International Journal of Pure and Applied Mathematical Sciences.
ISSN 0972-9828 Volume 10, Number 2 (2017), pp. 111-123

http://www.ripublication.com

A Fuzzy Approach to Co-Ordinated and Non Co-Ordinated Two Stage Supply Chain

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Abstract

This research paper focuses on the fuzziness factor of demand in a two-stage supply chain network. The general fuzzy number technique is applied to exhibit customer demand. This paper explores the optimization of the vertically integrated two stage supply chain under rational co-ordination between the wholesaler and retailer of any business firm. Even the non-coordination case is also discussed. A numerical example is illustrated to exhibit the optimum profit performance of both cases.

Keywords: Supply chain, Fuzzy Number, Wholesaler, Retailer, Co-ordination Fuzzy model.

1. INTRODUCTION

To tackle uncertainty of different integrities of supply chain design and planning, fuzzy approach is applied to traditional supply chain modelling giving emphasis on the randomness factor of uncertainty. Plethora of probabilistic modelling techniques has been evolved to tackle uncertainty but fuzzy modelling has been effectively applied in supply chain design and planning to yield best optimum results.

In actual supply chain design and planning, one of the most complex issues encountered by the decision makers is to forecast the market demand. In the recent past, extensive research endeavours have been focused to surmount this challenge.

The available research in the literature mostly explains the models from a probabilistic aspect, in which a premise is that requisite data is not available to make a reasonable conclusion to be made to the demand distribution and also past data are not always accessable or pertinent due to market volatility or technological growth.

Diminished product life cycles along with escalating innovation rather make the demand extremely fluctuating and needed statistical data is not forthcoming is today's competitive world. At this juncture standard probabilistic modelling that uniquely applies a repeated approach may not be the right choice. Thus an alternate presentation of uncertainty is required. One method is applying subjective probabilities that represent the degree of belief of the decision maker yet another approach that is applied here is fuzzy set theory, where the information available for values of market demand is interval- valued.

In the area of supply chain modelling with fuzzy demand there have been few research studies. Wang and Shu [11] developed fuzzy supply chain model by using six-point fuzzy numbers [3] to represent the fluctuating customer demand, uncertain processing time and unreliable supply delivery. They applied the genetic algorithm approach to determine the optimal stock order-up-to-levels, and the simulation approach to validate the developed concept. Recently, Aliev et al. [2] developed a fuzzy integrated multi-period and multi-product production and distribution model in supply chain, which is formulated in terms of fuzzy programming and the solution is provided by genetic algorithm. In all these papers listed above approximate algorithms were adopted to derive the solution of their models such as the simulation techniques used in [4, 9, 10] and the genetic algorithm in [11, 2].

In our proposed model we use general fuzzy number, namely, Left-Right (LR) -type fuzzy number, to describe the estimate for the external demand, and develop the decision models to study the coordination problem for two stage supply chain.

The closed form solution for the optimal profit of the vertically integrated two stage supply chain is obtained. The relationships of a wholesaler and retailer under both coordination and non-coordination scenarios, are considered and we prove that the maximum supply chain profit in the coordination situation is greater than that in the non-coordination situation. In order to share the obtained profit, we give a simple scheme such that both players can benefit from the coordination. Finally, numerical results are presented to demonstrate the performance of our techniques.

PRELIMINARIES

Basically most supply chain problems do not have enough historical data to construct the probability distribution hence in this paper we develop the decision models for single period two stage supply chain which consists of the wholesaler and retailerand we also enumerate the method to maximise their supply chain profit in the case of coordination and non-coordination of their business deals to overcome the aspect of demand uncertainty.

We use general fuzzy number namely LR- type fuzzy number to estimate the external demand.

