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Wave-Particle Duality Theorem solves Mystery without Bohr's Complementarity Principle, with Examples to Interaction-Free-Measurement, Counterfactual Communication, Duality Computer

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Abstract

An Axiom recently proposed by the author explained the wave-particle duality mystery without Niels Bohr's Complementarity Principle, so that the particle always remains particle and its wave function always remains wave, without mysterious transformation from particle to wave and vice versa depending on measurement. In this paper the Axiom is replaced by a Duality Theorem with proof, and the results are applied to intriguing Non-Interaction Measurements, Counter-Factual Quantum Communications and Duality Quantum Computers, significantly clarifying these phenomena without any mystical implications. No new assumptions are made, only new reasoning. A brief review of earlier work is included.

Early on, Albert Einstein and Niels Bohr heatedly debated wave-particle duality. To explain duality Bohr postulated his principle of complementarity, which is now widely accepted, but has mystifying metaphysical implications. Albert Einstein disagreed. Richard Feynman called it the "only mystery" in quantum mechanics. Ingenious experiments including those using entanglement have confirmed Bohr's complementarity. Earlier work using the Axiom explained results of reported experiments by showing the equivalence: Coherence and alignment ≡ Interference ≡ No "which way" observation; No coherence or alignment ≡ No interference ≡ "which way" observation That is, complementarity is redundant; conventional criteria of alignment and coherence alone suffice for interference.

Keywords: Quantum Mechanics; Wave-particle Duality; Duality Theorem; Interference; Complementarity; Entanglement; Causality; Locality; Interaction-Free-Measurement; Counterfactual Communication; Duality Computer.

I. INTRODUCTION

A brief review is presented first to provide the background leading to the Duality Theorem proved in the next section. Proposed by Niels Bohr [1], the widely accepted complementarity principle explanation of wave-particle duality is as follows: (a) if the experimental setup is for detecting the particle, then interference (its wave nature) is destroyed and the particle travels through the particular sensed path ("which way" observation), and (b) if the setup is for detecting interference (wave nature) with no "which way" observation, then particle nature does not hold, and the particle travels as a wave through both (multiple) paths for interference. Richard Feynman, an authority on quantum mechanics, called this "the only mystery" in quantum mechanics [2]. This mystery has also given rise to metaphysical conjectures that somehow the very intent of the experimenter (his or her consciousness) influences the particle's behavior, some even postulating supernatural influence from outside space-time itself [7]. More generally, early on, Erwin Schrodinger had considered interpreting the probabilistic nature of quantum mechanics to imply that the many trials underlying probability actually occur simultaneously in multiple universes, giving rise to the metaphysical concept of multi-verse which has been seriously considered by eminent scientists including Stephen Hawking, and discussed by philosophers. But, to this day, multi-verse remains merely a speculation by scientists.

Albert Einstein felt that the experimental setup to measure a quantity can in principle be independent of the measured quantity and so cannot determine something as fundamental as the wave or particle nature of the measured quantity. Note that here we are talking about not merely the inclusion of states of measuring instrument in the states of overall quantum system comprising the measured quantity plus the measuring instrument (analogous to the loading or termination effect of measuring instrument in classical networks and systems) which is of course required, but also the more fundamental wave versus particle behavior of measured object being determined by the measurement system.

Recent single photon interference experiments [4], [5], [6] have implemented John Wheeler's ingenious thought experiment [3] to test Bohr's complementarity principle. While confirming complementarity, some of these experiments have revealed the weirder phenomenon of retro-causality and quantum erasure which stretches the understanding of duality, complicated further when entangled photon pairs are involved.

All experiments to date confirm Bohr's complementarity. In a multi-path interferometer, the act of observing which path the particle took (which way) is thus believed today to cause the disappearance of the interference pattern, and so "which way" ("welcher-weg" in German) determination has become an accepted analysis and design consideration in multi-path quantum systems. The critical question of whether or not there exists a "which way" measurement implied in a given multipath interferometer system becomes difficult if not impossible as the complexity of the system increases such as in quantum communication systems and quantum computers. Thus it is of great value if the "which-way" determination – which potentially can include consciousness - can be avoided altogether.

The Theorem proposed and proved in this paper does not use any metaphysical "multiverse" or "consciousness" of the observer, and explains duality without complementarity or "which way" consideration or any "knowledge" on the part of the inanimate photon (particle) about experimental setup, and incidentally redeems Albert Einstein's view that measurement purpose may not influence wave-particle behavior.

Some of the more remarkable experiments reported use entanglement as a carrier of "which way" information, and so our discussions involve entanglement also, which must therefore be understood. Albert Einstein, troubled by the statistical nature of quantum mechanics, suggested a thought experiment in the famous E.P.R. paper [8] (1935) which he co-authored, which predicted action at a distance violating the locality constraint imposed by the relativistic speed limit of velocity of light, and therefore expressed the doubt: "Is quantum mechanics complete?" Erwin Schrodinger immediately responded [9] affirming that the phenomenon described necessarily follows from the wave function concept, and coined for it the term "entanglement". A hypothesis of non-verifiable hidden random variables (as the name implies) to explain entanglement was rendered verifiable by experiment by the landmark inequality test developed by J.S. Bell [10] (1964), improved upon by many others for example [11], and studied by experimenters gradually eliminating loop holes, to finally confirm recently [12] (2015) that there are no hidden variables, thus confirming action at a distance.

As a quick review of the evolution of wave function $\psi(\underline{\mathbf{r}}, t)$ in space $\underline{\mathbf{r}}$ and time t, which is central to the relationship (duality) between the particle and its wave function, for example for electron with mass m in potential field V ($\underline{\mathbf{r}}$) the Schrodinger wave equation is

$$i\cdot\hbar\cdot\frac{\partial}{\partial t}\,\psi(\underline{\mathbf{r}}, t) = H\cdot\psi(\underline{\mathbf{r}}, t) \qquad (1a)$$

where $H = (\underline{\mathbf{p}}\cdot\underline{\mathbf{p}}/(2\cdot m) + V)$ is the Hamiltonian = total energy E, $\underline{\mathbf{p}}$ is momentum, $i = \sqrt{(-1)}$ and $\hbar$ $(= \frac{h}{2\cdot\Pi})$ is the reduced Planck's constant. With operator interpretation of $\underline{\mathbf{p}}$ as $\underline{\mathbf{p}} = -i\cdot\hbar\cdot\nabla_r$ where $\nabla_r = (\frac{\partial}{\partial x}\cdot\underline{\mathbf{u}}_x + \frac{\partial}{\partial y}\cdot\underline{\mathbf{u}}_y + \frac{\partial}{\partial z}\cdot\underline{\mathbf{u}}_z)$, $\underline{\mathbf{u}}_x$, $\underline{\mathbf{u}}_y$, $\underline{\mathbf{u}}_z$ spatial unit vectors, and with operator interpretation of energy E as $i\cdot\hbar\cdot\frac{\partial}{\partial t}$ in $E = (\underline{\mathbf{p}}\cdot\underline{\mathbf{p}}/(2\cdot m) + V)$, we get

$$i\cdot\hbar\cdot\frac{\partial}{\partial t}\,\psi = -(\hbar^2/2m)\cdot\nabla_r^2\,\psi + V\psi \qquad (1b)$$

Note: Operator interpretation is implied in "derivation" of Schrodinger's equation starting with $\psi = e^{-i\cdot(E\cdot t - \underline{\mathbf{r}}\cdot\underline{\mathbf{p}})}$ as can be readily seen from partial derivatives of ψ with respect to time and space variables.

For photon $m = 0$ and so (1b) is not applicable. Using relativistic relationship $E^2 = m(0)^2 \cdot c^4 + \underline{\mathbf{p}} \cdot \underline{\mathbf{p}} \cdot c^2$ where rest mass m(0) does not appear in the denominator, with $m(0) = 0$, the operator interpretation results in

$$\delta^2\psi/\delta t^2 = c^2 \cdot \nabla^2_r \psi \qquad (2)$$

which is the quantum mechanical wave equation for photon, whose mathematical form is same as that of electromagnetic wave equation of classical electrodynamics, and so has similar solutions that propagate in space. The important difference being that spatial integral of $|\psi|^2$ is constrained to be 1 for quantum mechanical wave function, whereas there is no such constraint for the amplitude of classical electromagnetic wave.

We note that in general

(a) The operator interpretation of physical quantities links non-physical wave function to physical quantities.

(b) Either (1) or (2) results in causal evolution of ψ in space-time, from initial conditions of forward motion which result in ψ evolving only forward in time from the initial time of creation (components of backward propagation cancel out due to initial condition of forward motion, as in any wave motion) until annihilation.

