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A Didactic Note for Accurate Laboratory Determination of Planck's Constant from the Planck Radiation Theory

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Abstract

Using a simple experiment, we determined Planck's constant from the Planck radiation theory. We used a laboratory setup that consists, essentially, of a low cost lamp and a photodetector such that a light filter of known wavelength is interposed between the two pieces of apparatus. The lamp filament is the approximate blackbody of which radiation is described with the Planck's law of radiation. When the setup is in operation, a phenomenon of resistance-temperature filter effect is observed. Data collected have been subjected to interesting analyses. Usually, a pair of intensity values calculated from the observed phenomenon, would be required. Ratio of the intensities is then used in the exact Planck's law of radiation equations leading to a transcendental exponential equation. Solutions of this equation enable one to obtain an accurate value of the Planck's constant.

Keywords: photodetector, photoresistance, temperature, blackbody, filament, optical filer, Mathematica findroot

Introduction

From the study of several articles [1,2,3[every objective scientist would be convinced that Planck parameters are all-pervasive [1]. In fact, theoretically and experimentally, the Planck mass [1,2], should be seen as a basic particle for the realization of most elementary particles; these include electrons [3]. More fundamental, however, is the Planck's constant, h. The precision of h, which appears to be elusive, is the purpose of this study. We begin with a simple experiment of which data fit well into the Planck radiation equation [4,5]. Also, our aim is to illustrate, at various level of physics either in research or teaching, how theory is indispensable to experiment.

Based on different principles, there are several techniques for laboratory determination of the Planck's constant. The Photoelectric effect appears to be the commonest [4,5] with relevant well developed pieces of apparatus, for most physics laboratories. X-ray diffraction through Duane and Hunt relation [6,7] is less common. Use of the blackbody radiation through the Planck's law of radiation does not have specific apparatus for the purpose of didactic classroom illustrations. A few attempts [4,8,9] provide useful instructions on how to implement the law with commonly available apparatus; even other pieces that are not readily available can easily be devised. Here, the pieces of apparatus, when fitted and operated are amenable to precautions as far as initiations can sustain. However, in previous works, [4,8,9] the results show glaring systematic errors that are largely due to approximations and not due to lack of laboratory precautions and their setup.

From the laboratory experimental setup somewhat of a new phenomenon that is entirely due to the effect of an interposing filter between an electric filament and a photocurrent detector, is observed. At some point of recording data, it was noticed that the photocurrent depends on types of filter. This means that the amount of heat filtered is proportional to resistance, and consequently temperature of the filament. This is referred to in the text, as resistance-temperature filter effect, RTFE. By this phenomenon, it is also noted that even the glass bulb enclosing the filament behaves as a filter.

The RTFE is, thus, applied to the exact Planck radiation law to obtain a transcendental exponential equation of the form

$$e^{ah} - ce^{bh} + c - 1 = 0 \qquad (1)$$

where h is the Planck's constant; a, is a parameter that is related to the filter frequency, ν, the Boltzmann's constant, k, and an initial phototemperature, T_{pj}, b is also related to ν and k, and a second phototemperature T_{pl}; the parameter $c \equiv I_{pl}/I_{pj}$, $j \neq l$ where $I_{pj} = i_{pj}^2 R_{pj}$ /A and $I_{pl} = i_{pl}^2 R_{pl}$ /A are the photointensities, A, being the surface area of the filament, i_p is the photocurrent, and R_p is the photoresistance. Precisely, the relation of a to ν, k and T_{pj} is $a \equiv \nu/kT_{pj}$. Also, $b \equiv \nu/kT_{pl}$. Actually, the simple Mathematica FindRoot statement [10] is used to solve equation (1).

It is instructive to make all categories of students realize that the Planck's constant, h, through the Planck's law of radiation has serious technological applications [11]. Thermionic direct conversion of heat to electricity can also be explained with and developed from the law [12].

Theory

The Planck radiation law governs the energy distribution in blackbody radiation [6]. In units of intensity given by W/m^2 the law can be stated as

$$I(\nu,T)=\frac{8\pi h\nu^2}{c^2}\{\exp(h\nu/kT)-1\}^{-1} \tag{2}$$

where c is the speed of light.