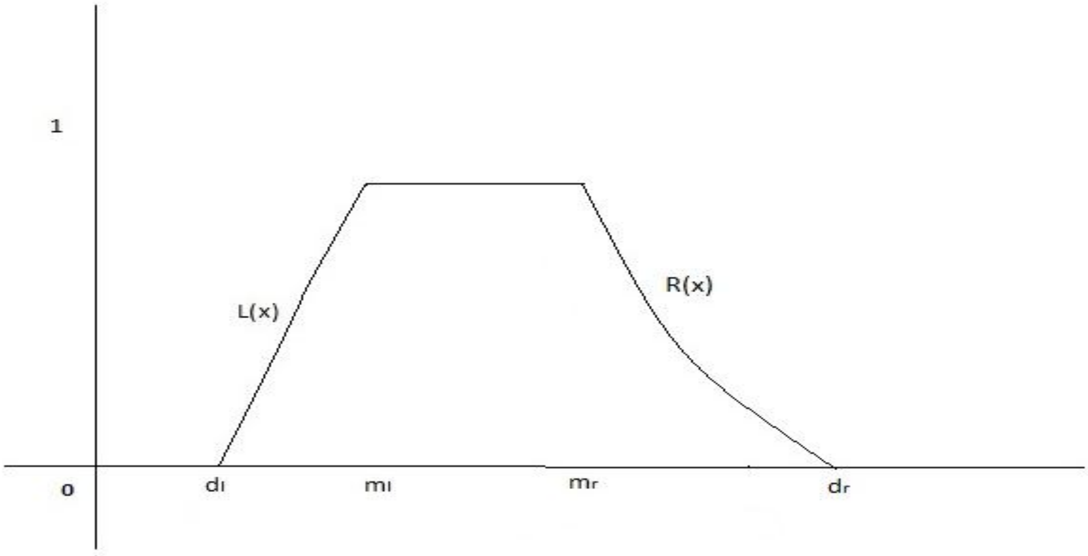

Figure 1: LR type fuzzy number

DEFINITION 1

An LR-type fuzzy number D can be described with the following membership function[18]:

$$D = \begin{cases} \mathrm{L(x)} & \text{if } \mathrm{d_l \le x < m_l} \\ 1 & \text{if } \mathrm{x \in [m_l, m_r]} \\ \mathrm{R(x)} & \text{if } \mathrm{m_r < x \le d_r} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $[m_l, m_r]$ is the range of most likely values of D; m_l and m_r are the lower and upper modal values; $L(x)$ and $\mathrm{R(x)}$ are the increasing and decreasing continuous functions respectively. The pictorial representation of the above membership function is fig1. The closure of the support of D is exactly $[d_1, d_2]$ and the $\lambda -$ level set of D can be denoted as $D_\lambda = [\mathrm{L}^{-1}(\lambda), \mathrm{R}^{-1}(\lambda)]$, $\lambda \in [0,1]$, which is the closed interval on real number set R. In order to measure the mean value of a fuzzy number D,we use the fuzzy mean introduced by Dubois and Prade[20]:

$$E(D) = \frac{E_*(D) + E^*(D)}{2} = \int_0^1 \frac{d_1(\lambda) + d_2(\lambda)}{2} d\lambda \qquad (2)$$

Where $[E_*(D), \mathrm{E}^*(D)]$ is the interval-valued expectation, and $[d_1(\lambda), d_2(\lambda)]$ is the $\lambda -$ level set of D.

NOTATIONS AND ASSUMPTIONS

In this proposed paper a single period two stage supply chain, namely the wholesalerand retailer supply chain setting is considered.Demand is fuzzy.To start with, the wholesaler fixes his unit price and then the retailer his unit price and then makes his own replacement policy to maximise his own profit.The wholesaler's profit will depend on the retailer's ordering quantity and in turn the retailer's profit depends on the consumer demand.Following are the notations used in our model:

p - Wholesaler's unit price

c - Wholesaler's unit production cost

w - Retailer's unit price

h - Retailer's unit holding cost

s - Retailer's unit shortage cost

q - Retailer's order quantity

D - Fuzzy demand

FORMULATON OF THE PROBLEM

Let the retailer's fore-casted demand be $D = (d_l, m_l, m_r, d_r)_{LR}$ then for order quantity q units the sales volume, holding and shortage quantity is denoted as $\min\{q - D\}$ $\max\{q - D, 0\}$ and $\max\{D - \mathrm{q}, 0\}$ respectively, by Zadeh'sextension principle .