As discussed in the next section, complex wave function ψ represents a probability amplitude, with $|\psi|^2$ a probability density function, and so it is a non-physical purely mathematical entity. H or E and $\underline{\mathbf{p}}$ in (1) or parameter c in (2) contain the physical parameters of the system, and therefore, non-physical wave function ψ propagates in space and time as per physical parameters, obeying locality constraint of speed limit of velocity of light in free space.

The fact that Schrodinger's wave equation works has been confirmed by all experiments and quantum systems. But "Why (not how) does Schrodinger's wave equation work?" remains the unanswered question of quantum mechanics, suggesting rephrasing accordingly Albert Einstein's question "Is quantum mechanics complete?"

Any approach to explain duality requires the understanding of the relationship between the particle and its wave function. Louis De Broglie and Erwin Schrodinger initially thought that the wave function was actually a physical wave associated with the particle, which led to problems because wave function is inherently complex and not real. This difficulty was removed by Max Born in 1926 by interpreting the physical wave as complex probability amplitude ψ, the wave function. Born states in his Nobel Prize acceptance speech [13] "… an idea of Einstein's gave me the lead. He had tried to make the duality of particles - light quanta or photons - and waves comprehensible by interpreting the square of the optical wave amplitudes as probability density for the occurrence of photons. This concept could at once be carried over to the ψ-function: $|\psi|^2$ ought to represent the probability density for electrons (or other particles)". Note that though the wave function is thus recognized as non-physical complex probability amplitude, it is viewed as an interpretation of a physical wave, especially for photon whose wave nature is more evident as physical electromagnetic wave, while for electron, particle nature is more evident as non-zero physical rest mass. This view of non-physical wave function as somehow being also some physical wave entity has persisted to this day, requiring co-location (coincidence) of particle and its wave function, changing from particle to wave and vice-versa depending on measurement, and this is at the heart of the duality mystery. The Duality Theorem stated and proved below removes this co-location (coincidence) and thereby explains duality without complementarity or "which way", physical particle always remaining particle and its wave function always remaining wave.

II WAVE-PARTICLE DUALITY THEOREM

Given that (1) Wave function $\psi(\underline{\mathbf{r}}, t)$ of a particle is a non-physical purely mathematical complex probability amplitude, $|\psi(\underline{\mathbf{r}}, t)|^2$ being the probability density function, that is, $|\psi(\underline{\mathbf{r}}, t)|^2 \cdot \delta v$ is the probability that the particle is in an infinitesimal volume δv at space-time point $(\underline{\mathbf{r}}, t)$ (2) Physical particle is indivisible, and (3) In the case of an extended (non-point) physical particle, by "position" of the particle we mean the position of some cardinal point of the particle such as its centroid, it follows that:

At any given time t, wave function ψ can be co-located (coincident) with its particle only at space-time point $(\underline{\mathbf{r}}_0, t)$ where $|\psi(\underline{\mathbf{r}}_0, t)|^2 = \delta(\underline{\mathbf{r}} - \underline{\mathbf{r}}_0, t)$, the unit Dirac delta function. At any space-time point $(\underline{\mathbf{r}}, t)$ where $0 < |\psi(\underline{\mathbf{r}}, t)|^2 < 1$, wave function cannot be co-located (coincident) with its particle.

Proof:

Because spatial integral of probability density function must be equal to 1 at any given time t (particle exists somewhere in space), if there is a space-time point $(\underline{\mathbf{r}}_1, t)$ where $0 < |\psi(\underline{\mathbf{r}}_1, t)|^2 < 1$, it means that there are more than one different space-time points $(\underline{\mathbf{r}}_1, t)$, $(\underline{\mathbf{r}}_2, t)$ … where $|\psi(\underline{\mathbf{r}}_2, t)|^2 > 0$ ... That is, there is non-zero probability that the particle may be at different points $(\underline{\mathbf{r}}_1, t)$, $(\underline{\mathbf{r}}_2, t)$, etc. But because the particle is indivisible, it (its cardinal point) cannot be at (more than one) different space points at the same time t, that is, the particle cannot be coincident with the wave function; wave-particle coincidence is possible if only if the wave function itself exists at only one point and is zero everywhere else. But the spatial integral of $|\psi(\underline{\mathbf{r}}, t)|^2$ must equal 1, which is possible only if probability density is Dirac delta function, that is, $|\psi(\underline{\mathbf{r}}_0, t)|^2 = \delta(\underline{\mathbf{r}} - \underline{\mathbf{r}}_0, t)$. The above logical reasoning is also illustrated in Figure 1.

Figure 1(a) Case of wave function and particle with non-point spatial spread having the same profile (case of spatial point is covered in 1(d)), representing the conventional view that wave function is probability amplitude interpretation of something physical associated with the particle, thereby requiring co-location of identical profiles of both wave and particle. Position of particle is represented by position of some cardinal point such as centroid. Because of the spread, there are other points where the physical centroid can be at the same time, not possible for indivisible physical particle.

Figure 1(b) Case of the wave function and particle having different non-point spatial spreads. There are several points where probability is not zero, and so spatial colocation of wave function and cardinal point of indivisible physical particle is not possible.

Figure 1(c) Case of multiple paths of wave function, each with non-zero probability. At a given time, cardinal point of indivisible physical particle can be at only one location. Co-location of wave function and particle is not possible. Wave function defines probabilities of multiple probable paths. Physical particle follows only one probable path.

Figure 1(d) Case of the wave function being a Dirac delta function. Only in this case wave function and cardinal point of indivisible particle can coincide, co-location is possible.

Figure 1(e) Example of emission of a single physical particle detected by only one detector, while spherical wave function defines non-zero probabilities for detectors at other locations on the wave front.

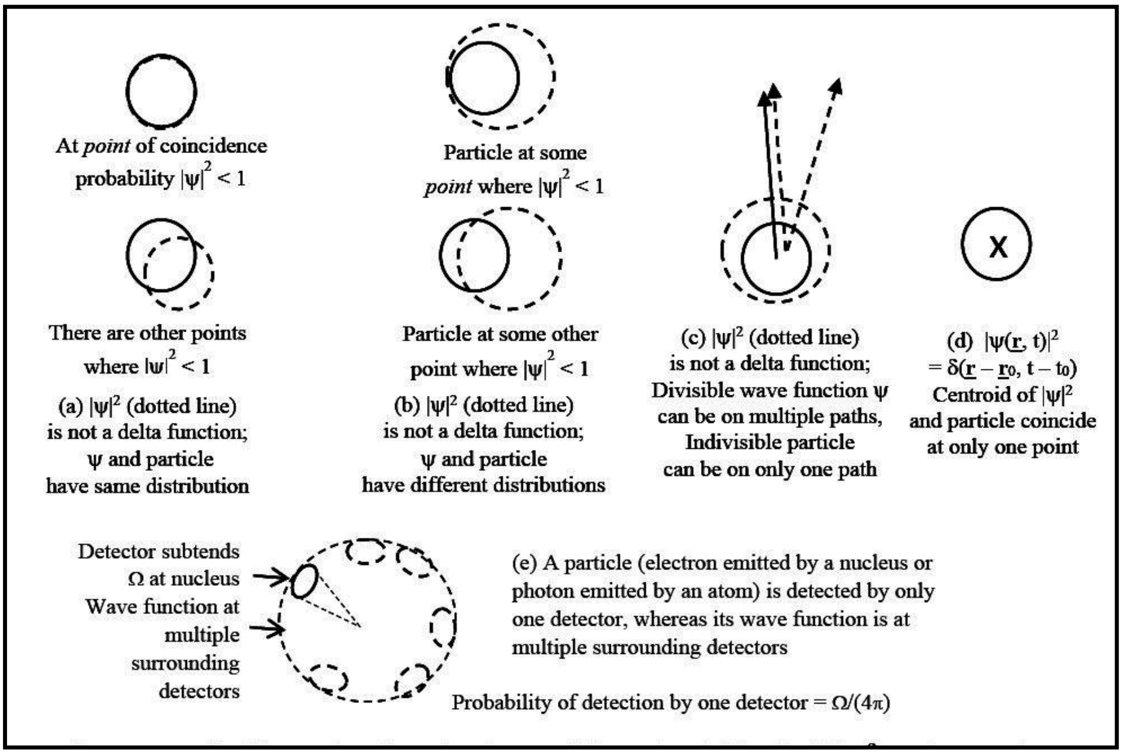


Figure 1. Coincidence / Colocation is possible at ($\underline{r}_0$, t) if only if $|\psi|^2 = \delta(\underline{r} - \underline{r}_0, t)$

Comments:

1. A photon is indivisible except when it passes through a device such as parametric down converter in which it splits into two photons each of less energy. Single photons in all interference experiments such as Young's double slit experiment (which was the subject of heated debates between Albert Einstein and Niels Bohr), and in all experiments that have been conducted to test Bohr's complementarity, and the signal photons in most quantum communication systems and quantum computers, are all indivisible between the time they are created (such as at the output of a parametric down converter source of entangled pair) till the time they are detected by absorption (annihilation) in a detector. Between the time of creation and the time of annihilation the photon may interact with optical media and optical components such as beam splitters which may change its state such as polarization, but it remains physically indivisible. An electron is similarly indivisible unless it is of high energy and may disintegrate into multiple particles, which is not the case in most quantum systems of interest for quantum computers and quantum communications. Such indivisibility of photon and electron in conditions described, in the interference systems of interest such as Young's double slit experiment and in most quantum communications and quantum computer systems, is an experimentally established fact.