For a fixed frequency, ν, at different temperatures T_j and T_l the ratio of any two intensities $I_j(T_j)$ and $I_l(T_l)$ leads to

$$\exp(h\nu/kT_j)-\frac{I_l}{I_j}\exp(h\nu/kT_l)+\frac{I_l}{I_j}-1=0 \tag{3}$$

where $j \neq l = 1,2,3,\ldots$, so that for measured values of intensities I_j and I_l, and known values of T_j, T_l and ν, equation (3) is a form of transcendental equation which does not have a closed form of solution for h/k to be found [9]. It is, however, an interesting exercise to apply anyone numerical procedure such as the Newton's Method or the fixed-point iteration method [13].

In the visible range of frequencies used, the approximate form of equation (3) based

on hν >> kT so that exp(hν/kT) >> 1 is [4]

$$\ln\left(\frac{I_j}{I_l}\right)=\frac{h\nu}{k}\left(\frac{1}{T_l}-\frac{1}{T_j}\right) \tag{4}$$

The laboratory setup involves measurements of series of voltages V_j with their corresponding electric current, i_j, of a light bulb filament as shown in Fig. 1. To determining the intensities I_j and temperatures T_j one requires to use the Stefan-Boltzmann law which can be expressed as [8]

$$W \equiv P/A = \sigma T^4 \tag{5}$$

where σ is the Stefan-Boltzmann constant and P is the power emitted.

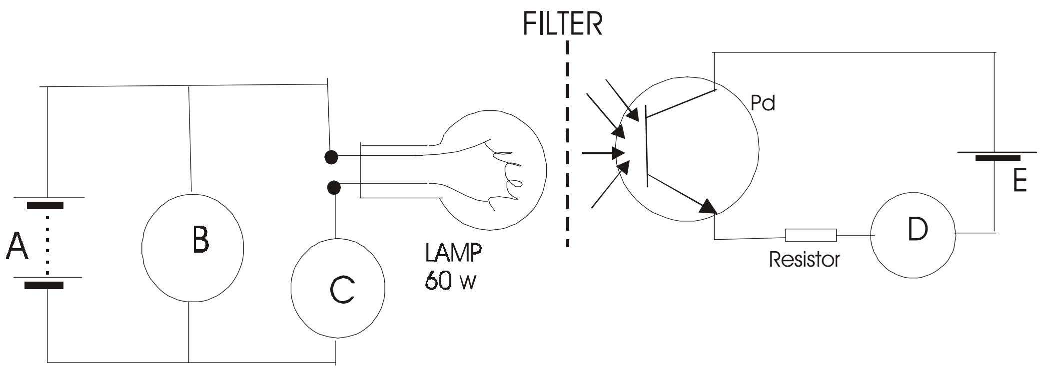

Figure 1: Laboratory setup of electric circuit. A and E should be stabilized sources and not ordinary batteries which drain every second. See text for further explanation

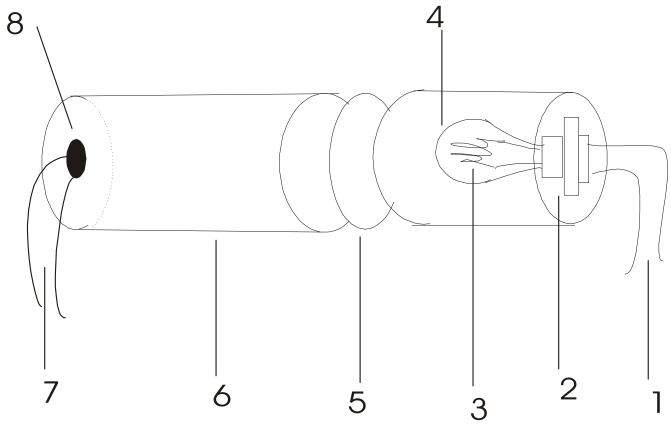

Figure 2: Schematic drawing of essential pieces of apparatus: 1. Connecting cords to variable General Purpose (Philip Harris) power supply, 2. Bulb socket, 3. Lamp filament, 4. Lamp bulb, 5. PHYWE light filter, 6. Polyvinyl tube, 7. Connecting cords to biasing voltage (i.e., PHYWE stabilized power source), 8. RS 651-995 Photodetector

