

## Q(A) - Balance Super Edge Magic Graphs Results

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### Abstract

Let  $G$  be  $(p,q)$ -graph in which the edges are labeled  $1,2,3,4,\dots,q$  so that the vertex sums are constant, mod  $p$ , then  $G$  is called an edge magic graph. Magic labeling on  $Q(a)$  balance super edge-magic graphs introduced [5]. In this paper extend my discussion of  $Q(a)$ - balance super edge magic labeling(BSEM) to few types of special graphs. **AMS 2010:** 05C

**Keywords:** edge magic graph, super edge magic graph,  $Q(a)$  balance edge-magic graph.

### 1.INTRODUCTION

Magic graphs are related to the well-known magic squares, but are not quite as famous. Magic squares are an arrangement of numbers in a square in such a way that the sum of the rows, columns and diagonals is always the same.

All graphs in this paper are connected, (multi-)graphs without loop. The graph  $G$  has vertex – set  $V(G)$  and edge – set  $E(G)$ . A labeling (or valuation) of a graph is a map that carries graph elements to numbers(usually to the positive or non- negative integers). Edge magic graph introduced by Sin Min Lee, Eric Seah and S.K Tan in 1992. Various author discussed in edge magic graphs like Edge magic  $(p,3p-1)$ -graphs, Zykov sums of graphs, cubic multigraphs, Edge-magicness of the composition of a cycle with a null graph. In 2007 Sin-Min Lee and Thomas Wong and Sheng-Ping Bill Lo introduced two types of magic labeling on  $Q(a)$ - Balance Super Edge –magic Graphs of complete bipartite and proved several conjectures [5]. In this paper  $Q(a)$ - BSEM of Cubical graph, Haar graph, Fan graph, Book graph, Complete bipartite graph, Complete tripartite graph, Taylor graph, Cycle graph, Multi graph, Cubical Prism graph, Utility graph and Helm graph.

## 2. PRELIMINARIES:

**Definition 1.1** A graph  $G$  is a  $(p, q)$ -graph in which the edges are labeled by  $1, 2, 3, \dots, q$ . So that the vertex sum are constant, mod  $P$ , then  $G$  is called an **Edge Magic Graph** (for simplicity we denote EM). The concepts of edge-magic graphs are introduced by Lee, Seah and Tan. It is obvious that Edge Magic Graph is not necessary super magic. A necessary condition for a  $(p, q)$  –graph to be edge magic is  $q(q + 1) \equiv 0 \pmod{p}$ . However, there are infinitely many connected graphs such as trees cycle that satisfy the necessary condition but not edge magic.

### Definition 1.2

For  $a \geq 1$  we denote,

$$Q(a) = \begin{cases} \{\pm a, \dots, \pm(a - 1 + (\frac{q}{2}))\}, & \text{if } q \text{ is even} \\ \{0, \pm a, \dots, \pm(a - 1 + (\frac{q-1}{2}))\}, & \text{if } q \text{ is odd} \end{cases}$$

A  $(p, q)$ - graph  $G$  in which the edges are labeled by  $Q(a)$  so that the vertex sum is a constant, is called  $Q(a)$  balance super edge-magic.

**Cubical graph:** The Cubical graph is the platonic graph corresponding to the connectivity of the cube. It has 12 distinct (directed) Hamiltonian cycles. The Cubical graph has 8 nodes, 12 edges, vertex connectivity 3, edge connectivity 3, graph diameter 3, graph radius 3 and girth 4.

**Haar graph:** A Haar graph  $H(n)$  is a bipartite regular vertex-transitive graph indexed by a positive integer and obtained by a simple binary encoding of cyclically adjacent vertices. Haar graphs may be connected or disconnected. There are  $2^{k-1}$  Haar graphs on  $2k$  vertices, so the vertex count of  $H(n)$  is

$$|H(n)| = 2(1 + \lfloor \log_2 n \rfloor) = 2[1 + \log_2 n].$$

**Fan graph:** A Fan graph  $F_{m,n}$  is defined as the graph join  $\overline{k_m} + p_n$ , where  $\overline{k_m}$  is the empty graph on  $m$  nodes and  $p_n$  is the path graph on  $n$  nodes. The case  $m = 1$  corresponds to the usual fan graphs while  $m = 2$  corresponds to the double fan graphs.