2. At the space-time point of creation $|\psi(\underline{r}, t)|^2$ is a Dirac delta function, from which point the wave function evolves per Schrodinger's wave equation.

3. At the space-time point of annihilation (absorption) the "collapse" of the wave function can be viewed as $|\psi(\underline{r}, t)|^2$ collapsing into a Dirac delta function.

4. The novelty of Duality Theorem lies in that it completely does away with complementarity and "which way" (welcher-weg) criterion, and also does not require any "observer" in a measurement process or any "intelligence" on the part of the particle. This has not been done before except in precedent paper by the author [16] with Axiom.

NOTE1: The widely accepted definition of probability is the Von Mises definition as the $\text{Lim}_{N\to\infty}$ (n/N) where n is the number of times the outcome occurs in N hypothetical trials, see [14] p 8-9. Thus the propagation of wave function along all possible paths is hypothetical, corresponding to various hypothetical trials.

NOTE2: The uncertainty in position (due to Heisenberg's uncertainty principle) can be taken into account by including the position uncertainty in the profile used above to define the region, the centroid of which is taken to be point $\underline{\mathbf{r}}$ in $\psi(\underline{\mathbf{r}}, t)$.

An important consequence of the Duality Theorem, which removes the conventional co-location of wave function and particle, is that the wave function hypothetically explores all possible paths defining probabilities for each probable path, that is, the wave function is divisible, whereas the indivisible physical particle follows only one probable path, illustrated in Figure 2 for two important cases: (a) reflection and refraction and (b) Single photon Young's double slit experiment. Note that the configuration may be changed dynamically at any instant of time, and wave function propagates according to new configuration from that instant of time onwards.

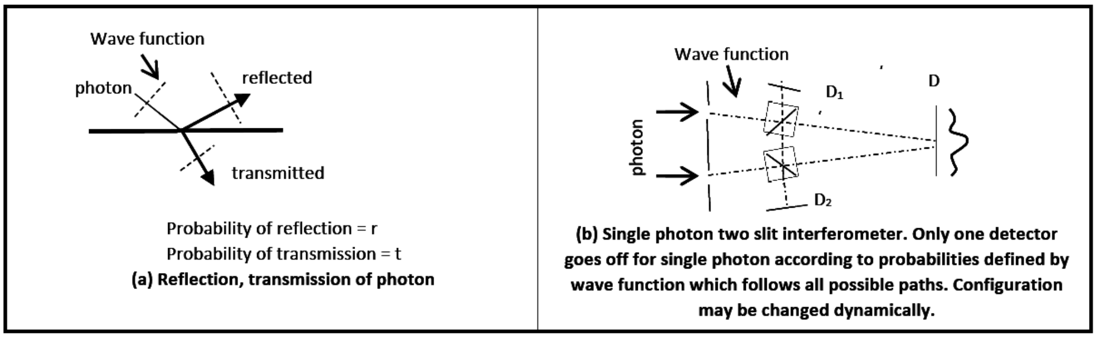


Figure 2. Divisible wave function explores all possible paths defining probabilities, Indivisible physical particle follows only one probable path.

Because propagation of wave function is determined by physical parameters as pointed out earlier, the phenomenon of reflection or refraction of wave function at physical surfaces is governed by interactions with atoms defining the surface and the media. See for example [15] R.P. Feynman "QED the strange theory of light and matter" for the geometrical construction of resultant wave function amplitude as due to wavelets from each point (atom) of the surface (medium). As long as the amplitudes of wave function components in such reflections and refractions (or in general in any medium of propagation or scattering phenomena) remain non-zero, the wave function continues to propagate in such systems. The state of the wave function, such as the state of polarization of photon, or spin of electron, may be altered due to interactions with the medium. Thus the wave function, which is non-physical probability amplitude, carries with it the probability of the state of the particle due to probable interactions of the physical particle with the physical medium.

ENTANGLEMENT: Because probability is defined axiomatically as a frequency measure based on hypothetical trials (Papoulis [14] page 7), for any given configuration which may vary with time, wave function ψ propagates hypothetically along all possible paths to determine various probabilities, without physical propagations. Which probable path / outcome actually occurs is found by the measurement. In classical picture the selection of outcome is associated with some random variable prior to measurement. However, in the quantum picture of entanglement it has been demonstrated that there is no random variable selection prior to measurement (no hidden variable), and it is only the measurement that finds the outcome. A pair of particles are entangled if their joint probability density is not factorable as product of individual probability densities, and there is thus a constraint of conditional probability, such as a constraint of polarization between two polarization-entangled photons. In such cases, the outcome found by measurement must necessarily involve measurement of both particles, which may occur at different space-time points, regardless of temporal sequence of the two measurements. For clarity, let us call the measurement of the two entangled particles as one joint measurement, completed only when the last one is measured (to satisfy entanglement constraint). Note that for entangled pair, one joint measurement finds an outcome for both in the pair out of many probable pair-outcomes.

There are no two separate pair-measurements, and so there is really no "erasure" of a prior measurement.

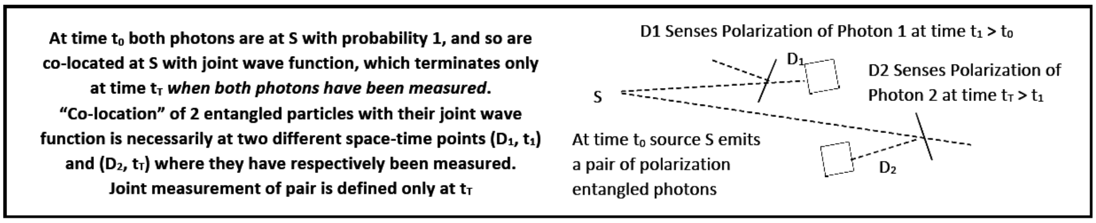


Figure 3. Joint measurement of entangled pair is defined only when both particles have been measured. Co-location (coincidence) with joint wave function only at source S and detectors D_1 and D_2, not elsewhere.

Joint measurement and co-location of entangled particle pair with joint wave function is illustrated in Figure 3. Joint wave function magnitude squared is a unit Dirac delta function at Source S at creation time t_0, and partial Dirac delta function at detector D_1 at time t_1 (partial collapse) and at detector D_2 at time t_T, overall integral being 1.

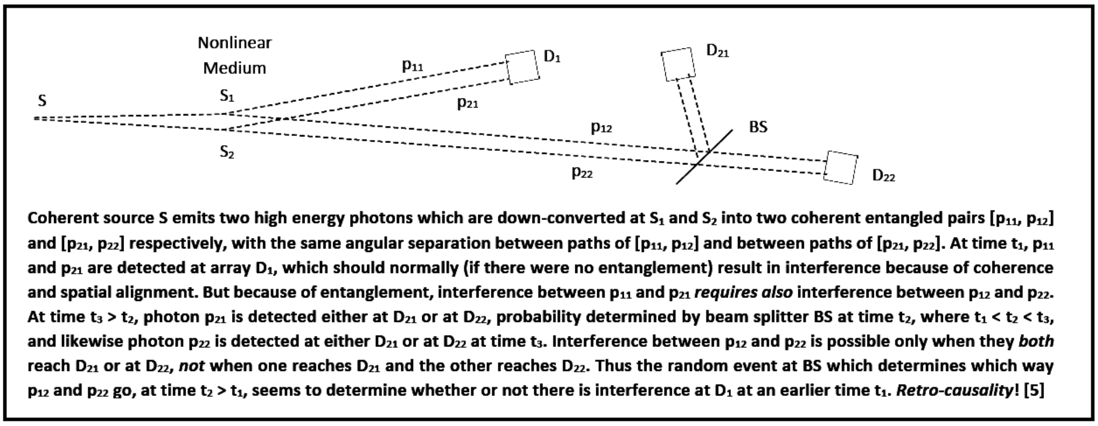


Figure 4. Interference between two pairs of entangled photons – apparent retro-causality.