Hence the retailer's total profit $R(q)$ becomes

$$R(q) = \mathrm{w}\min\{\mathrm{q}, \mathrm{D}\} - h\max\{\mathrm{q} - \mathrm{D}, 0\} - s\max\{\mathrm{D} - q, 0\} - pq \qquad (3)$$

As D is LR - fuzzy number, so is $R(q)$ we have to solve the optimisation problem to obtain the optimal order quantity q so as to get the maximum profit. Three cases arise for the optimisation problem $\max_{q \geq 0} E(R(q))$ which are as follows:-

Case 1: $d_l \le q \le m_l$

In this case the λ - level sets of fuzzy sales volume,holding and shortage quantity can be respectively expressed as

$$(\min\{q,D\}) = \begin{cases} [L^{-1}(\lambda),q] & \text{for } 0<\lambda \le L(q) \\ [q,q] & \text{for } L(q)<\lambda \le 1 \end{cases}$$

$$(\max\{q-D,0\}) = \begin{cases} [0,q\text{-}[L^{-1}(\lambda)] & \text{for } 0<\lambda \le L(q) \\ [0,0] & \text{for } L(q)<\lambda \le 1 \end{cases}$$

$$(\max\{D-q,0\}) = \begin{cases} [0,R^{-1}(\lambda)\text{-}q] & \text{for } 0<\lambda \le L(q) \\ [L^{-1}(\lambda)\text{-}q,R^{-1}(\lambda)\text{-}q] & \text{for } L(q)<\lambda \le 1 \end{cases}$$

Similarly the λ - level sets of $R(q)$ are calculated as follows:

If $0 \le \lambda \le L(q)$,then

$$\begin{aligned} R(q) &= w\min(q,D) - h\max(q-D,0) - s\max(D-q,0) - pq \\ &= w[L^{-1}(\lambda),q] - h[0,q-L^{-1}(\lambda)] - s[0,R^{-1}(\lambda)-q] - [pq,pq] \\ &= [(w+h)L^{-1}(\lambda) - sR^{-1}(\lambda) - (h-s+p)q.(w-p)q] \end{aligned}$$

If $L(q) \le \lambda \le 1$,then

$$\begin{aligned} R(q) &= w[q,q] - h[0,0] - s[L^{-1}(\lambda)-q, R^{-1}(\lambda)-q] - [pq,pq] \\ &= [(w+s-p)q - sR^{-1}(\lambda), (w+s-p)q - sL^{-1}(\lambda)] \end{aligned}$$

It follows that the fuzzy mean of $R(q)$ by (2), is

$$\begin{aligned} E(R(q)) &= \frac{1}{2}\int_0^{L(q)} [(w+h)L^{-1}(\lambda) - sR^{-1}(\lambda) - (h-s+p)q + (w-p)q]\,d\lambda \\ &\quad + \frac{1}{2}\int_{L(q)}^{1} [(w+s-p)q - sR^{-1}(\lambda) + (w+s-p)q - sL^{-1}(\lambda)]\,d\lambda \\ &= \frac{1}{2}\int_0^{L(q)} [(w+h)L^{-1}(\lambda) - sR^{-1}(\lambda)]\,d\lambda \\ &\quad - \frac{1}{2}\int_{L(q)}^{1} (sR^{-1}(\lambda) + L^{-1}(\lambda))\,d\lambda + (w+s-p)q - \frac{1}{2}(w+h+s)qL(q) \end{aligned}$$

Find the first and second derivatives of $E(R(q))$ with respect to q as follows:

$$\frac{dE(R(\mathrm{q}))}{dq} = (w+s-p) - \frac{1}{2}(\mathrm{w}+h+s)L(q) \tag{4}$$

$$\frac{d^2E(R(\mathrm{q}))}{dq^2} = -\frac{1}{2}(\mathrm{w}+h+s)L'(q) \tag{5}$$

Using first order conditions, we have

$$L(q_l) = \frac{2(w+s-p)}{(\mathrm{w}+h+s)} \tag{6}$$

Clearly $\frac{2(w+s-p)}{(\mathrm{w}+h+s)} > 0$ Since $w > p$ So, if $\frac{2(w+s-p)}{(\mathrm{w}+h+s)} \le 1$, namely $w+s \le \mathrm{h}+2\mathrm{p}$

