

## Measurement of Zhariphium Energy Distribution Function

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### Abstract

The energy distribution function of a particle called Zhariphium is measured from the second derivative of the electron current at the post sheath potential. It is found that the Zhariphium energy distribution function has a Lorentzian profile. The mean life of the Zhariphium is measured to be  $(1.28 \pm 0.23) \times 10^{-16}$  second.

**Key words:** Plasma Physics, Quantum Mechanics, Statistical Mechanics, Atomic Physics.

### Introduction

According to the best of my knowledge there are no reports in the literature studying post-sheath potential [1] effect on the energy distribution function. Druyvesteyn proved that the electron energy distribution function can be obtained from the second derivative of the electron current [2].

In the following report we are going to study the second derivative of the electron current in order to identify the energy distribution functions. Also, we are going to measure the energy distribution function of a particle, which we call "Zhariphium", and determine its mean life-time.

### Quantum Mechanical Unstable State

Let us denote the quantum mechanical wave function of the unstable state by  $\psi_0$ , and the states into which it can decay by  $\psi_1, \psi_2, \psi_3, \dots, \psi_n, \dots$ .

The time-dependent Schrödinger wave equation is given by

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi . \quad (1)$$

The Hamiltonian of the system  $\hat{H}$  can be written as

$$\hat{H} = \hat{H}_n + \hat{H}(\vec{r}), \quad (2)$$

where  $\hat{H}_n$  is the Hamiltonian of the system which has an exact energy eigenstate:

$$\hat{H}_n \psi_n(\vec{r}) = E_n \psi_n(\vec{r}). \quad (3)$$

We have to note that  $\psi_0$  is not an exact energy eigenstate: if it were, it would not decay. Thus the state  $\psi(\vec{r}, t)$  of the system, which is  $\psi_0$  at  $t = 0$ , develops admixture of the final states. We can express  $\psi(\vec{r}, t)$  as the super position of the states  $\psi_n(\vec{r})$  and write

$$\psi(\vec{r}, t) = \sum_{n=0}^{\infty} a_n(t) \psi_n(\vec{r}) \exp\left(-\frac{iE_n t}{\hbar}\right). \quad (4)$$

If all the states were exact eigenstates of  $\hat{H}$ , the coefficients  $a_n(t)$  are time independent [3].

Substituting from Eq.(4) into Eq.(1), and applying Eq.(3), then after manipulations, we will have

$$i\hbar \sum_{n=0}^{\infty} \dot{a}_n(t) \psi_n(\vec{r}) \exp\left(-\frac{iE_n t}{\hbar}\right) = \sum_{n=0}^{\infty} a_n(t) \hat{H}(\vec{r}) \psi_n(\vec{r}) \exp\left(-\frac{iE_n t}{\hbar}\right). \quad (5)$$

Multiply the above equation from the left by  $\Psi_k^*(\vec{r}) d\tau$ , integrate and use the orthogonality condition

$$\int \Psi_k^*(\vec{r}) \psi_n(\vec{r}) d\tau = \delta_{kn}$$

the above equation reduces to

$$\dot{a}_k(t) = \frac{1}{i\hbar} \sum_{n=0}^{\infty} a_n(t) \exp\left(-\frac{i(E_n - E_k)t}{\hbar}\right) H_{kn}, \quad (6)$$

where the matrix elements are defined by

$$H_{kn} = \int \Psi_k^*(\vec{r}) \hat{H}(\vec{r}) \psi_n(\vec{r}) d\tau.$$

The matrix elements  $H_{kn}$  are in general non vanishing for  $k \neq n$ .

So far the equations given in Eq.(6) are exact. The initial conditions at  $t = 0$  are  $a_0(0) = 1$ ,  $a_n(0) = 0$  for  $n \geq 1$ .

If we set  $n = 0$ , then for  $k \geq 1$ , Eq.(6), reduces to

$$\dot{a}_k(t) = \frac{1}{i\hbar} a_0(t) \exp\left(-\frac{i(E_0 - E_k)t}{\hbar}\right). \quad (7)$$

The state  $\psi_0$  is unstable state. Let us assume the following postulate

$$a_0(t) = \exp\left(-\frac{\Gamma}{2\hbar} t\right), \quad (8)$$

so that

$$\|a_0(t)\|^2 = \exp\left(-\frac{t}{\tau}\right), \quad (9)$$

where

$$\tau = \frac{\hbar}{\Gamma}. \quad (10)$$

The insertion of Eq.(8) into Eq.(7) produces

$$\dot{a}_k(t) = \frac{1}{i\hbar} H_{k0} \exp\left(-\frac{i}{\hbar}\left[E_0 - E_k - \frac{i\Gamma}{2}\right]t\right). \quad (11)$$