For more details see prior paper [16], where it is shown that for all experimental results with or without entanglement as for example Young's double slit experiment, John Wheeler's thought experiment and [4], [5], [6],

Coherence and spatial alignment ≡ Interference ≡ indistinguishable paths, no "which way"

No coherence or spatial alignment ≡ No interference ≡ distinguishable paths, "which way"

That is, "which way" determination of measurement (Complementarity Principle) is redundant, and so can be dispensed with, thereby avoiding unnecessary (unscientific) metaphysical speculations of mystical involvement of the consciousness of the observer also as part of the quantum system. This concludes review of prior archived unpublished paper [16] with Axiom replaced by Duality Theorem. The following results were presented at SPIE Photonics West 2019 conference [17] but not yet published.

III. APPLICATION OF DUALITY THEOREM TO INTERACTION-FREE-MEASUREMENT (IFM)

Interaction-free measurements with potential to preserve the state of a quantum signal particle, if feasible, would be valuable in quantum communications, as quantum measurements usually change the state of the measured quantum object (signal). In 1981, R.H. Dicke proposed "interaction-free measurement" in a thought experiment shown in Figure 10, as perhaps a paradox. For details see his paper[18].

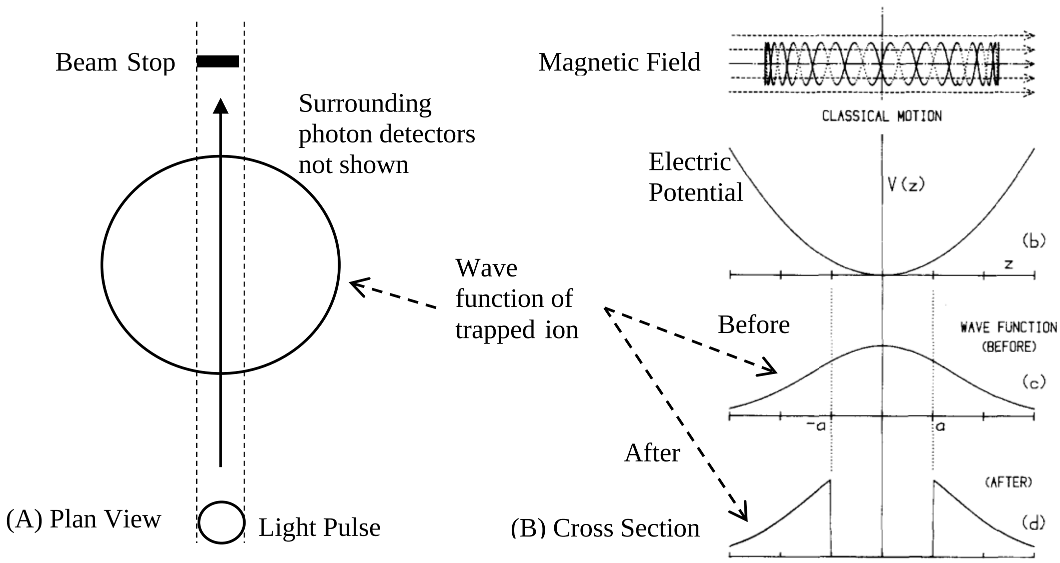


Figure 5. Thought experiment of R.H.Dicke[18] for interaction-free quantum measurement

(Reuse of Figure 3 of R.H. Dicke's paper per Creative Commons License Attribution 4.0 terms of use at https://www.scitations.org; https://aapt.scitation/doi/10.1119/1.12592;)

An intense light pulse is sent through the wave function of a trapped ion and any photons scattered by the ion are monitored by detectors completely surrounding the wave function (detectors are not shown in the figure). If no scattered photons are detected, it is assumed that the wave function in the path of light pulse can be zeroed out, thus claiming that the negative result of non-interaction results in a modification of the wave function.

R.M. Angelo[19] points out the weakness in Dicke's reasoning as due to not recognizing the wave function as a probability function. But Dicke does recognize wave function as a probability function but seems to consider it as a probability interpretation of a physical aspect of the particle (ion), just as most scientists have viewed duality all along.

Applying our Duality Theorem: Because wave function of ion is not a Dirac delta function at any point in the path of the light pulse, and is not zero outside the path of the light pulse, we cannot expect the particle to be in the path probed by the light pulse with certainty. More light pulses will eventually interact with the ion, provided the ion-photon interaction cross section is not zero. The zeroing of the wave function in the probe path due to negative result of the first light pulse probe, thereby expecting the same negative result for any number of more light pulses, is actually a contradiction, not a paradox: If an arbitrarily large number of intense light pulses were all to produce negative result, then the assumption of the shape of the original wave function of ion itself is wrong.

This example illustrates how the persisting notion that wave function is a probability amplitude interpretation of something physical associated with the particle is misleading and wrong. The wave function is simply not a physical entity, it is divisible, and it is not coincident with indivisible particle unless it is a Dirac delta function at that point.

3.1 Elitzur and Vaidman (EV) scheme for interaction-free measurement

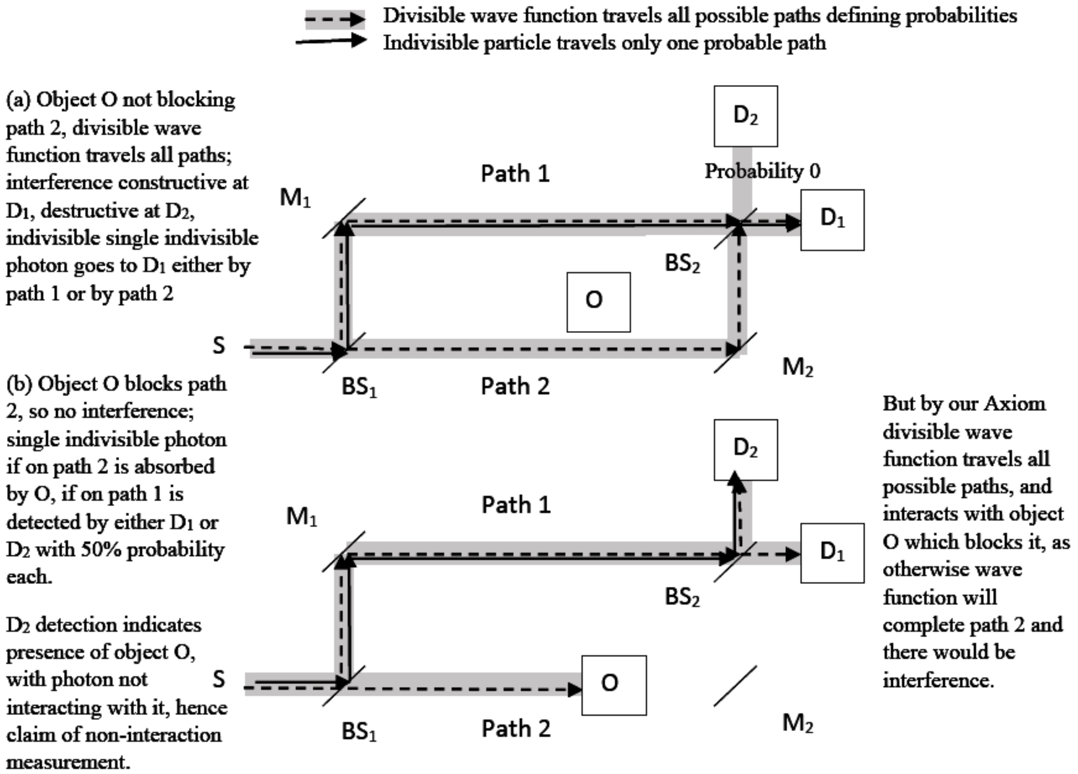


Figure 6. Elitzur and Vaidman (EV) proposal for interaction-free measurement[20]

Figure 6 shows another thought experiment for interaction-free quantum measurement, based on Mach-Zehnder interferometer, proposed by A.C. Elitzur and L. Vaidman[20]. This attracted considerable debate and also attention for some forms of counterfactual communication which we shall discuss shortly. Vaidman has extensively reviewed these discussions[21].

