge *rms* radius of ^3He.

Results and Conclusion

Figure 1 compares the p-^3He reaction cross section calculated using the analytical expression (10) with experiment. The open circles with statistical error bars represent the available experimental data [14] in the energy range about 20 to 50 *MeV*. The parameters σ and γ of Γ_{NN} at the desired energy are determined by suitably averaging the corresponding *pp* and *pn* forward scattering amplitude. We use the following parameterizations [10] for σ_{pp} and σ_{pn} which reproduce the observed values in the energy range 10 to 1000 *MeV* quite well:

$$\sigma_{pp} = 13.73 - 15.04/v + 8.76/v^2 + 68.67v^4 \quad (11)$$

$$\sigma_{pn} = -70.67 - 18.18/v + 25.26/v^2 + 113.85v, \quad (12)$$

where v denotes the velocity (in unit of c) of the incident proton in lab system, σ_{pp} and σ_{pn} are in *mb*. To calculate γ_{pp} and γ_{pn}, we use the parameterizations of Ahmad *et al* [15]:

$$\gamma_{pp} = -0.386 + 1.224\, e^{-\frac{1}{2}\left(\frac{k-0.427}{0.178}\right)^2} + 1.01\, e^{-\frac{1}{2}\left(\frac{k-0.592}{0.638}\right)^2} \quad (13)$$

$$\gamma_{pn} = -0.666 + 1.437\, e^{-\frac{1}{2}\left(\frac{k-0.412}{0.196}\right)^2} + 0.617\, e^{-\frac{1}{2}\left(\frac{k-0.797}{0.291}\right)^2}, \quad (14)$$

where k is the incident nucleon lab momentum in *GeV/c*. As far as the parameter β^2 is concerned, it is an energy dependent quantity and the literature shows that very many values have been used. We, following Charagi and Gupta [10] choose 0.423 fm^2, which successfully reproduces the σ_R data over a wide energy range. Finally the size parameter α_0^2 has been determined from their charge *rms* radius [16] after correcting for the finite proton charge distribution. Using the relation $\alpha_0^2 = 1.5\alpha_m^2$, the model size parameter α_m^2 comes out to be 0.462 *fm*$^{-2}$.

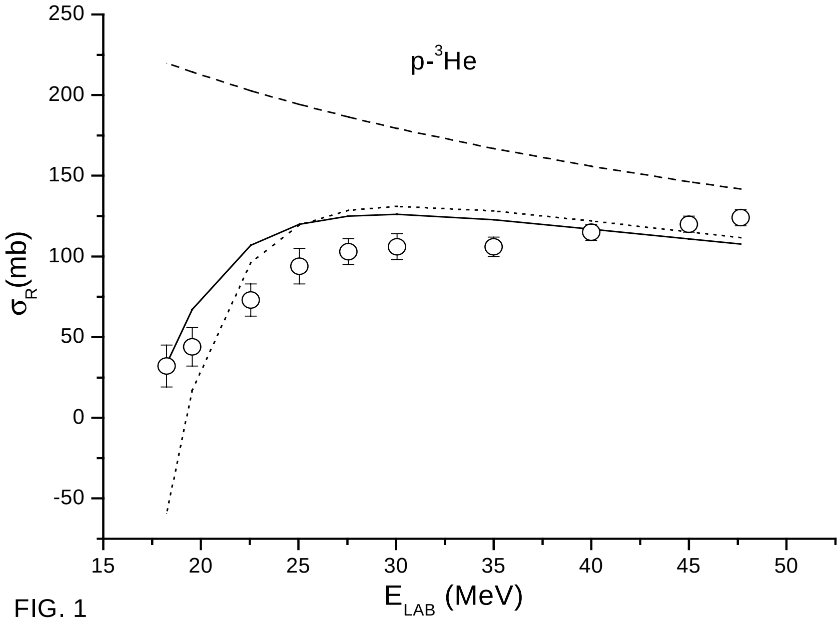

FIG. 1

Figure 1 : Proton-^{3}*He* total reaction cross-section as a function of energy. Solid and dashed curves show the result of our microscopic method and of *OLA* respectively. The dotted curve: without considering the Coulomb correction factor. Open circles with error bars represent the experimental data from Carlson [30].

The dotted curve shows the prediction of our microscopic calculation of σ_R without considering the deviation of projectile trajectory due to Coulomb field between the proton and the ^{3}*He* nucleus. It is seen that there is a large disagreement with the experimental data throughout the energy range. In fact, it becomes negative at lower energies, which is highly unrealistic. The solid curve gives the results of σ_R when the correction due to Coulomb field is included. We see that in general our calculated reaction cross section agrees nicely with the experimental data. Apart from the fact that that Glauber model is not very well placed at low energies, the slight disagreement may be partly due to consideration of interaction radius *R* (an energy dependent parameter) to be equal to charge *rms* radius. We have also calculated σ_R in the usual optical limit approximation. The result is displayed by the dashed curve, which shows complete failure of *OLA* in reproducing the data.

To summarize our work, we using the independent single particle model for the density of ^{3}*He*, have derived an analytical expression to calculate *p-*^{3}*He* total reaction cross section by applying Coulomb correction factor to the Glauber multiple scattering model. In the absence of any known study of *p-*^{3}*He* total reaction cross section, we are unable to compare the success of our closed expression that reproduces the available experimental data fairly well.

Acknowledgements

The author is grateful to Dr. Faraj Al-Hazmi, Chairman, Department of Physics for providing the facilities to complete this work. The author is also grateful to Prof. Ahmad Bin A Al-Ghamdi for constant encouragement.

References

[1] Ozawa A., Nucl. Phys. **A738,** 38(2004).
[2] Ramsey C. R., *et al*, Phys. Rev. **C57,** 982(1998).
[3] Zheng T., *et al*., Nucl. Phys. **A709**,103(2002) and references therein.
[4] Auce A., *et al.* , Phys. Rev. **C71**, 064606(2005).
[5] Carlson R. F., *et al.* , Phys. Rev. **C49**, 3090(1994).
[6] Koning A .J. and Delaroche J .P., Nucl. Phys. **A713**, 213(2003).
[7] Amos K. and Deb P. K., Phys. Rev. **C66**, 024604(2002).
[8] Deb P.K., Karataglidis K., S., Chadwick M.B. and Madland D.G., Phys. Rev. Lett**.** **86**, 3248(2001).
[9] Alvi M.A., Nucl. Phys. **A789**, 73(2007) and references therein.
[10] Charagi S.K. and Gupta S.K., Phys. Rev. **C41**,1610(1990).
[11] Madani J.H., Int. J. Mod. Phys. **E11**, 475(2002).
[12] Ahmad I., Abdulmomen M.A. and Madani J.H, Indian J. Phys. **71A**, 61 (1997).
[13] Alvi M. A., Communicated to Physica Scripta (2007).
[14] Carlson R.F., At. Data and Nucl. Data Tables **63**, 93(1996).
[15] Ahmad I., Abdulmomen M A. and Al-Khattabi L.A., Int. J. of Mod. Phys. **E10**, 43(2001).
[16] de Vries H., de Jager C.W. and de Vries C., At. Data and Nucl. Data Tables **36**, 495(1987).