**Book graph:** The  $m$  –book graph is defined as the graph cartesian product  $s_{m+1} + p_2$  where  $s_m$  is a star graph and  $p_2$  is the path graph on two nodes. The generalization of the book graph to  $n$  –stacked pages is the  $(m, n)$  –stacked book graph.

**Complete bipartite graph:** A complete bipartite graph is a bipartite graph ( i.e., a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent) such that every pair of graph vertices in the two sets are adjacent.

If there are  $p$  and  $q$  graph vertices in the two sets, the complete bipartite graph (sometimes also called a complete biography) is denoted  $k(p, q)$ .

**Complete tripartite graph:** A complete tripartite graph is the  $k = 3$  case of a complete  $k$  – partite graph. In other words, it is a tripartite graph (i.e., a set of graph vertices decomposed into three disjoint sets such that no two graph vertices within the same set are adjacent) such that every vertex of each set graph vertices is adjacent to every vertex in the other two sets.

If there are  $p, q$  and  $r$  graph vertices in the three sets, the complete tripartite graph (sometimes also called a complete trigraph) is denoted  $k_{p,q,r}$ .

**Taylor graph:** A Taylor graph is a distance-regular graph with an intersection array  $\{k, \mu, 1; 1, \mu, k\}$ . A Taylor graph with these parameters has  $2(k + 1)$  vertices.

**Cycle graph:** A cycle graph  $c_n$ , sometimes simply known as an  $n$ -cycle is a graph on  $n$  nodes containing a single cycle through all nodes.

A cycle graph of a group is a graph which shows cycle of a group as well as the connectivity between the cycles.

**Multi graph:** The term multigraph refers to a graph in which multiple edges between nodes are either permitted.

Multigraph possesses no graph loops and the multigraph to mean a graph containing either loops or multiple edges.

**Utility Graph:** The Utility graph is the graph also known as the Thomsen graph and in the more formal parlance of graph theory is known as the complete bipartite graph and is also equivalent to the circulant graph.

### 3. MAIN RESULTS

#### Theorem 3.1

If the Cubical graph is Strong  $Q(a)$  -balance super edge -magic for all  $a \geq 1$ .

#### Proof:

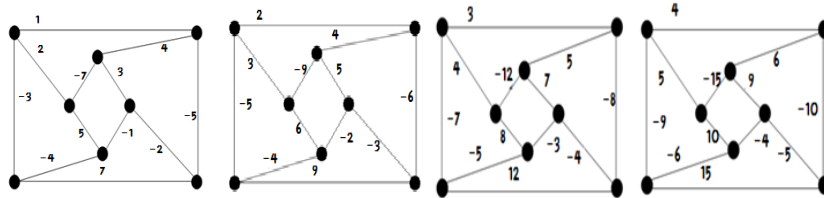
If  $n = 8, q = 12$ . Cubical graph is Strong  $Q(a)$  -Balance Super Edge Magic for  $1, 2, 3, 4, 5, 6, 7, 8$ .

For  $a \geq 1$  we denote

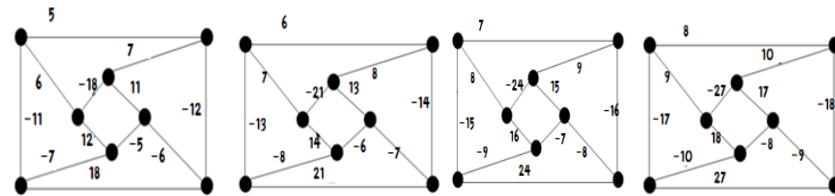
$$Q(a) = \begin{cases} \{\pm a, \dots, \pm(a - 1 + \left(\frac{q}{2}\right))\}, & \text{if } q \text{ is even} \\ \{0, \pm a, \dots, \pm(a - 1 + \left(\frac{q - 1}{2}\right))\}, & \text{if } q \text{ is odd} \end{cases}$$

Here  $q$  is even following types 1 to 6 graphs shows that  $Q(a)$ -Balance Super Edge Magic.