Referring to Figure 6, S is a source of single photons. M_1 and M_2 are ideal mirrors. BS_1 and BS_2 are 50% ideal beam splitters. D_1 and D_2 are ideal single photon detectors. Without the absorbing object O in path2, the path lengths are such that constructive interference occurs at D_1 (count) and destructive interference at D_2 (no count). If now path2 is blocked by an absorbing object O, then D_1 or D_2 will register count each with 25% probability (50% probability at BS_1 for path1 and 50% probability at BS_2 for either D_1 or D_2). When detection of photon at D_2 via path1 occurs, the presence of object O is thus sensed or "measured", without the photon interacting with object O, and thus it is claimed to be an interaction-free measurement.

Applying our Duality Theorem: In (b), travelling both paths from BS_1, divisible wave function does interact with object O in path2. Thus it is incorrect to say that this is a truly interaction-free measurement.

Figure 7 shows a Michelson interferometer version of the Mach-Zehnder interferometer based interaction-free measurement scheme of Elitzur – Vaidman (Figure 6). This will lead us to so-called counter factual communications to which we shall apply our Duality Theorem to show limitations and gain a clearer understanding.

Source S sends a single photon into the interferometer. After going through beam splitters 2 and 1, path1 is to mirror M_1 then back to beam splitter BS_1 and then to detector D_1. After beam splitters, path2 is to mirror M_2 then back to BS_1 then to BS_2 and then to detector D_2. Path lengths are such that constructive interference occurs at D_1 (count) and destructive interference at D_2 (no count). If now an absorbing object O is placed in path2, then interference is destroyed and either D_1 or D_2 registers count in the event photon does not go through path2 and get absorbed (if photon is absorbed by O neither D_1 nor D_2 can register count.). Thus, when D_2 registers count object O is sensed ("measured") without photon interacting with the object, which may be claimed to be interaction-free measurement.

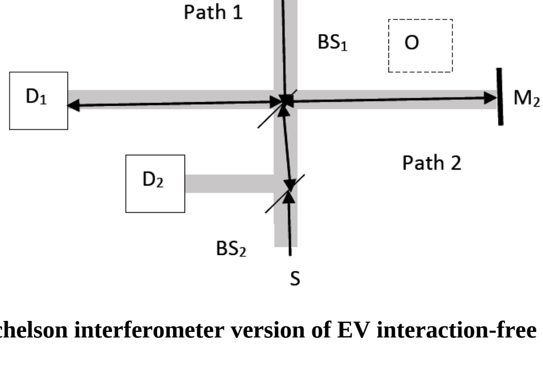


Figure 7. Michelson interferometer version of EV interaction-free measurement

Applying our Duality Theorem, divisible wave function does travel path2 and interact with object O. Thus it is not exactly interaction-free, just as in the case of Elitzur-Vaidman Mach-Zehnder scheme in Figure 6.

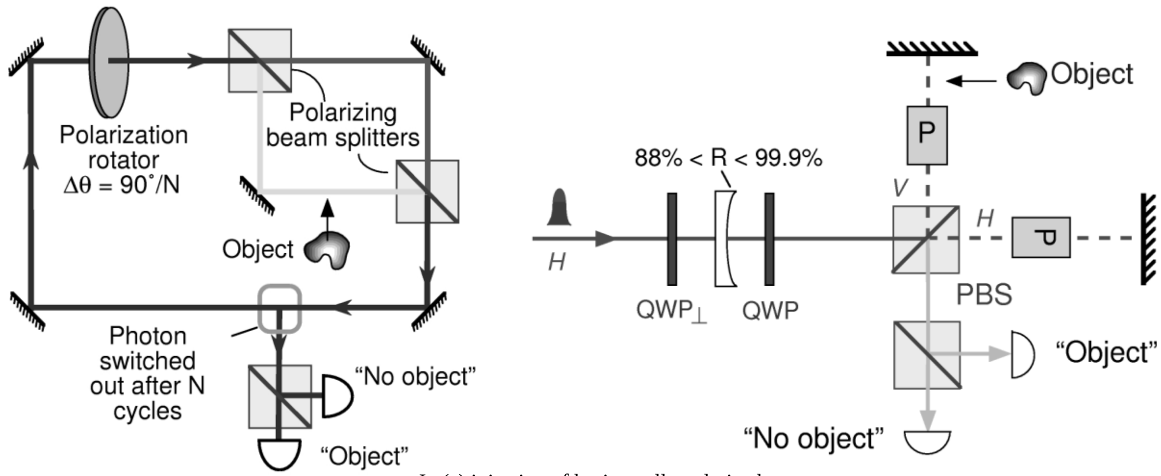
In an attempt at better terminology to describe the wave function, recently A. Shenoy, R. Srikanth[22] have proposed to characterize the wave function as "real but non-physical". While this draws our attention to the dilemma, it is a bit confusing to say it is real but non-physical, because wave function is complex, not real. As we shall see shortly, recognition of this important fact - non-physical wave function does interact with physical media - throws a damper on the claims of counterfactual quantum communication also, which nevertheless are interesting and worth studying – but with some significant clarifying qualifications.

3.2 Increased probability interaction-free measurement using quantum Zeno effect

In the Elitzur-Vaidman (EV) thought experiment for interaction free measurement (IFM) shown in Figure 6, the probability of IFM is 25% of all trials, and 50% of trials in which detector D_2 records count. In 1999, P.G. Kwiat, A.G. White, J.R. Mitchell, O. Nairz, G. Weihs, H. Weinfurter and A. Zeilinger [23] reported a way to significantly boost the probability of IFM by using an optical version of Quantum Zeno Effect (concept discussed in 1977 by B. Misra and E.C.G. Sudarshan[24]), and verified it experimentally. Their concept and experimental setup are shown in Figure 8 (a) and (b) respectively, with permission to reuse by APS.

Zeilinger et al prefer to call their measurement as quantum interrogation instead of interaction free measurement, the basic idea being the same, namely sensing (interrogation or measurement of) an absorbing object without the physical particle interacting with the object. Here we shall limit our discussion to aspects that are made clearer by application of our Duality Theorem. Note that their concept 8(a) is based on Mach-Zehnder interferometer for convenience of explanation, while their experiment 8(b) is based on Michelson interferometer version for practical convenience. They are essentially equivalent. To understand the concept, a horizontally polarized photon is injected into the system at top left (injection not shown in figure 8(a)) and its plane of polarization is rotated by a small angle $\Delta\theta = \pi/(2\cdot N)$, where N is a large number. H component probability amplitude $\cos(\Delta\theta)$ is transmitted by PBS while V component probability amplitude $\sin(\Delta\theta)$ is reflected. Note that by our Theorem it is the wave function ψ (which defines probability amplitudes for various paths) that is split, not the particle (photon) which follows only one path.

Now, (i) when the absorbing object does not obstruct the path, the V and H components of probability amplitude coherently combine (assuming path lengths are equal and stable) to form the resultant ψ at the output of second PBS with the same polarization angle as at the input of the first PBS, which is $\Delta\theta$ for the first time around the loop. For the second time around the loop the polarization angle is $2\Delta\theta$ and so on, for the m-th cycle, $m < N$, it is $m\Delta\theta$. After N cycles, when polarization angle is $\pi/2$, the path to detectors is switched on for the wave function (and the photon), and because polarization is now V, the “No object” detector must record a count with probability 1.



(a) Concept

In (a) injection of horizontally polarized particle at top left is not shown. Polarizing beam splitter transmits H and reflects V.

(b) Experiment

Figure 8. P.G. Kwiat, A. Zeilinger et al increase of IFM probability using Quantum Zeno Effect

(Figures 1 and 2 of their paper[23] reused under APS License RNP/18/NOV/009928)

Note: Wave function does interact with the object, as otherwise object cannot affect interferometer.

On the other hand, (ii) when the absorbing object blocks the path, in the first cycle the small V component probability amplitude sin(Δθ) of wave function is absorbed by the object, and so it does not reach the second PBS, and the H component probability amplitude cos(Δθ) of wave function is transmitted as such through the second PBS. In the second cycle, *this* H component gets rotated by Δθ and so the angle of polarization remains at Δθ. Indeed, angle of polarization remains at Δθ through all cycles – this state of polarization as defined by angle of polarization remaining unaltered is termed Zeno effect, named after the paradox discussed by ancient Greek philosopher Zeno[25]. Also, after N cycles when the path to detectors is switched on, due to the H polarization of wave function, it is transmitted to “Object” detector which must record a count with probability $\cos^N(\Delta\theta)$, which tends to 1 in the limit as N tends to ∞.