Then we have

$$q_l = L^{-1}\frac{2(w+s-p)}{(\mathrm{w}+h+s)} \tag{7}$$

On the other hand, if $\frac{d^2E(R(\mathrm{q}))}{dq^2} < 0$ since $L(q)$ is increasing in $[d_l, m_l]$.

Then the function $E(R(\mathrm{q}))$ reaches its maximum at q_l if $w+s \le h+2p$, and the maximum value is

$$E(R(\mathrm{q})) = \frac{1}{2}\left[(w+h)\int_0^{\frac{2(w+s-p)}{(\mathrm{w}+h+s)}} \mathrm{L}^{-1}(\lambda)\,\mathrm{d}\lambda - s\left(\int_0^1 R^{-1}(\lambda)\,\mathrm{d}\lambda + \int_{\frac{2(w+s-p)}{(\mathrm{w}+h+s)}}^1 \mathrm{L}^{-1}(\lambda)\,\mathrm{d}\lambda\right)\right] \tag{8}$$

Case2: $m_l \le q \le m_r$

For q lying in the range of m_l and m_r we have

$$(\min\{q, D\}) = [\mathrm{L}^{-1}(\lambda), q]$$

$$(\max\{q-D, 0\}) = [0, q-\mathrm{L}^{-1}(\lambda)]$$

$$(\max\{\mathrm{D}-\mathrm{q}, 0\}) = [0, R^{-1}(\lambda)-\mathrm{q}]$$

It follows that the λ level set of retailer's fuzzy profit:

$$R(\mathrm{q}) = \mathrm{w}\min\{\mathrm{q},\mathrm{D}\} - h\max\{q-D,0\} - s\max\{D-q,0\} - pq$$

$$= w[\mathrm{L}^{-1}(\lambda), q] - h[0, q-\mathrm{L}^{-1}(\lambda)] - s[0, R^{-1}(\lambda)-\mathrm{q}] - \mathrm{pq}$$

$$= [(\mathrm{w}+\mathrm{h})\mathrm{L}^{-1}(\lambda) - \mathrm{s}\,R^{-1}(\lambda) + (\mathrm{s}-\mathrm{h}-\mathrm{p})\mathrm{q}, (\mathrm{w}-\mathrm{p})\mathrm{q}]$$

Now, the fuzzy mean of retailer's profit is given by

$$E(R(\mathrm{q})) = \frac{1}{2}\int_0^1 [(w+h)\,\mathrm{L}^{-1}(\lambda) - sR^{-1}(\lambda)]\,\mathrm{d}\lambda + (w+s-h-2p)q]$$

And the first derivative

$$\frac{dE(R(\mathrm{q}))}{dq} = \frac{1}{2}(\mathrm{w}+s-h-2p) \tag{9}$$

If $\mathrm{w}+s-h-2p>0$, then $E(R(\mathrm{q}))$ gets its maximum at $q=m_r$;if $\mathrm{w}+s-h-2p<0$, then $E(R(\mathrm{q}))$ reaches its maximum at $q=m_l$; and if $\mathrm{w}+s-h-2p=0$, then $E(R(\mathrm{q}))$ reaches its maximum for any $q\in[m_l,m_r]$