**Type 1:**

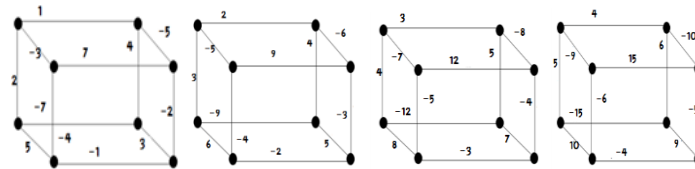


$Q(1) - BSEM$        $Q(2) - BSEM$        $Q(3) - BSEM$        $Q(4) - BSEM$

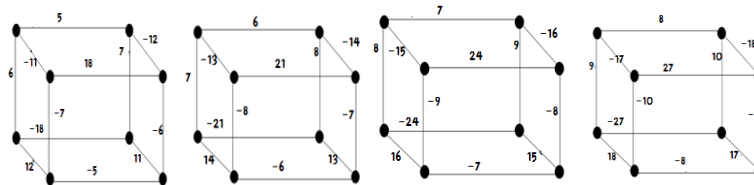


$Q(5) - BSEM$        $Q(6) - BSEM$        $Q(7) - BSEM$        $Q(8) - BSEM$

**Type 2:**

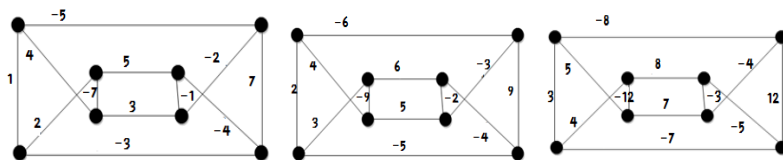


$Q(1) - BSEM$        $Q(2) - BSEM$        $Q(3) - BSEM$        $Q(4) - BSEM$



$Q(5) - BSEM$        $Q(6) - BSEM$        $Q(7) - BSEM$        $Q(8) - BSEM$

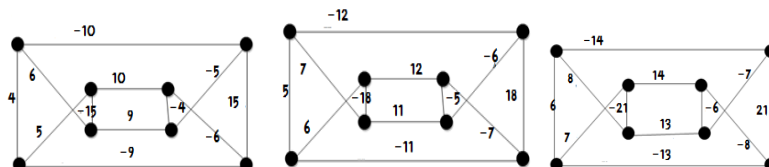
**Type 3:**



$Q(1) - BSEM$

$Q(2) - BSEM$

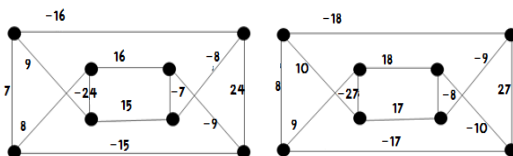
$Q(3) - BSEM$



$Q(4) - BSEM$

$Q(5) - BSEM$

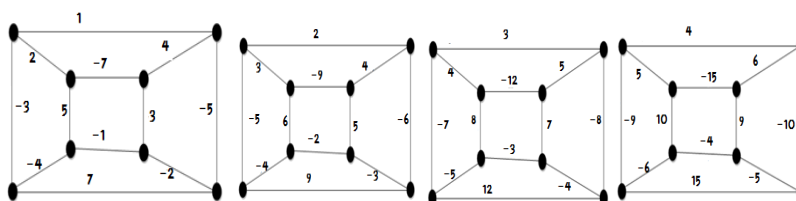
$Q(6) - BSEM$



$Q(7) - BSEM$

$Q(8) - BSEM$

**Type 4:**

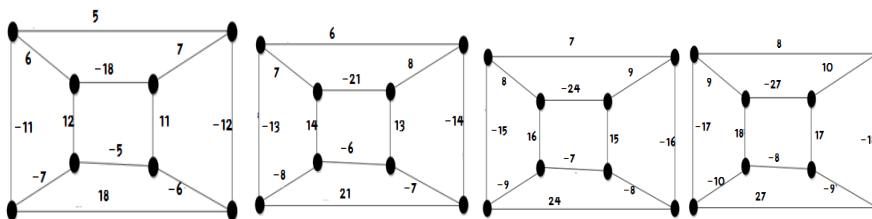


$Q(1) - BSEM$

$Q(2) - BSEM$

$Q(3) - BSEM$

$Q(4) - BSEM$



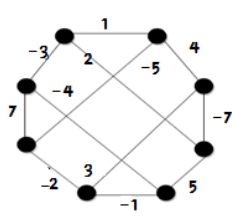
$Q(5) - BSEM$

$Q(6) - BSEM$

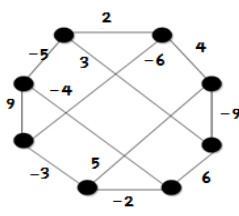
$Q(7) - BSEM$

$Q(8) - BSEM$

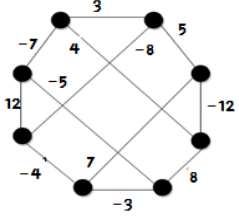
**Type 5:**



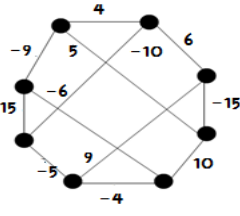
$Q(1) - BSEM$



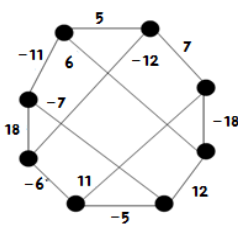
$Q(2) - BSEM$



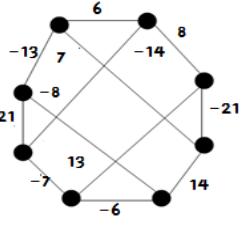
$Q(3) - BSEM$



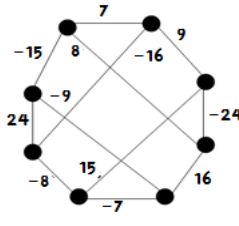
$Q(4) - BSEM$



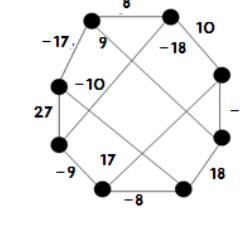
$Q(5) - BSEM$



$Q(6) - BSEM$

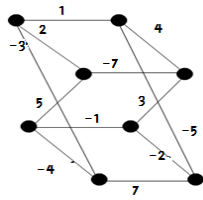


$Q(7) - BSEM$

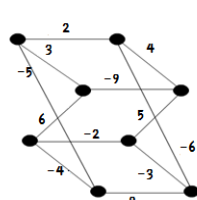


$Q(8) - BSEM$