The integration of Eq.(11) gives

$$\begin{aligned} a_k(t) &= \frac{1}{i\hbar} H_{k0} \int_0^t \exp\left(-\frac{i}{\hbar}\left[E_0 - E_k - \frac{i\Gamma}{2}\right]t'\right) dt' \\ &= H_{k0} \frac{\exp\left(-\frac{i}{\hbar}\left[E_0 - E_k - \frac{i\Gamma}{2}\right]t\right) - 1}{E_0 - E_k - i\Gamma/2} \end{aligned}$$

For times  $t \gg \hbar/\Gamma$ , the exponent term  $\exp(-\Gamma t/2\hbar) \rightarrow 0$ , vanishes and for such times, we have

$$a_k(t) = \frac{H_{k0}}{E_k - E_0 + i\Gamma/2}. \quad (12)$$

Thus the probability that the state  $\psi_0$  will decay to the state  $\psi_n$  is

$$\|a_k(t)\|^2 = \frac{2\pi}{\Gamma} \|H_{k0}\|^2 P(E_k - E_0), \quad (13)$$

where

$$P(E_k - E_0) = \frac{\Gamma}{2\pi} \frac{1}{(E_k - E_0)^2 + \Gamma^2/4}. \quad (14)$$

The function  $P(E_k - E_0)$  is a probability distribution in energy for the unstable state  $\psi_0$  [4]. The factor  $\Gamma/2\pi$  has been inserted so that

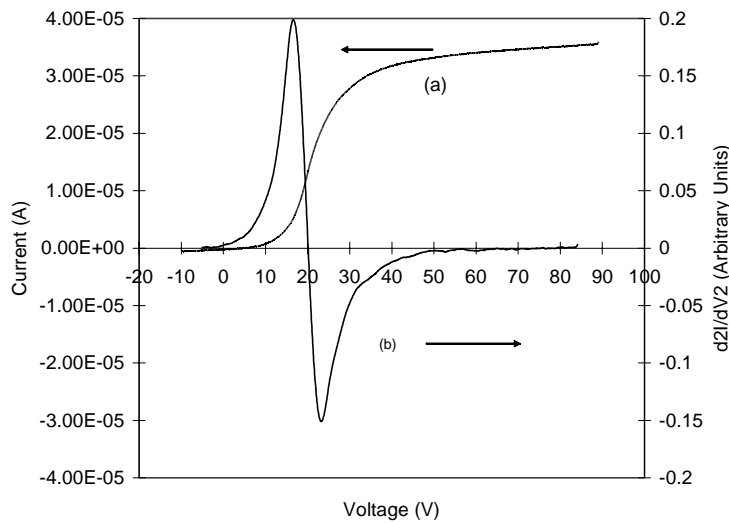
$$\int_{-\infty}^{\infty} P(E_k - E_0) dE = 1. \quad (15)$$

This function has a Lorentzian profile [5], which is symmetric about the peak energy  $E_0$ . This distribution function has a narrow peak. Thus the energy of the final state  $E_k$  is not identically equal to  $E_0$ . We have to note that  $\Gamma$  in Eq.(14) is known as the full width at half maximum of the energy distribution.

## Experimental Results

The apparatus where these measurements are taken is described in a paper written by the same author of this paper [1].

As shown in Figure 1, the plasma potential is at 20 V. At the plasma potential there is negative electrons and positive ions interacting according to Coulomb law. The second derivative of the probe current as shown in Figure 1 shows a curve on the upper half of the voltage axis between -10 V and 20 V. This curve is called an energy distribution function [6]. This energy distribution function is a mixture of electron energy distribution function and an ion energy distribution function.