Case 3: $m_r \le q \le d_r$

Hence again, the λ- level sets of fuzzy sales volume, holding and shortage quantity can be respectively expressed as

$$\min\{\mathrm{q},\mathrm{D}\}) = \begin{cases} [L^{-1}(\lambda), q] & \text{for } 0<\lambda \le R(\mathrm{q}) \\ [\mathrm{L}^{-1}(\lambda), R^{-1}(\lambda)] & \text{for } R(\mathrm{q})<\lambda \le 1 \end{cases}$$

$$(\max\{\mathrm{q}-\mathrm{D},0\}) = \begin{cases} [0, \mathrm{q}-L^{-1}(\lambda)] & \text{for } 0<\lambda \le R(\mathrm{q}) \\ [q-R^{-1}(\lambda), \mathrm{q}-\mathrm{L}^{-1}(\lambda)] & \text{for } R(\mathrm{q})<\lambda \le 1 \end{cases}$$

$$(\max\{\mathrm{D}-\mathrm{q},0\}) = \begin{cases} [0, R^{-1}(\lambda)-\mathrm{q}] & \text{for } 0<\lambda \le R(\mathrm{q}) \\ [0,0] & \text{for } R(\mathrm{q})<\lambda \le 1 \end{cases}$$

Similarly the λ - level sets of $R(\mathrm{q})$ are calculated as follows:

$$R(\mathrm{q}) = \begin{cases} [(\mathrm{w}+\mathrm{h})\,\mathrm{L}^{-1}(\lambda) - sR^{-1}(\lambda) + (\mathrm{s}-\mathrm{h}-\mathrm{p})\,\mathrm{q}, (\mathrm{w}-\mathrm{p})\,\mathrm{q}] & \text{for } 0<\lambda \le R(\mathrm{q}) \\ [(\mathrm{w}+\mathrm{h})\,\mathrm{L}^{-1}(\lambda) - (\mathrm{h}+p)q, (w+h)R^{-1}(\lambda) - (\mathrm{h}+\mathrm{p})\,\mathrm{q}] & \text{for } R(\mathrm{q})<\lambda \le 1 \end{cases} \tag{10}$$

Now the fuzzy mean of $R(\mathrm{q})$ is calculated as follows:

$$E(R(\mathrm{q})) = \frac{1}{2}\int_0^{R(\mathrm{q})} [(\mathrm{w}+\mathrm{h})\,\mathrm{L}^{-1}(\lambda) - \mathrm{s}\,R^{-1}(\lambda) + (\mathrm{w}+s-h-2p)q]\,\mathrm{d}\lambda$$

$$+\frac{1}{2}\int_{R(q)}^{1} (\mathrm{w}+h)(\mathrm{L}^{-1}(\lambda)+R^{-1}(\lambda)) - 2(\mathrm{h}+p)q]\,d\lambda$$

$$=\frac{1}{2}\left[\int_0^1 (w+h)\,\mathrm{L}^{-1}(\lambda)\,\mathrm{d}\lambda - \int_0^{\mathrm{R}(q)} sR^{-1}(\lambda)\,d\lambda + \int_{\mathrm{R}(q)}^{1} (\mathrm{w}+h)R^{-1}(\lambda)\,d\lambda\right]$$

$$-(h+p)q+\frac{1}{2}(w+h+s)qR(q) \tag{11}$$

The first and second order derivatives of $E(R(\mathrm{q}))$ with respect to q are as follows:

$$\frac{dE(R(\mathrm{q}))}{dq}=\frac{1}{2}(\mathrm{w}+h+s)R(q)-(h+p) \tag{12}$$

$$\frac{d^2E(R(\mathrm{q}))}{dq^2}=\frac{1}{2}(\mathrm{w}+h+s)R'(q)<0 \tag{13}$$

If $\frac{dE(R(\mathrm{q}))}{dq}=0$ then we get

$$R(\mathrm{q_r})=\left(\frac{2(\mathrm{h}+\mathrm{p})}{w+h+s}\right) \tag{14}$$

If $\frac{2(\mathrm{h}+\mathrm{p})}{w+h+s}\le 1$, namely, $w+s\ge h+2p$, then the Retailer optimal order quantity will be as follows:

$$q_r=R^{-1}\left(\frac{2(\mathrm{h}+p)}{(\mathrm{w}+h+s)}\right) \tag{15}$$

And the retailer's optimal profit is

$$E(R(q_r))=\frac{1}{2}\left[(w+h)\left(\int_0^1 \mathrm{L}^{-1}(\lambda)d\lambda+\int_{\frac{2(\mathrm{h}+p)}{\mathrm{w}+h+s}}^{1} R^{-1}(\lambda)d\lambda\right)-s\int_0^{\frac{2(\mathrm{h}+p)}{\mathrm{w}+h+s}} R^{-1}(\lambda)d\lambda\right]$$