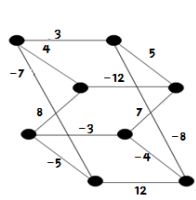
**Type 6:**



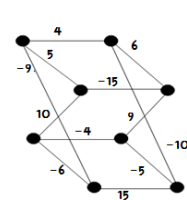
$Q(1) - BSEM$



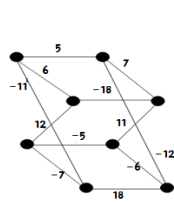
$Q(2) - BSEM$



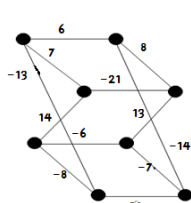
$Q(3) - BSEM$



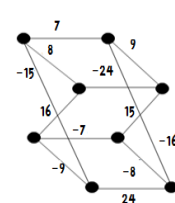
$Q(4) - BSEM$



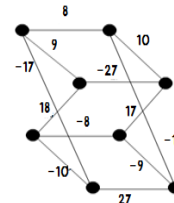
$Q(5) - BSEM$



$Q(6) - BSEM$



$Q(7) - BSEM$



$Q(8) - BSEM$

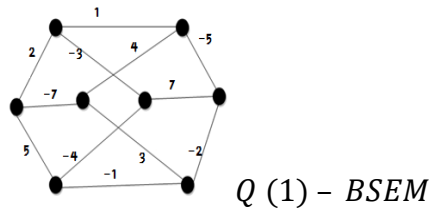
**Theorem 3.2**

If the Haar graph is Strong  $Q(a)$ -balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $n = 8, q = 12$ . Haar graph is Strong  $Q(a)$ -balance super edge-magic for 1,2,3,4,5,6,7,8.

Here  $q$  is even following graph shows that  $Q(a)$ -balance super edge-magic.



- $Q(1) - BSEM\{1, -3, 2, -7, 5, -4, -1, 3, -2, 7, -5, 4\}$
- $Q(2) - BSEM\{2, -5, 3, -9, 6, -4, -2, 5, -3, 9, -6, 4\}$
- $Q(3) - BSEM\{3, -7, 4, -12, 8, -5, -3, 7, -4, 12, -8, 5\}$
- $Q(4) - BSEM\{4, -9, 5, 6 - 15, 10, -6, -4, 9, -5, 15, -10, 6\}$
- $Q(5) - BSEM\{5, -11, 6, -18, 12, -7, -5, 11, -6, 18, -12, 7\}$
- $Q(6) - BSEM\{6, -13, 7, -21, 14, -8, -6, 13, -7, 21, -14, 8\}$
- $Q(7) - BSEM\{7, -15, 8, -24, 16, -9, -7, 15, -8, 24, -16, 9\}$
- $Q(8) - BSEM\{8, -17, 9, -27, 18, -10, -8, 17, -9, 27, -18, 10\}$

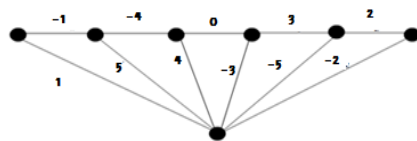
**Theorem 3.3**

If the Fan graph  $F_6$  is Strong  $Q(a)$ - balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $p = 7, q = 11$ .  $F_6$  is Strong  $Q(a)$  balance super edge-magic for 1,2,3,4,5,6,7.