If a power law of the form $T \propto R^{\gamma}$ is assumed, an empirical relation between resistance of the filament, R_e, and temperature T_e, through the power dissipated can be expressed as

$$P_e = i_e^2 R_e \equiv A\sigma T_e^4 = \beta R_e^{4\gamma} \tag{6}$$

in which β and γ are constants, and from equation (6) the empirical relation of R_e and T_e is [4]

$$T_e = \left(\frac{R_e}{R_0}\right)^{\gamma} T_0 \tag{7}$$

where R_0 denotes the filament resistance at temperature T_0. The room temperature was used as T_0 for which R_0 was measured using a digital resistance meter [14]. Using equation (6) in (7) yields

$$\ln\left(i_e^2 R_e\right) = \ln C_1 + 4\gamma \ln R_e; \quad \text{where} \quad C_1 = A\sigma\left(\frac{T_0}{R_0^{\gamma}}\right)^4 \tag{8}$$

so that a plot of $\ln(i_e^2 R_e)$ on the vertical axis against $\ln(R_e)$ on the horizontal axis is expected to give a straight line in order to determine the power, γ, using the least squares fit.

Fig. 1 consists of two circuits: the heat emitting circuit and the detector circuit. Although the pair of circuits operates in complete analogy to thermionic conversion circuits of heat to electricity [12], the later has the photodetector fundamentally of which work is to detect the amount of current that is proportional to the filtered heat.

In the filtering process, therefore, there is some content of temperature which is proportional to the detected current. The photopower content $i_p^2 R_p$, where i_p and R_p denote photocurrent and photoresistance, should have the relation of Stefan-Boltzmann law as expressed by equation (6) for the same power value, γ, already determined. That is, for the detected current, i_p, the empirical relation with R_p and T_p is

$$P_p = i_p^2 R_p \equiv A\sigma T_p^4 = \beta R_p^{4\gamma} \tag{9}$$

It should be noticed that i_{pj} are recorded simultaneously with V_{ej} and i_{ej} in the raw data. Equation (9) which arises from RTFE enables us to calculate R_p and T_p. For example, by taking the ratio of equation (6) to equation (9) one gets

$$R_p = R_e \left(\frac{i_p}{i_e} \right)^{2/(4\gamma - 1)} \tag{10}$$

By expressing T_p in the same form similar to equation (7), $T_p = (R_p/R_0)T_0$ and then taking the ratio of this expression to equation (7), T_p is obtained in terms of i_p, i_e and T_e as

$$T_p = T_e \left(\frac{i_p}{i_e} \right)^{2\gamma/(4\gamma - 1)} \tag{11}$$

It is pertinent to realise that the photointensity of the filament due to RTFE for anyone measured set of V_{ej}, i_{ej} and i_{pj} is $I_{pj} = i_{pj}^2 R_{pj}/A$. Another set of raw data values V_{el}, i_{el} and i_{pl} gives photointensity $I_{pl} = i_{pl}^2 R_{pl}/A$ where R_{pj} and R_{pl} are calculated using equation (10). With the fixed value of frequency (specified for the filter), ν, expressions of the applicable Planck radiation law given by equation (2) can be written for I_{pj} and I_{pl}. Ratio of the photointensities taken would lead to

$$\exp\left(h\nu/kT_{pj}\right) - \frac{I_{pl}}{I_{pj}} \exp\left(h\nu/kT_{pl}\right) + \frac{I_{pl}}{I_{pj}} - 1 = 0 \tag{12}$$

where T_{pj} and T_{pl} are obtainable from equation (11). Theoretically, one deduces that $h\nu >> kT_{pj}$ is an improved approximation over $h\nu >> kT_{ej}$, since $T_{ej} > T_{pj}$, so that $\exp(h\nu/kT_{pj}) >> 1$ is, by implication, more reliable. Now, a form of equation (4) when RTFE is invoked is

$$\ln\left(\frac{I_{pj}}{I_{pl}} \right) = \frac{h\nu}{k} \left(\frac{1}{T_{pl}} - \frac{1}{T_{pj}} \right) \tag{13}$$

Measurement of the filament surface area, A, according to the theory so far [4], is not necessary.