Thus we see that this system, without particle interacting with the object, detects absence of object with probability 1, and presence of object with probability close to 1, tending to 1 as N → ∞, which is truly remarkable. Moreover, it accomplishes this with arbitrarily small amplitude of wave function reaching the object as N is made arbitrarily large. Here we have a paradoxical situation that in the theoretical limit the object has a significant effect on the system with not only the particle, but also with the wave function, not interacting with it. Researchers have noted this paradoxical situation in theory, though in practice it is difficult to make N arbitrarily large.

We shall now show that it is impossible, even in theory, to make the number of cycles N arbitrarily large. Let L_0 be the optical path length of the loop, the distance traveled by wave function (and photon) in one cycle. Then time taken for N cycles is $T_N = N \cdot L_0/c$, where c is velocity of light. Therefore, for given time T_N per measurement,

$$N \leq T_N \cdot c/L_0 \tag{3}$$

Because L_0 cannot be made arbitrarily small, N cannot be made arbitrarily large in any finite time T_N. As a consequence of N being finite, wave function always interacts with the object when it blocks the path. There is no paradox. We note that this resolution is not possible if one assumes that the particle (photon) and its wave function are one and the same, as in conventional view of duality per complementarity principle. This resolution and clarity is possible only due to our Duality Theorem which does away with complementarity principle and maintains that wave function always remains a wave (defining probabilities of evolution along various paths and superposition and interference) and particle (photon) always remains a particle, following only one particular path. We shall see next that this clarification significantly limits claims of "counterfactual" quantum communication also.

IV. APPLICATION OF DUALITY THEOREM TO "COUNTERFACTUAL" COMMUNICATIONS

In classical communication, information (bit) is encoded in a physical entity such as macro electro-magnetic wave carrier sent from sender to receiver. In quantum communication, the information (qbit) is encoded in the state of a physical object such as a particle, usually a photon, which is transmitted from sender (Alice) to receiver (Bob). In recently developed "counterfactual" quantum communication using ingenious schemes, communication is claimed to be achieved without any physical particle passing from sender (Alice) to receiver (Bob) through the communication channel. Because physical particle is susceptible to be intercepted by eavesdropper (Eve), it is claimed that "counterfactual" quantum communication holds promise for increased security. We show that these claims fall short.

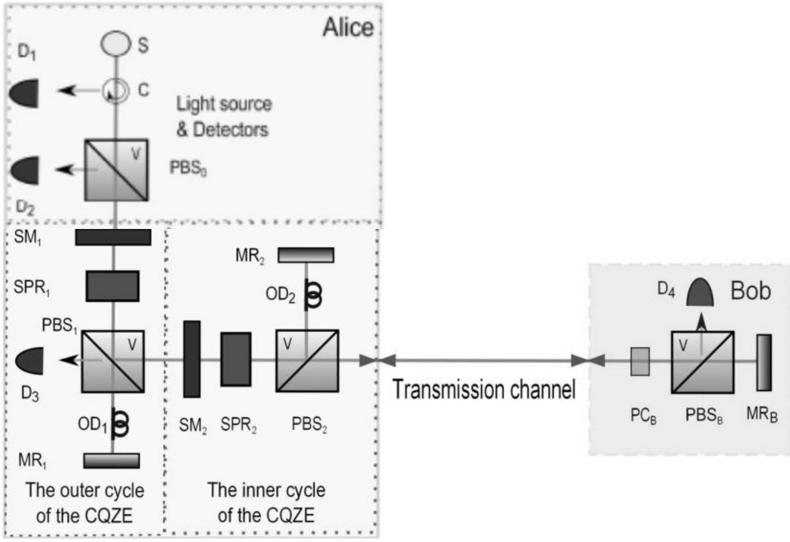


Figure 1 of SLAZ paper

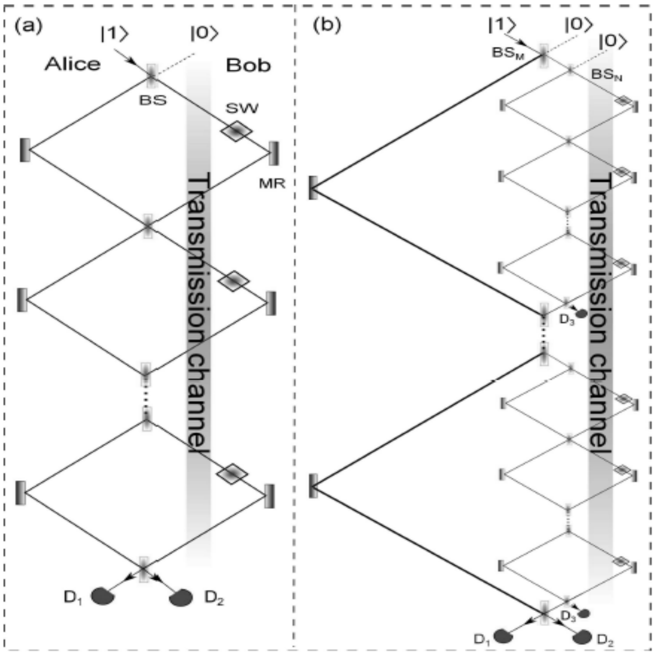


Figure 2 of SLAZ paper

NOTE: By Duality Theorem: Wave function always travels across transmission channel and interacts with objects in Bob's terminal, as otherwise there will be no interferometer. Wave function in transmission channel can be intercepted to provide alternate paths, and thereby system can be altered.

Figure 9. SLAZ (Salih, Li, Amri, Zubairy) scheme for counterfactual communication

(Reuse of Figures 1 and 2 of SLAZ paper[26] under APS license RNP/18/NOV/009925)

In 2013, H. Salih, Z.H. Li, M. Al-Amri and M.S. Zubairy[26] (SLAZ) proposed a thought experiment for counterfactual communication using interferometers, conceptually similar to the quantum interrogation (interaction free measurement) scheme using quantum Zeno effect proposed in 1999 by Kwiat and Zeilinger[23] which we have already discussed, but with the function of "switch" at the end of N cycles (at which time photon passes to final detectors) replaced with an additional M cycles for each of the main N cycles (claimed to avoid photon traversing communication channel at the end of N cycles). Their scheme, shown in Figure 9, has come under strong criticism[27, 28]. In 2017 Cheng-Zhi Peng, Jian-Wei Pan et al[29] reported implementing a limited version of SLAZ experimentally and claim to have successfully verified their results against predictions. Also, SLAZ has been patented[30] in USA.

In Figure 9 on left is the Michelson interferometer version proposed for practical implementation. On right is the Mach-Zehnder version with opened up loops for conceptual understanding, (a) without inner cycles wherein photon is said to traverse communication channel at the end, (b) with inner cycles wherein photon is claimed not to ever traverse the communication channel as $N\rightarrow\infty$. Details of controversial claims made is outside the scope of this paper. We shall limit ourselves to how the clarity of this concept (or any such concept) is improved by our Theorem.

For example, SLAZ paper[26] states "with the help of pockel cell PC_B Bob can either switch the polarization of incoming H photon to a V photon or keep the polarization state unchanged. The PBS_B reflects V photons to a detector D_4 (effectively blocking the communication channel) and allows H photons to be reflected back by the mirror MR_B. Bob can send a stream of logic 0's and 1's by either keeping the polarization state H unchanged (logic 0) or switching it to polarization state V (logic 1). Bob's choice of logic 0 and 1 leads to a click at detectors D_1 and D_2 respectively with almost unit probability and with almost no photon in the transmission channel, thus leading to counterfactual communication" – there is the incoming photon to Bob, but almost no photon in the communication channel. This contradiction is due to the conventional view of duality that particle changes to wave and vice versa, and can be cleared by applying our Theorem, by which wave function always remains wave traveling all possible paths defining various probabilities while particle (photon) always remains particle traveling a single particular path to D_1 or D_2 without traversing communication channel, assuming validity of probability computations – debate of which is outside scope of this paper. Thus it is the wave function and not particle that is incoming to Bob during the many ($N\cdot M$) cycles defining the probabilities for the photon which takes a single particular path to detector D_1 or D_2.

Note that Bob has to keep his selection of 0 or 1 for the entire duration T_N of N outer cycles for each bit ($N\cdot M$ inner cycles), $T_N = N\cdot L_0/c$ where L_0 is the optical path length per one outer cycle and c is velocity of light. This means the bit rate is limited to $1/T_N$ which is $c/(N\cdot L_0)$ which tends to zero as $N\rightarrow\infty$. This is a limitation of SLAZ scheme or any other scheme using interferometric quantum Zeno effect for quantum communication. Also, timing by Alice must be synchronized with Bob's, and path differences must be held stabilized throughout to a small fraction of wavelength.