Here  $q$  is odd following graph shows that  $Q(a)$  balance super edge-magic.



$Q(1) - BSEM$

- $Q(1) - BSEM\{1, -1, 5, -4, 4, 0, -3, 3, -5, 2, -2\}$
- $Q(2) - BSEM\{2, -2, 7, -5, 5, 0, -4, 4, -7, 3, -3\}$
- $Q(3) - BSEM\{3, -3, 9, -6, 6, 0, -5, 5, -9, 4, -4\}$
- $Q(4) - BSEM\{4, -4, 11, -7, 7, 0, -6, 6, -11, 5, -5\}$
- $Q(5) - BSEM\{5, -5, 13, -8, 8, 0, -7, 7, -13, 6, -6\}$
- $Q(6) - BSEM\{6, -6, 15, -9, 9, 0, -8, 8, -15, 7, -7\}$

$$Q(7) - BSEM\{7, -7, 17, -10, 10, 0, -9, 9, -17, 8, -8\}$$

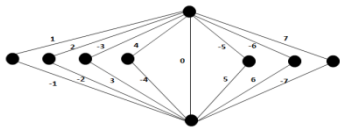
**Theorem 3.4**

If the Book graph  $Book(3,7)$  is strong  $Q(a)$  balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $p = 9, q = 15$ . Book graph  $Book(3,7)$  is Strong  $Q(a)$ - balance super edge-magic for  $1,2,3,4,5,6,7,8,9$ .

Here  $q$  is odd following graph shows that  $Q(a)$ - balance super edge-magic.



$$Q(1) - BSEM$$

$$Q(1) - BSEM\{1, -1, 2, -2, -3, 3, 4, -4, 0, -5, 5, -6, 6, 7, -7\}$$

$$Q(2) - BSEM\{2, -2, 3, -3, -5, 5, 6, -6, 0, -7, 7, -8, 8, 9, -9\}$$

$$Q(3) - BSEM\{3, -3, 4, -4, -7, 7, 8, -8, 0, -9, 9, -10, 10, 11, -11\}$$

$$Q(4) - BSEM\{4, -4, 5, -5, -9, 9, 10, -10, 0, -11, 11, -12, 12, 13, -13\}$$

$$Q(5) - BSEM\{5, -5, 6, -6, -11, 11, 12, -12, 0, -13, 13, -14, 14, 15, -15\}$$

$$Q(6) - BSEM\{6, -6, 7, -7, -13, 13, 14, -14, 0, -15, 15, -16, 16, 17, -17\}$$

$$Q(7) - BSEM\{7, -7, 8, -8, -15, 15, 16, -16, 0, -17, 17, -18, 18, 19, -19\}$$

$$Q(8) - BSEM\{8, -8, 9, -9, -17, 17, 18, -18, 0, -19, 19, -20, 20, 21, -21\}$$

$$Q(9) - BSEM\{9, -9, 10, -10, -19, 19, 20, -20, 0, -21, 21, -22, 22, 23, -23\}$$

**Theorem 3.5**

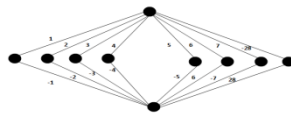
If the Complete bipartite graph  $k(2,8)$  is Strong  $Q(a)$  balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $p = 10, q = 16$ . Complete bipartite graph  $k(2,8)$  is Strong  $Q(a)$ - balance super edge-magic for  $1,2,3,4,5,6,7,8,9,10$ .

Here  $q$  is even following graph shows that  $Q(a)$ - balance super edge-magic.