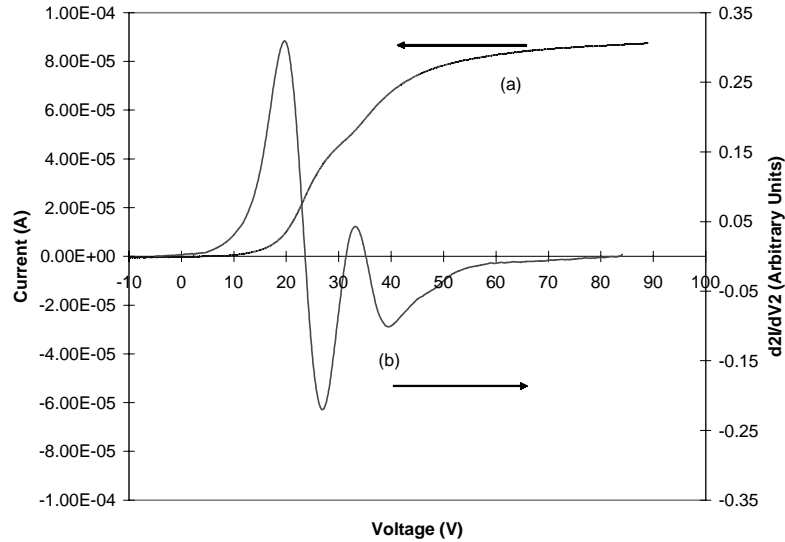


**Figure 1:** (a) The probe current, the grid was fixed at 20 V. (b) The second derivative. There is only one energy distribution on the upper half of the voltage axis between -10 V and 20 V.

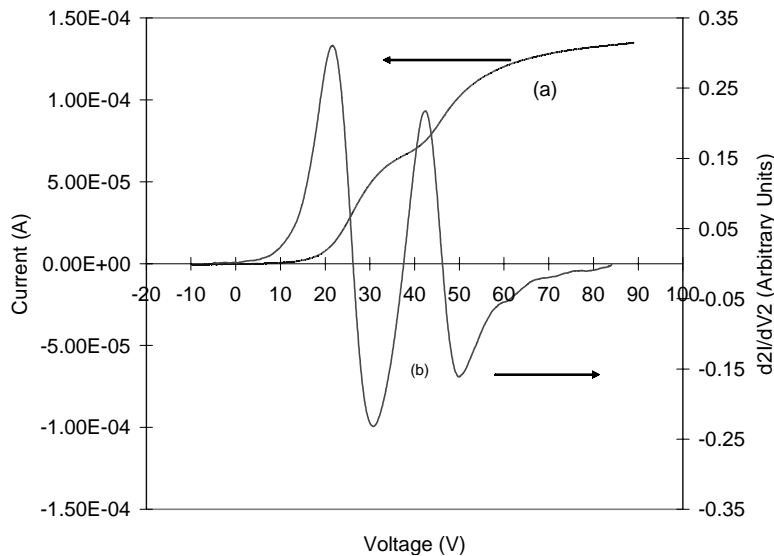
To obtain electron energy distribution function only we must bias the grid by positive voltage higher than the plasma potential in order to reject positive ions.

Therefore, by biasing the grid at  $V_g = 30$  V which is higher than the plasma potential at 20 V, we find that in the second derivative there are two energy distribution functions on the upper half of the voltage axis, Figure 2. One energy distribution function is the electron energy distribution function. The other is an energy distribution function of a particle which is neither an electron nor a positive ion because we have already rejected positive ions by positive grid. The distribution between -10 V and 23.6 V is the electron energy distribution function. This electron energy distribution function is exponentially increasing from -10 V to 20 V. The shape of this distribution has a Maxwellian profile. In this paper we are not going to study the electron energy distribution function. The other energy distribution function is symmetric between 31.5 V and 35.1 V and has a peak at 33 V. The shape of this distribution has a Lorentzian profile as given by Eq.(14). The full width at half maximum of this energy distribution is  $\Gamma = 3$  V. That is, the mean life of this particle,

according to Eq.(10), is given by  $\tau = \hbar/\Gamma = 2.2 \times 10^{-16}$  second.