Following conventional view of duality (that particle changes to wave and vice versa) the paper[29] on successful experimental realization of SLAZ with M = 4, N = 2, states that “photon reaching Bob is discarded when absorber is selected by Bob”, thus implying that photon does traverse the communication channel, contradicting their statements “when single photons are used the counterfactual property is preserved in the case of logic 0 for a finite M and N” and “(even when N is small) the counterfactual property is preserved for the case of logic 1 in all practical scenarios”. This contradiction is resolved by our Theorem, by which it is the wave function, not particle that is absorbed. As shown in Figure 3 of their paper[29] the measured probability (for 0 or 1 signal) drops to about 82% for M = 4, N = 2, which is an impressive improvement from 50% without quantum Zeno effect. However, it took more than 5 hours to transmit 10 kilobits, slowed not only by the minimum time per bit ($T_N = N \cdot L_0/c$) but also by the 52 dB channel loss which required Alice to repeat each bit several times. Also, stabilization of optical path differences to small fraction of wavelength necessitates auxiliary active optical control loops, a challenge even between adjacent optical benches, a formidable task over long distance communication links.

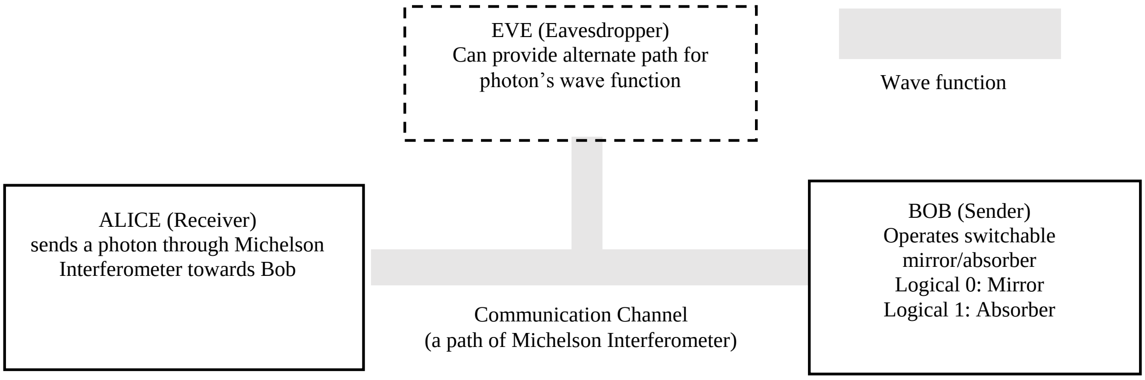
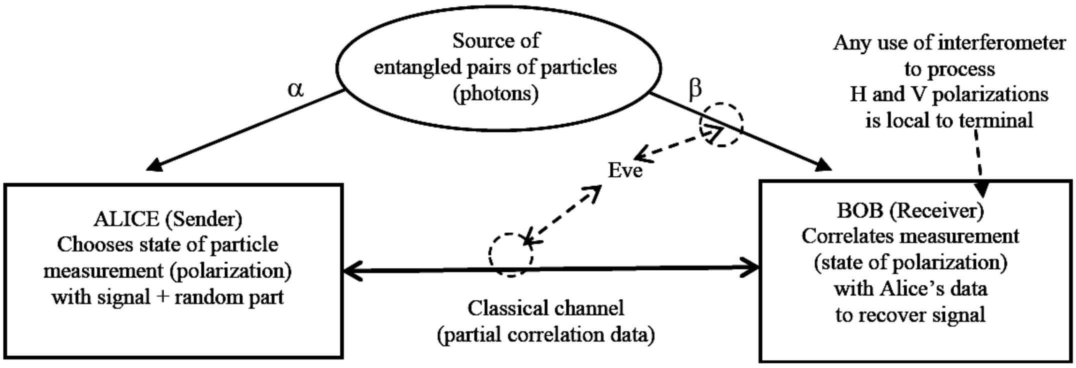


Figure 10. Susceptibility of “counterfactual” communication

As shown generically in Figure 10, due to the fact that the non-physical wave function does interact with physical objects, a whole class of such counterfactual schemes including the SLAZ are really not interaction-free or in general free from Eve’s eavesdropping interference. Given that Eve can access the channel (otherwise eavesdropping is impossible), such as by a beam splitter, Eve can provide an alternate path for the non-physical wave function (which explores all possible paths) even when physical photon is not traversing the channel, and thereby change the characteristics of the interferometer, possibly duplicate Alice. This susceptibility, combined with the formidable challenge of maintaining tight interferometric tolerances on path differences over long distances, would seem to limit practical use. However, a review paper on quantum communication[26] reports counterfactual communication (with interferometric tolerances) over a few km, a significant achievement.

4.1 Quantum communication using entanglement for secure communication

Quantum communication using entanglement has become a main focus of researchers, with impressive results such as the experimental satellite link between China and Austria which currently holds the record for longest quantum communication link. Nevertheless, there are important implications due to Duality Theorem that can help analysis and design of such systems, and their security noting that divisible wave function travels all paths.



Eve can disrupt communication but cannot decode the entangled communication.

Figure 11. Quantum communication using entanglement for secure communication

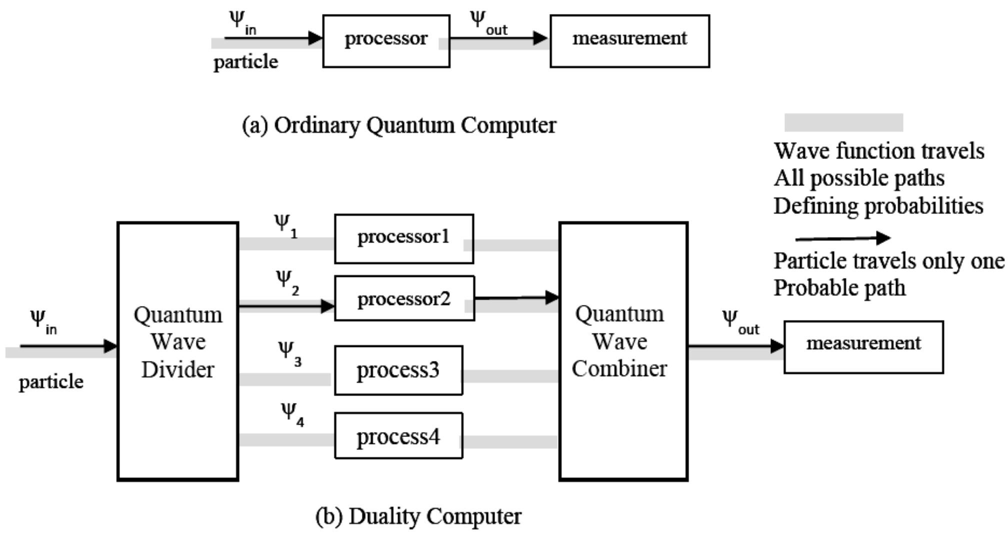
But duality issues arise in multi-paths within terminals

Experimental investigations of entanglement require sufficient physical separation of Bob's terminal from Alice's to avoid any possibility of classical communication. Recently a joint China – Austria team has demonstrated quantum communication using entanglement via a satellite link[31-35], clearly demonstrating feasibility over very long distances. For our discussion, which is limited to treatment of duality that may arise in such systems, Figure 11 shows a basic generic quantum communication system using entanglement. The point to be made here is that any use of interferometer is local to terminal and so there is no challenge of maintaining optical path lengths to interferometric tolerances over communication channel, in sharp contrast to SLAZ and other counterfactual communication systems.

Eve's interception will be a disturbance sensed by Bob in which case he voids data. Thus Eve can disrupt communication but not eavesdrop. If interferometer is used to process H and V polarizations locally in the terminal, then duality "which way" complementarity issue inevitably arises. In all such cases application of our Duality Theorem helps avoid "which way" observation complementarity issues, as wave function always remains wave defining probabilities for alternate paths while particle remains particle following a particular path out of the many probable path. It is hoped that this will clarify and simplify analysis and design of future terminal systems.