$Q(1) - BSEM$

$Q(1)BSEM\{1, -1, 2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7, -28, 28\}$

$Q(2)BSEM\{2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8, -35, 35\}$

$Q(3)BSEM\{3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9, -42, 42\}$

$Q(4)BSEM\{4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9, 10, -10, -49, 49\}$

$Q(5)BSEM\{5, -5, 6, -6, 7, -7, 8, -8, 9, -9, 10, -10, 11, -11, -56, 56\}$

$Q(6)BSEM\{6, -6, 7, -7, 8, -8, 9, -9, 10, -10, 11, -11, 12, -12, -63, 63\}$

$Q(7)BSEM\{7, -7, 8, -8, 9, -9, 10, -10, 11, -11, 12, -12, 13, -13, -70, 70\}$

$Q(8)BSEM\{8, -8, 9, -9, 10, -10, 11, -11, 12, -12, 13, -13, 14, -14, -77, 77\}$

$Q(9)BSEM\{9, -9, 10, -10, 11, -11, 12, -12, 13, -13, 14, -14, 15, -15, -84, 84\}$

$Q(10)BSEM\{10, -10, 11, -11, 12, -12, 13, -13, 14, -14, 15, -15, 15, -15, -84, 84\}$

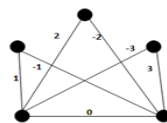
**Theorem 3.6**

If the Complete tripartite graph  $k_{1,1,n}, n \geq 3$  is Strong  $Q(a)$ - balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $n = 3$   $p = 5, q = 7$ . Complete tripartite graph  $k_{1,1,3}$  is strong  $Q(a)$  balance super edge-magic for 1,2,3,4,5.

Here  $q$  is odd following graph shows that  $Q(a)$  balance super edge-magic.



$Q(1) - BSEM$

$Q(1) - BSEM\{1, -1, 2, -2, -3, 3, 0\}$

$Q(2) - BSEM\{ 2, -2, -3, 3, -5, 5, 0\}$

$Q(3) - BSEM\{3, -3, 4, -4, -7, 7, 0\}$

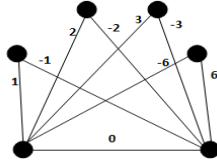
$Q(4) - BSEM\{4, -4, 5, -5, -9, 9, 0\}$

$Q(5) - BSEM\{5, -5, 6, -6, -11, 11, 0\}$

If  $n = 4$   $p = 6, q = 9$ .

Complete tripartite graph  $k_{1,1,4}$  is strong  $Q(a)$ - balance super edge-magic for 1,2,3,4,5,6.

Here  $q$  is odd following graph shows that  $Q(a)$ - balance super edge-magic.



$Q(1) - BSEM$

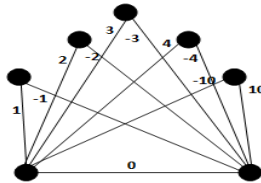
$Q(1) - BSEM\{1, -1, 2, -2, 3, -3, -6, 6, 0\}$        $Q(2) - BSEM\{2, -2, 3, -3, 4, -4, -9, 9, 0\}$

$Q(3) - BSEM\{3, -3, 4, -4, 5, -5, -12, 12, 0\}$        $Q(4) - BSEM\{4, -4, 5, -5, 6, -6, -15, 15, 0\}$

$Q(5) - BSEM\{5, -5, 6, -6, 7, -7, -18, 18, 0\}$        $Q(6) - BSEM\{6, -6, 7, -7, 8, -8, -21, 21, 0\}$

If  $n = 5$   $p = 7, q = 11$ . Complete tripartite graph  $k_{1,1,5}$  is Strong  $Q(a)$ - balance super edge-magic for 1,2,3,4,5,6,7.

Here  $q$  is odd following graph shows that  $Q(a)$ - balance super edge-magic.



$Q(1) - BSEM$

$Q(1) - BSEM\{1, -1, 2, -2, 3, -3, 4, -4, -10, 10, 0\}$        $Q(2) - BSEM\{2, -2, 3, -3, 4, -4, 5, -5, -14, 14, 0\}$

$Q(3) - BSEM\{3, -3, 4, -4, 5, -5, 6, -6, -18, 18, 0\}$        $Q(4) - BSEM\{4, -4, 5, -5, 6, -6, 7, -7, -22, 22, 0\}$

$Q(5) - BSEM\{5, -5, 6, -6, 7, -7, 8, -8, -26, 26, 0\}$        $Q(6) - BSEM\{6, -6, 7, -7, 8, -8, 9, -9, -30, 30, 0\}$

$Q(7) - BSEM\{7, -7, 8, -8, 9, -9, 10, -10, -34, 34, 0\}$

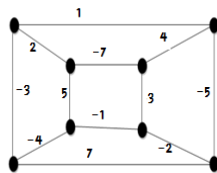
**Theorem 3.7**

If the Taylor graph is Strong Q (a) - balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $n = 8, q = 12$ . Taylor graph is Strong Q (a) - balance super edge-magic for 1,2,3,4,5,6,7,8.