Combining all the 3 cases we have the following inferences for optimal order quantity and its corresponding optimal profit of the Retailer which is as follows:

$$q^*=\begin{cases}\mathrm{L}^{-1}\left(\frac{2(\mathrm{w+s-p})}{\mathrm{w+h+s}}\right) & \text{if } \mathrm{w+s}<h+2p\\ [\mathrm{m}_l,\mathrm{m_r}] & \text{if } \mathrm{w+s}=h+2p\\ \mathrm{R}^{-1}\left(2\left(\frac{\mathrm{h+p}}{\mathrm{w+h+s}}\right)\right) & \text{if } \mathrm{w+s}>h+2p\end{cases} \tag{16}$$

$$E(R(q^*))=\begin{cases}\frac{1}{2}\left[(w+h)\int_0^{\frac{2(w+s-p)}{w+h+s}}L^{-1}(\lambda)\text{-s}\left(\int_0^1 R^{-1}(\lambda)d\lambda+\int_{\frac{2(w+s-p)}{w+h+s}}^1 L^{-1}(\lambda)\,\mathrm{d}\lambda\right)\right] & \text{if } \mathrm{w+s}<h+2p\\ \frac{1}{2}\int_0^1[(\mathrm{w+h})\mathrm{L}^{-1}(\lambda)\text{-sR}^{-1}(\lambda)]\mathrm{d}\lambda & \text{if } \mathrm{w+s}=h+2p\\ \frac{1}{2}\left[(w+h)\left(\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda+\int_{2\left(\frac{h+p}{w+h+s}\right)}^1 R^{-1}(\lambda)\,\mathrm{d}\lambda\right)-s\int_0^{2\left(\frac{h+p}{w+h+s}\right)}R^{-1}(\lambda)d\lambda\right] & \text{if } \mathrm{w+s}>h+2p\end{cases} \quad (17)$$

Case of non-coordinated supply chain

Both the wholesaler and the retailer are independent in their decisions of how much to place the orders and fix their costs or profits accordingly once they take independent decisions the wholesaler has to supply the order quantity required by the retailer and in that case first the wholesaler's profit is calculated as follows:

$$\mathrm{E_N}(\mathrm{W}(q^*))=(p-c)q^*=\begin{cases}(\mathrm{p-c})\mathrm{L}^{-1}\left(\frac{2(\mathrm{w+s-p})}{w+h+s}\right) & \text{if } \mathrm{w+s}<h+2p\\ (\mathrm{p-c})[\mathrm{m}_l,\mathrm{m_r}] & \text{if } \mathrm{w+s}=h+2p\\ (\mathrm{p-c})\mathrm{R}^{-1}\left(2\left(\frac{\mathrm{h+p}}{w+h+s}\right)\right) & \text{if } \mathrm{w+s}>h+2p\end{cases} \quad (19)$$

The corresponding whole supply chain profit is

$$T_N(q^*)=\mathrm{E}(R(q^*))+E_N(\mathrm{W}(q^*)) \quad (20)$$

Case of coordinated supply chain

In this case the wholesaler and retailer coordinate and jointly discuss their demands and requirements so as to maximise their combined profit and hence in this scenario the external demand, wholesaler unit production cost, retailer's holding and penalty cost is same for both of them hence the whole supply chain profit is now calculated as follows:

$$q_T=\begin{cases}\mathrm{L}^{-1}\left(\frac{2(\mathrm{w+s-c})}{w+h+s}\right) & \text{if } \mathrm{w+s}<h+2c\\ [\mathrm{m}_l,\mathrm{m_r}] & \text{if } \mathrm{w+s}=h+2c\\ \mathrm{R}^{-1}\left(2\left(\frac{\mathrm{h+c}}{w+h+s}\right)\right) & \text{if } \mathrm{w+s}>h+2c\end{cases} \quad (21)$$