Equation (13) is an improved version of equation (4) that has been reported [4]. However, the two equations do not give practically dependable results. Advanced degree students would find it stimulating to attempting solutions of equations (3) and (12) using anyone of several numerics [13]. Even then, there are problems. One problem concerns the large value of exp(hν/kT_{ej}) or exp(hν/kT_{pj}). Another one is that due to the presence of exponents; either equation has several solutions depending on starting input values when the Newton's Method is applied. In this wise a simple Mathematica statement of FindRoot [10] helps a lot. Overall, analyses show that equation (12) contains an exact value of h, the Planck's constant. That is, a recommended value of h, is subsumed in the series of solutions of (12).

Laboratory Setup of The Experiment

As indicated in Fig. 1, A is a stabilized variable d.c power source, of range 0.0 V to 350.0 V; B and C are digital multimeters [14] for measurements of voltage and current. D happens to be an analog microammeter as it is an important precaution to avoid saturation currents which may likely occur if measurement of detected currents are in miliampers. E is a PHYWE stabilized a.c./d.c power source of which the d.c source was operated at a constant value of 11.0 V to biasing the RS 651-995 Photodetector which was chosen because of its large area and flexibility to various optical filter frequencies up to a peak wavelength of 900nm.

The setup consists of two circuits (see Fig. 1): the heat emitting circuit through a 60W light bulb, and the heat (current) detector circuit essentially by the RS photodetector Pd. The two circuits communicate by one way from the lamp to the photodetector. It is necessary to understand that the detected current is a property of the filament enclosed in the bulb. However, the detected current, i_p, is measured with a microammeter D and the function of the resistor, 1.0 kΩ, is to dissipate the detected power. By increasing the d.c. voltage, A, in the heat emitting circuit, the filament resistance, R_e, and consequently its temperature T_e, increase. The voltage across the filament is measured by B and denoted as V_e while the electric current through it is i_e measured by C. Spontaneously, Pd, detects the heat converted to photocurrent, i_p, measured by D of which value depends on the interposing filter (see Figs. 1 and 2). It is therefore, conformable to reason that the detected current has some content of resistance R_p, and consequently temperature T_p filtered from R_e and T_e. This process we refer to as resistance-temperature filter effect, RTFE, resulting into equations (10) and (11). V_e, i_e, and i_p are the physical quantities of the raw data shown in table 1; other quantities recorded are calculated as indicated, through γ of equation (8) determined by the least squares fit of Fig. 3.

It is found that avoiding darkening the laboratory is both precautionary [4] and applicable to most other experiments in our didactic programme. A blackened-inside polyvinyl tube of diameter 8.0 cm was used inside of which the communication of lamp bulb and photodetector takes place thus avoiding other sources of interfering radiation. A simple thermometer was used to measure T_0, the room temperature, and R_0, the resistance at T_0, was measured with the digital multimeter [14]. T_0 and R_0 are required in equation (7). The data recorded in table 1 are for the PHYWE yellow light filter of wave length, λ_f = 578.0 nm.

Table 1: Raw experimental data and calculated filament resistance, power, temperature and photocurrent; other quantities are calculated data due to RTFE: photoresistance, photopower, and phototemperature. Parameters used are: T_0 (Room temperature) = 310 K, R_0 (resistance at T_0) = 64.5 Ω, bulb wattage = 60W, tube length, L_t = 10.0 cm, biasing voltage, V_b = 11.0 V, filter wavelength, λ_f = 578.0 nm. See Fig. 3 for determination of γ.