V. APPLICATION OF DUALITY THEOREM TO DUALITY COMPUTER

"Duality Computer"[36] shown conceptually in Figure 12 is claimed to be more powerful than regular quantum computers because it utilizes parallel processing of "sub-waves" of the wave function. Stan Gudder[37] provides a mathematical treatment of the duality computer concept. The ordinary quantum computer ideally processes a single particle (such as single photon or electron) in such a way that the output measurements contain the result of computations (In reality the process is repeated to establish correlations of probabilistic outcomes). In the duality computer the input wave function ψ_{in} of the particle is passed through a quantum wave divider, which is typically a set of slits (not beam splitters which can alter the state of input wave function) which outputs multiple attenuated copies ψ_1, ψ_2, ψ_3 etc of the input wave function without altering its state which are then processed by respective quantum processors the outputs of which are superposed in quantum wave combiner and measured. This parallelism of multiple processors processing the input wave function gives corresponding increase in computing power. It may be noted that there is no cloning involved in the quantum wave divider which is typically slits as in Young's double slit experiment. It may also be noted that this is not same as running several ordinary quantum computers in parallel, because here exactly the same input state is provided to each processors, which is not possible without cloning in separate ordinary quantum processors.



Parallelism exists only for divisible wave function, as indivisible particle travels only one probable path.

To assemble results of all parallel processing a sequence of multiple particles needs to be input.

Figure 12. Duality Computer concept of Gui Lu Long[36]

However, in describing his duality computer concept for a three slit case, author Gui Lu Long says on page 5 of his paper[36] : "For instance a three slits wall will divide a wave into 3 parts, each with $1/3|\psi>_i$ where the subscript i indicates the path number. However this information should not be available at the detector, otherwise the interference pattern will disappear". The last sentence is a consequence of conventional complementarity view of duality, namely "which way" path information destroys interference, which is not only imprecise, bordering on mystical subjectivity than objective science, it makes systematic analysis and design of multi path systems complicated and confusing, if not impossible. It is in such situations, as complexity of future quantum computers inevitably increases with multitude of paths, that our Duality Theorem finds useful application: Interference depends only on coherence and alignment (including alignment of polarizations) which can be systematically analyzed and designed for, there is no need to determine if "which way" path knowledge is implied in the measurement system.

VI DUALITY IN NANO-SCALE PHOTONIC QUANTUM COMPUTER CHIPS

Reported major R&D results in quantum computers seem to fall under two main categories: (1) Electronic quantum computer using up/down spin of electron as the basic quantum state, but requiring strong magnetic fields and cryogenic temperatures, which point to highly centralized processing to afford the infrastructure (2) Photonic quantum computer using polarization and angular momentum of photon as the basic quantum states, but with poor linear inter-photon interaction. However, newly emerging topological photonics seems to provide answer, quoting Dr Alberto Peruzzo of Australian Center of Excellence for Quantum Computation and Communication (CQC2T)[38] about their topological photonic chip: "Topological photonics have the advantage of not requiring strong magnetic fields, and feature intrinsically high-coherence, room temperature operation and easy manipulation".

Australian CQC2T is also reported[39] to be developing electronic 10-qubit quantum integrated circuit prototype in Silicon to be accomplished by 2022, but it requires superconducting magnets to provide the magnetic field needed to flip the spin states of electrons, and cryo temperature to reduce noise. In either case (electronic or photonic) CQC2T developments are examples of major advances in integrated quantum circuits (chips) at the nano-scale just as classical computer chips are at the nano-scale. Understanding what happens at the nano-scale (without the benefit of usual discrete components on an optical bench) becomes crucial. Research examples[40,41] discuss multi-quantum dot structures.

Our discussion is limited to duality issues that will inevitably arise as complexity of quantum computers increases using such chips. A basic component in photonic quantum computer is the beam splitter (CQC2T's topological photonic chip is claimed to replicate the functionality of beam splitters) which is typically used to separate the paths of incoming polarized photon on H/V basis, and later to combine paths for interferometric superposition of states. There are also many single photon detectors in the system. According to conventional complementarity view of duality, the all-important interference (superposition) critically depends on whether or not some of the detectors constitute sensing "which way" the photon went, a task that can become highly complicated as the number of beam splitters, beam combiners and detectors increase, as one would expect in integrated quantum circuits (chips). Our Duality

Theorem completely eliminates this complexity, as non-physical wave function remains wave throughout defining various probabilities of various paths and at various detectors, while physical photon follows only one particular path out of the many probable paths, with the particular probability. The equivalence established

Coherence and alignment ≡ interference ≡ no "which way" observation;

No coherence or alignment ≡ no interference ≡ "which way" observation

allows us to avoid troublesome "interference ≡ no "which way" observation" which can also involve observer's subjectivity, and work with objective "coherence and alignment ≡ interference" enhancing clarity in analysis/design.

A comparison of salient aspects of conventional electronic computers with those of quantum computers is attempted in table in Table 1. On the hardware side clearly quantum computers are in very early stages of development, with major challenges, especially cryo versus room temperature operation and integrated "chips" with nano-scale photonics. On the "software" side one can only wonder if the quantum computer "software" development will parallel that of conventional electronic computers, namely "instruction set" to "machine language" to "kernels" to "operating systems" to "applications software" and "input/output" to "user interfaces" and graphical "windows". We may recall that programmability is the defining characteristic of what we have come to call a "computer". A mere computing machine such as an abacus does not qualify to be a "computer" because it cannot be readily re-programmed to solve a very different problem. Analog computers needed to be rewired to solve a different problem. It is programmability enabled by a judicious instruction set that allows the same piece of computer hardware to perform an almost infinite variety of scientific, business, entertainment, communication and other endless applications.

Table 1. Development of quantum computers compared to electronic computers

Electronic Computer Development		Quantum Computer Development	
Analog-Hybrid parallelism	1940-1970	Parallelism	2000-
Discrete technology	1940-1970	Discrete technology	2000-
Integrated circuits	1970-	Integrated quantum circuits ("chips")	2018*-
Programmability, operating systems, general purpose s/w	1940-	Programmability, operating systems, general purpose s/w	????
Bits	Binary	Bits	Qbits
Operating temperature	Room temp	Operating temperature	Cryo (Room temp?)**
Number of particles per bit	> 100	Number of particles per bit	1
Results	Deterministic	Results	Probabilistic

** Brian Wang[39]*
*** [38]*

At present quantum computers seem to be designed for specific uses such as search algorithms and factoring large numbers for cryptography, and programmability is not yet in sight, a vast open field for development awaiting to be explored. Perhaps user interface will always be through conventional computer systems, thus evolving hybrid computer systems reminiscent of analog-digital hybrids of distant past wherein powerful parallelism of analog computers was combined with programming versatility of sequential digital computers. A fundamental difference seems to be that electronic digital computers (many particles per bit) are essentially deterministic (a digital quantum number generator with a given seed produces exactly the same random number sequence every time) whereas quantum computers (single particle per qbit) are essentially probabilistic (results are usually correlations) so it is difficult to produce repeatable results at a single particle level, and so may require multiple trials.

VII CONCLUSION AND DISCUSSION

1. The Duality Theorem is shown to provide much needed clarification in treating wave-particle duality issues in all quantum systems including quantum communication systems and quantum computers, by noting with rationale that particle and its wave function cannot be coincident or co-located except at space-time points where the wave function is a Dirac delta function such as at instant of creation and at instant of annihilation. This avoids "which way" (welcher-weg) complementarity criterion and resulting complexity in analysis and design of quantum communication systems and quantum computer systems. A result of the Duality Theorem which rejects the notion that particle mysteriously turns to wave and vice versa, ensures that (a) wave function always remains wave exploring all possible paths defining probability amplitudes for various paths, and (b) particle always remains particle following one particular path out of all probable paths, greatly simplifies analysis and design of quantum systems, especially in future integrated photonic chips that hold promise for room temperature quantum computers and communication systems.

2. We have shown in the precedent paper [16] that in single particle interference phenomena

> Coherence and alignment (including alignment of polarization) ≡ interference ≡ no "which way"
>
> No coherence or alignment (including alignment of polarization) ≡ no interference ≡ "which way"

Thus, "which way" observation is redundant and unnecessary. Traditional analysis of coherence and alignment applied to wave function suffices. This greatly simplifies analysis and design of multi-path quantum systems and also avoids unnecessary confusion involving "consciousness" of observer and other mystical metaphysical conjectures.

3. By doing away with complementarity and "which way" observation to explain duality, this paper and precedent paper [16] redeem Albert Einstein's view in his heated debates with Niels Bohr on wave-particle issue, namely that measuring instruments

cannot influence the fundamental wave–particle behavior: Wave always remains wave, and particle always remains particle, no mysterious conversion back and forth.

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