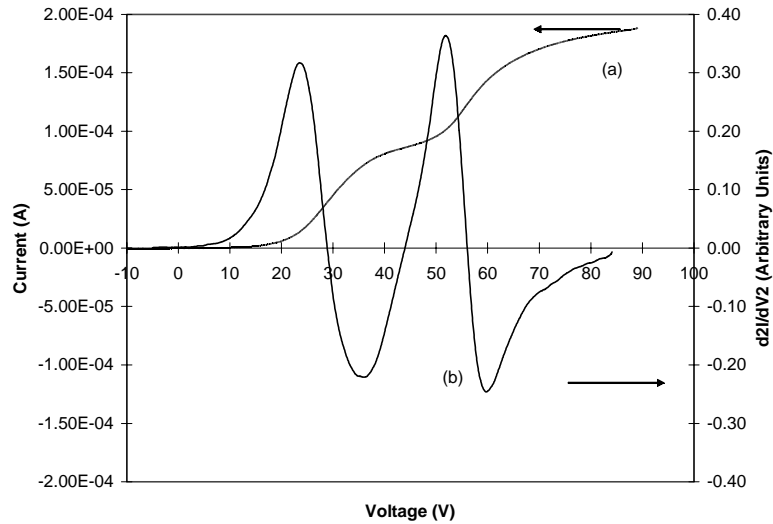


**Figure 2:** (a) The probe current, the grid was fixed at 30 V. (b) The second derivative. There are two energy distribution functions. One of them is extended between -10 V and 23.6 V. This distribution has a Maxwellian profile. The other energy distribution is symmetric and extended between 31.5 V and 35.1 V. This energy distribution function has a Lorentzian profile.



**Figure 3:** (a) The probe current, the grid was fixed at  $V_g = 40$  V. (b) The second derivative. There are two energy distribution functions. One of them is extended between -10 V and 26.2 V. This distribution has a Maxwellian profile. The other energy distribution is extended between 37.6 V and 45.6 V. This energy distribution function has a Lorentzian profile.

Figure 3 shows the grid bias voltage at  $V_g = 40$  V. On the upper half of the voltage axis there are two energy distributions. One of them extended between -10 V and 26.2 V. This is the electron energy distribution function. The other energy distribution has a Lorentzian profile extended between 37.6 V and 45.6 V and has a peak energy at 42 V. The full width at half maximum of this Lorentzian profile is  $\Gamma = 6.2$  V. That is, the mean life of this particle is  $\tau = 1.1 \times 10^{-16}$  second.



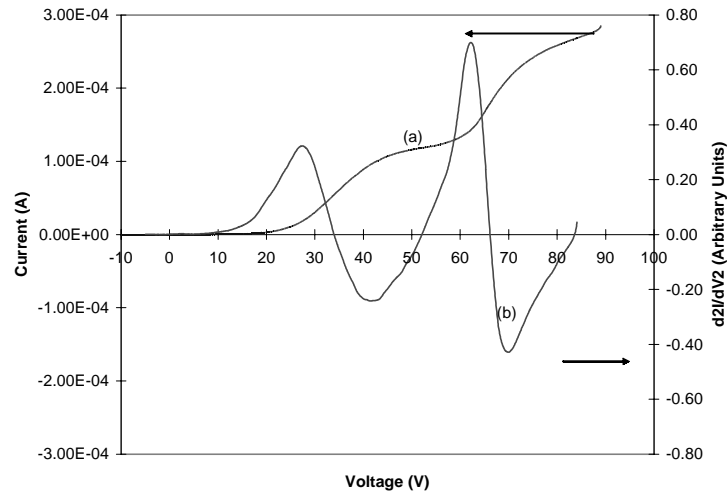
**Figure 4:** (a) The probe current, the grid was fixed at  $V_g = 50$  V. (b) The second derivative. There are two energy distribution functions on the upper half of the voltage axis. One of them is extended between -10 V and 28.8 V. This energy distribution function has a Maxwellian profile. The other energy distribution is extended between 43.9 V and 55.9 V. This energy distribution has a Lorentzian profile.

Increasing the grid bias voltage to 50 V we find that there are two distributions as shown in Figure 4. One of them is extended between -10 V and 28.8 V. This distribution is the electron energy distribution function. The other energy distribution is extended between 43.9 V and 55.9 V and has a peak at 52 V. This has a Lorentzian profile with full width at half maximum of  $\Gamma = 6.7$  V and a mean life of  $\tau = 9.9 \times 10^{-17}$  second.