Here  $q$  is even following graph shows that  $Q(a)$ - balance super edge-magic.



$Q(1) - BSEM$

- $Q(1) - BSEM\{1, 2, -3, -4, 7, -2, -5, 4, 3, -7, 5, -1\}$
- $Q(2) - BSEM\{2, 3, -5, -4, 9, -3, -6, 4, 5, -9, 6, -2\}$
- $Q(3) - BSEM\{3, 4, -7, -5, 12, -4, -8, 5, 7, -12, 8, -3\}$
- $Q(4) - BSEM\{4, 5, -9, -6, 15, -5, -10, 6, 9, -15, 10, -4\}$
- $Q(5) - BSEM\{5, 6, -11, -7, 18, -6, -12, 7, 11, -18, 12, -5\}$
- $Q(6) - BSEM\{6, 7, -13, -8, 21, -7, -14, 8, 13, -21, 14, -6\}$
- $Q(7) - BSEM\{7, 8, -15, -9, 24, -8, -16, 9, 15, -24, 16, -7\}$
- $Q(8) - BSEM\{8, 9, -17, -10, 27, -9, -18, 10, 17, -27, 18, -8\}$

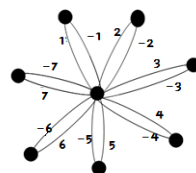
**Theorem 3.8**

If the Cycle graph is Strong Q(a)- balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $p = 8, q = 14$ . Cycle graph  $c_2 \times c_2 \times c_2$  is strong Q (a) balance super edge-magic for 1,2,3,4,5,6,7,8.

Here  $q$  is even following graph shows that Q (a) -Balance Super Edge Magic.



$Q(1) - BSEM$

- $Q(1) - BSEM\{1, -1, 2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7\}$

$$Q(2) - BSEM\{2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8\}$$

$$Q(3) - BSEM\{3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9\}$$

$$Q(4) - BSEM\{4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9, 10, -10\}$$

$$Q(5) - BSEM\{5, -5, 6, -6, 7, -7, 8, -8, 9, -9, 10, -10, 11, -11\}$$

$$Q(6) - BSEM\{6, -6, 7, -7, 8, -8, 9, -9, 10, -10, 11, -11, 12, -12\}$$

$$Q(7) - BSEM\{7, -7, 8, -8, 9, -9, 10, -10, 11, -11, 12, -12, 13, -13\}$$

$$Q(8) - BSEM\{8, -8, 9, -9, 10, -10, 11, -11, 12, -12, 13, -13, 14, -14\}$$

**Theorem 3.9**

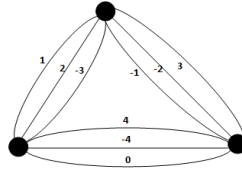
If the Multi graph is Strong  $Q(a)$ - balance super edge-magic for all  $a \geq 1$ .

**Proof:**

If  $p = 3, q = 9$ .

So, Multi graph is strong  $Q(a)$ - balance super edge-magic for 1,2,3.

Here  $q$  is odd following graph shows that  $Q(a)$ - balance super edge-magic.



$Q(1) - BSEM$

$$Q(1) - BSEM\{1, 2, -3, -1, -2, 3, 4, -4, 0\} \quad Q(2) - BSEM\{2, 3, -5, -2, -3, 5, 6, -6, 0\}$$

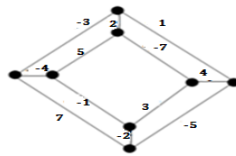
$$Q(3) - BSEM\{3, 4, -7, -3, -4, 7, 8, -8, 0\}$$

**Theorem 3.10**

If the cubical prism  $Y(4,1)$  graph is strong  $Q(a)$ -BSEM.

**Proof:**

If  $p = 8, q = 14$  Cubical prism graph is Strong  $Q(a)$  balance super edge-magic for 1,2,3,4,5,6,7,8. Here  $q$  is even following graph shows that  $Q(a)$  balance super edge-magic.