V_e (V)	i_e (A)	$R_e=V_e/i_e$ (Ω)	$R_e i_e^2$ P_e= (W)	T_e (K)	i_p (μA)	R_p (Ω)	$R_p i_p^2$ P_p= (pW)	T_p (K)
85.6	0.173	494.798	14.8088	1425.20	4.0	0.010466	0.16746	0.46476
130.1	0.2108	617.173	27.4251	1680.60	6.0	0.016099	0.57955	0.64077
154.4	0.2298	671.889	35.4811	1790.52	8.0	0.021472	1.37420	0.79431
167.9	0.2395	701.044	40.2120	1848.15	10.0	0.026911	2.69110	0.94001
180.3	0.2493	723.225	44.9488	1891.60	12.0	0.032043	4.61424	1.07072
191.2	0.2562	746.292	48.9854	1936.42	14.0	0.037577	7.36510	1.20580
204.4	0.2643	773.364	54.0229	1988.57	16.0	0.043176	11.05305	1.33742
212.8	0.2755	772.414	58.6264	1986.75	18.0	0.046571	15.08891	1.41509
223.5	0.2778	804.536	62.0883	2048.05	20.0	0.053494	21.39766	1.56921
232.4	0.2816	825.284	65.4438	2087.32	22.0	0.059587	28.84018	1.70067
248.7	0.2956	841.340	73.5157	2117.53	24.0	0.063151	36.37468	1.77597
258.7	0.2968	871.631	76.7822	2174.14	26.0	0.070634	47.74878	1.93069

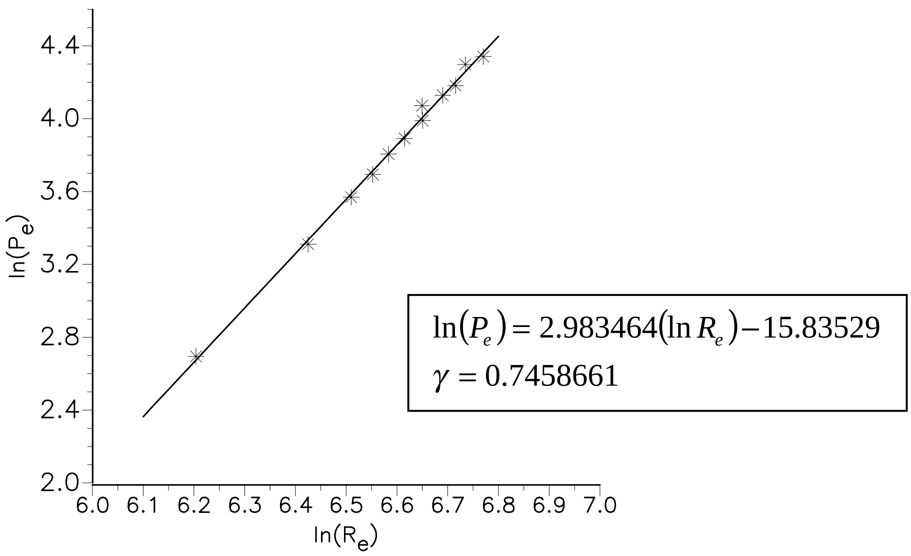

Figure 3: Graph of *ln* P_e (emitted power) against *ln* R_e (filament resistance) to determine γ. See table 1 for calculations of P_e and R_e.

Results are also reported here of data collected (see table 2) for PHYWE green and blue light filters at the same T_0, R_0 and other parameters recorded in table 1. Actually, the data discussed here constitute just one set of a very large number of sets of data collected and analysed over a period of two years up to now. The effort has so far been to diligently observe any drastic deviation from the RTFE. To still test the RTFE, we subjected data reported in table 1 of reference [4] to analyses using our present numerical experiments with equation (12).

Table 2: Results obtained for Planck's constant using the setup of Figs. 1 and 2 for the first three filters with the same values of other parameters given in table 1, and with the same numerical experiments explained in the text. Data for the near infrared filter will be found on page 820 of ref. [4].

Filter Color	Filter Wavelength (nm)	Planck's constant, h (Js) $\times 10^{-34}$
Blue	436.00	6.62573 ± 0.00033
Green	546.00	6.62434 ± 0.00046
Yellow	578.00	6.62321 ± 0.00052
Near Infrared	751.36	6.26387 ± 0.11248