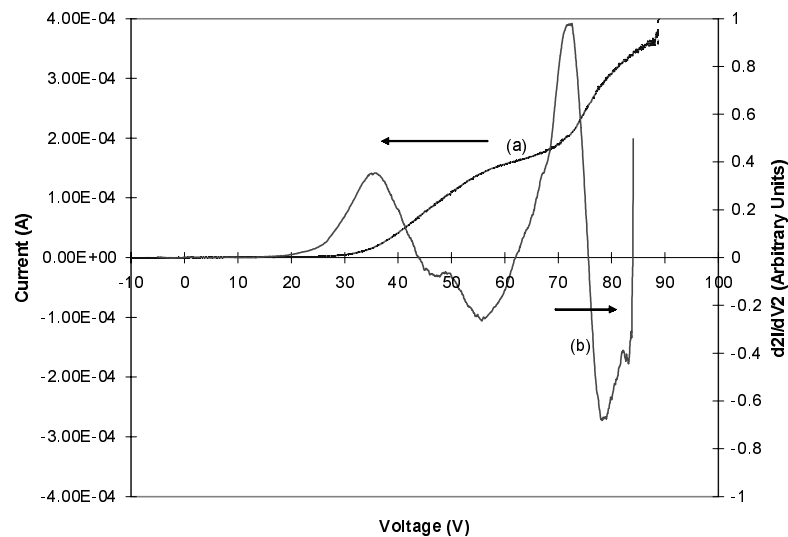
Also, if the grid bias was at 60 V, then Figure 5 shows that in the second derivative there are two energy distribution functions. One of them has a Maxwellian profile extended between -10 V and 33.8 V. The other distribution has a Lorentzian profile and extended between 43.9 V and 55.8 V and has a peak at 62 V. The full width at half maximum of this profile is  $\Gamma = 6.3$  V and a mean life of  $\tau = 1.1 \times 10^{-16}$  second.

Finally, when the grid is at 70 V, it is found that there are two energy distributions one of them extended between -10 V and 43.7 V. This energy distribution has a Maxwellian profile. The other distribution is symmetric and extended between 62.1 V

and 75.5 V and has a peak at 72 V. This distribution has a Lorentzian profile with full width at half maximum at  $\Gamma = 6.1$  V and a mean life of  $\tau = 1.1 \times 10^{-16}$  second.



**Figure 5:** (a) The probe current, the grid was fixed at  $V_g = 60$  V. (b) The second derivative. There are two energy distribution functions. One of them is extended between -10 V and 33.8 V. This energy distribution function has a Maxwellian profile. The other energy distribution is extended between 52.2 V and 65.9 V. This energy distribution has a Lorentzian profile.



**Figure 6:** (a) The probe current, the grid was fixed at  $V_g = 70$  V. (b) The second derivative. There are two energy distribution functions on the upper half of the voltage axis. One of them is extended between -10 V and 43.7 V. This energy distribution function has a Maxwellian profile. The other energy distribution is extended between 43.9 V and 55.9 V. This energy distribution has a Lorentzian profile.

## Discussions

In order to obtain an electron energy distribution function to study thermal electron properties such as electron temperature, electron density and the anisotropy of the plasma, we need to separate electrons from ions. The best way to separate electrons from ions is the electrical separation. Therefore, a gridded probe, with one grid only, was biased with positive voltage to reject ions inserted inside the plasma and an electron current was obtained. Figures 2-6 show there are two energy distribution functions on the second derivative of the electron current. One energy distribution function is exponentially increasing. This is an indication that this energy distribution function has a Maxwellian profile. Because this is at the plasma potential this must be the electron energy distribution function. The aim of this paper is not to study the electron energy distribution function. The other distribution is located at the grid bias voltage or the post sheath potential. This distribution has a peak at  $E_0$  and the curve is symmetric about this peak. This energy distribution function can be represented by a Lorentzian profile which is characterized by its full width at half maximum  $\Gamma$ . The mean life is obtained from Eq.(10). Since, we have a Lorentzian profile and mean life then this energy distribution function represents a particle. This particle is not an electron because all electrons are in the energy distribution function that has a Maxwellian profile. Also, this particle is not positive ion because all positive ions are rejected by positive grid bias voltage. Finally, this particle is not secondary electrons emitted from the driven electrode, because secondary electrons must have the sheath energy [7]. This leads us to the conclusion that this is a new particle and we must identify this particle, by the following definition.

**Definition:** *The particle with an energy distribution function that has a Lorentzian profile and characterized by a mean life in the order of  $(1.28 \pm 0.23) \times 10^{-16}$  second is called Zhariphium. The meaning of Zharif is very beautiful in Arabic.*

## Conclusion

A gridded probe was used to study the electron current and electron energy distribution function, but a new particle called Zhariphium has been discovered by studying the Zhariphium energy distribution function. We cannot observe the Zhariphium energy distribution function by the cylindrical or planar probe. This proves that the gridded probe is a very powerful tool in characterizing the plasma and even in finding one of its missing parameters. Also, a quantum mechanical unstable state is proved to be a very powerful method in studying the Zhariphium energy distribution function and the measurement of Zhariphium mean life time.

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