$Q(1) - BSEM$

$Q(1) - BSEM\{1, -1, 2, -2, 3, -3, 4, -4, 5, -5, 7, -7\}$

$Q(2) - BSEM\{2, -2, 3, -3, 5, -5, 4, -4, 6, -6, 9, -9\}$

$Q(3) - BSEM\{3, 4, -7, -5, 12, -4, -8, 5, 7, -12, 8, -3\}$

$Q(4) - BSEM\{4, 5, -9, -6, 15, -5, -10, 6, 9, -15, 10, -4\}$

$Q(5) - BSEM\{5, 6, -11, -7, 18, -6, -12, 7, 11, -18, 12, -5\}$

$Q(6) - BSEM\{6, 7, -13, -8, 21, -7, -14, 8, 13, -21, 14, -6\}$

$Q(7) - BSEM\{7, 8, -15, -9, 24, -8, -16, 9, 15, -24, 16, -7\}$

$Q(8) - BSEM\{8, 9, -17, -10, 27, -9, -18, 10, 17, -27, 18, -8\}$

**Theorem 3.11**

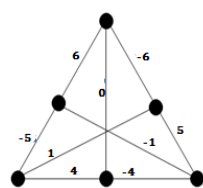
If the Utility graph is Strong  $Q(a)$ -Balanced Super Edge Magic for all  $a \geq 1$

**Proof:**

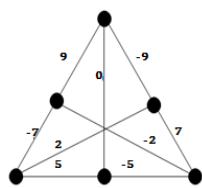
If  $p = 6, q = 9$ . Utility graph is Strong  $Q(a)$  balance super edge-magic for  $1, 2, 3, 4, 5, 6$ .

Here  $q$  is odd following types 1 & 2 shows that  $Q(a)$  balance super edge-magic.

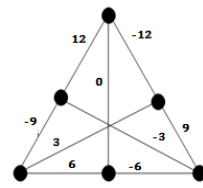
Type 1



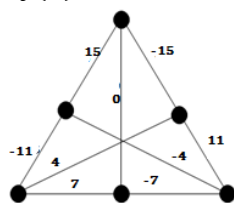
$Q(1) - BSEM$



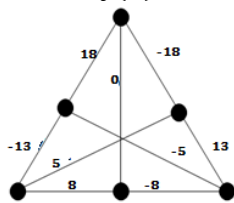
$Q(2) - BSEM$



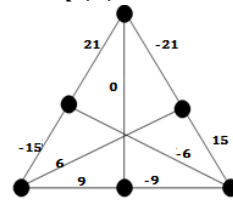
$q(3) - BSEM$



$Q(4) - BSEM$

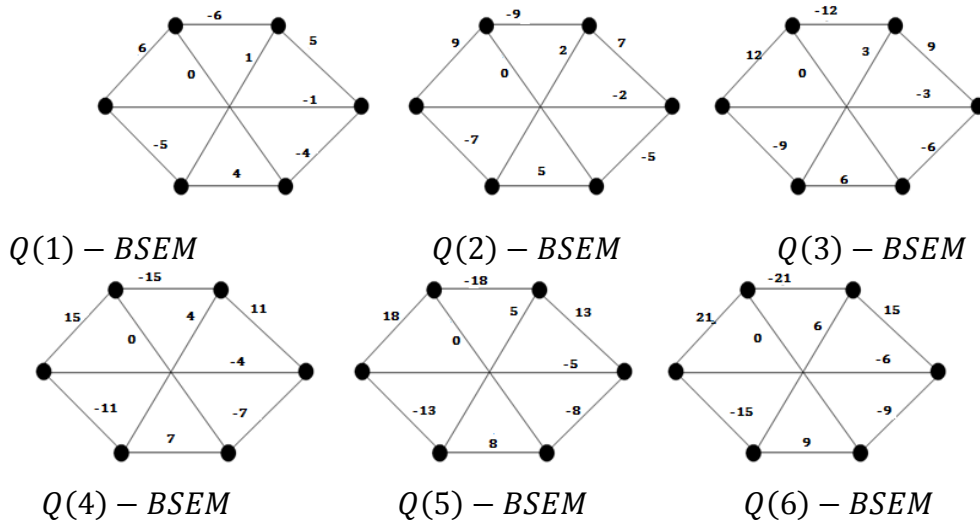


$Q(5) - BSEM$



$Q(6) - BSEM$

**Type 2:**



**4. CONCLUSION:**

Author interested to prove the results P(b)-Super Edge-Gracefulness of Hypercubes.

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