Results and Discussion

Based on the simple Mathemat ica FindRoot statement (see Appendix A) [10], our results are given in table 2. The result obtained for blue filter is of interest for one reason: the range of the multimeters used did not allow more than three data points, and the microammeter used for measurement of photocurrent is analogue. If we used a digital microammeter, or even better nanoammeter, it would be possible to divide the interval into at least five data points. Actually, working with microammeter of photocurrent or lower units would be preferable since we are dealing with a quantum phenomenon [6]. In deed, under the same conditions of ambient temperature, T_0, the Planck's constant value, h, obtained for the three filters, if only three data points were used for computation, are 6.62534×10^{-34} Js for yellow filter, and 6.62540×10^{-34} Js for green and blue filters when the photointensity ratio is I_{p3}/I_{p2} (or I_{p2}/I_{p3}). The same order of magnitude is observed for the cases of I_{p2}/I_{p1} or I_{p1}/I_{p2}. This implies that, three data points give results that are devoid of systematic errors when equation (12) is used, once all laboratory precautions [4] have been applied. We exhausted the range of the multimeters simply because of analyses of the numerical experiments. Even then, one could see that the result is accurate for the yellow filter, if by accuracy one understands to mean the degree of closeness of the computed value of h to the recommended value [15].

Result of the near infrared filter is also given in table 2. One should recall that the data belong to reference [4] which are clearly under different ambient conditions.

Although, our analysis shows improvement, on the results of [4], the error here, might be due to observation specifically, the saturation current in the units of miliamperes.

Conclusion

Figures 1 and 2 show the laboratory setup for the measurement of Planck's constant, h, [4]. The implications of our analyses are noteworthy. Laboratory precautions are very important in any experiments whether for classroom demonstrations or for researches. More necessary is that correct theory and analyses should be encouraged.

We circumvented systematic errors that affected results of previous works [4,8,9] by avoiding approximations in theory. It was found, for example, that the approximations proved to be better when variables T_{pj} were used in $\exp(h\nu/kT_{pj}) >> 1$ instead of variables T_{ej} in $\exp(h\nu/kT_{ej}) >> 1$ [4] where j = 1,2,3,…,12; 12 being the number of data points in the laboratory measurements. That is because $T_{ej} > T_{pj}$. It would be recalled that T_{pj} define temperatures calculated using equation (11) which is one of the results of RTFE already explained in the text. T_{ej} are the temperatures of the emitted heat by filament, which would subsequently be filtered to produce T_{pj} as observed, since, the filtered current (i.e., the photocurrent) i_{pj} would always be less than the emitted current, i_{ej} (i.e., $i_{pj} < i_{ej}$).

For our final analysis, it was found to be correct by solving equation (12) numerically with a simple Mathematica FindRoot statement (see Appendix A) [10]. The equation has several solutions, each depending on the input or starting value. However, with currect relation between photointensity I_{pj} and photocurrent i_{pj} which is $I_{pj} = i_{pj}^2 R_{pj}/A$, A being the surface area of the filament enclosed inside the bulb, we found that (see table 2) recommended value of the Planck's constant, h, is subsumed in the series of solutions of equation (12). In particular, if it is assumed that the Planck's constant, h, allowed to be used in any calculation, is in the range of 6.60 $\times 10^{-34}$ Js to 6.63 $\times 10^{-34}$ Js, then, our results shown in table 2 are reliably accurate, where accuracy is understood to meaning the degree of closeness of the calculated value of h, to the recommended or exact value [15].

Acknowledgement

We thank A. A. Makinde, the chief technologist of Physics Department, FUTY, for lending us his RS Photodetector and his frequent technical assistance.

Appendix A

Essential statements of Mathematica FindRoot are given here. Data for computation are obtained from table 1. We used the parameters that correspond to l = 3 of equation (12). Answers returned by Mathematica are numbered.

```
E^• c1 • x• • • • I2 • I1• • • E^• c2 • x• • • •• • I2 • I1• • 1.0• • ; •
 • c1 • •8.083195• • 10^37• ; c2 • •4.7295• • 10^37• ;
  I1 • •1.674605• • 10^• 13• ; I2 • •1.374199• • 10^• 12• • •

  7.20611• •8.20611 • ‡.7295• 10^37 x • • 8.0832• 10^37 x
```

(A1)

```
FindRoot• % • •0, • x, 6.6279• • 10^• 34• ; 4.2• • 10^• 34• ; 7.44• • 10^• 34• • • • •

  • x • •6.62604• 10^• 34• •
```

(A2)

The final answer is given by equation (A2). It should be noticed in the statement immediately before equation (A2) that 6.6279×10^{-34} Js is the starting value that was chosen and the other two values correspond to an interval within which results are desired. After giving a notice of accuracy Mathematica returns only one value as the answer given by equation (A2).

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