Corresponding optimal profit is

$$E_c(\mathrm{W}(q_T))=\begin{cases}\frac{1}{2}\left[(w+h)\int_0^{\frac{2(w+s-c)}{w+h+s}}L^{-1}(\lambda)\,\mathrm{d}\lambda\text{-s}\left(\int_0^1 R^{-1}(\lambda)d\lambda+\int_{\frac{2(w+s-c)}{w+h+s}}^1 L^{-1}(\lambda)\,\mathrm{d}\lambda\right)\right] & \text{if } \mathrm{w+s}<h+2c\\ \frac{1}{2}\int_0^1[(\mathrm{w+h})\mathrm{L}^{-1}(\lambda)\text{-sR}^{-1}(\lambda)]\mathrm{d}\lambda & \text{if } \mathrm{w+s}=h+2c\\ \frac{1}{2}\left[(w+h)\left(\int_0^1 L^{-1}(\lambda)\,\mathrm{d}\lambda+\int_{2\left(\frac{\mathrm{h}+c}{w+h+s}\right)}^1 R^{-1}(\lambda)\,\mathrm{d}\lambda\right)-s\int_0^{2\left(\frac{\mathrm{h}+c}{w+h+s}\right)}R^{-1}(\lambda)d\lambda\right] & \text{if } \mathrm{w+s}>h+2c\end{cases}$$

In continuation to the above two cases discussed, we can also enumerate a sharing scheme between the wholesaler and the retailer to get the maximum total profit as follows:

Let $A=\dfrac{E(R(q^*))}{T_N(\mathrm{q}^*)}$, $B=\dfrac{E_N(\mathrm{W}(q^*))}{T_N(\mathrm{q}^*)}$ where 0<A,B<1 and A+B=1.

Clearly we have $A(E_c(W(q_T))>\mathrm{E(R}(q^*))$ and $B(E_c(W(q_T))>\mathrm{E_N}(W(q^*))$

Hence in our inferences we will be concluding that in a vertically integrated two stage supply chain the totalprofit forecasted in coordinated case is greater than that in the non-coordinated case which we will prove in the following numerical example

Numerical Illustration

Let us assume a electronics wholesale dealer forecasts the demand for its rechargeable battery components which he represents as trapezoidal fuzzy number as $\mathrm{D}=(45000,50000,60000,65000)$ and the all the costs per unit are given by $w=100,\mathrm{h}=20,\mathrm{s}=80,\mathrm{c}=30$

In the non- coordinated case, suppose the wholesaler sets the its price as $p=60$ then the retailer's optimal order quantity is

$$\mathrm{q}^*=R^{-1}\left(\frac{2(h+p)}{w+h+s}\right)=R^{-1}\left(\frac{4}{5}\right)=61000 \qquad (22)$$

And the optimal fuzzy mean profit

$$\mathrm{E}(R(q^*))=\frac{1}{2}(w+h)\left(\int_0^1 L^{-1}(\lambda)d\lambda+\int_{\frac{2(h+p)}{w+h+s}}^1 R^{-1}(\lambda)d\lambda\right)-\frac{s}{2}\int_0^{\frac{2(h+p)}{w+h+s}}R^{-1}(\lambda)d\lambda=1{,}560{,}000 \qquad (23)$$

Since $w+s>h+2p$. Correspondingly, the wholesaler's profit is

$$E_N(W(q^*)) = (p-c)q^* = 1{,}830{,}000 \quad (24)$$

Therefore, the whole supply chain profit under non-coordination situation is

$$T_N(q^*) = E(R(q^*)) + E_N(W(q^*)) = 3{,}390{,}000 \quad (25)$$

On the other hand, in the coordination situation, the optimal order quantity and corresponding optimal supply chain profit can be computed

$$(\mathrm{q}_T) = 625{,}000$$

and

$$E_c(W(q_T)) = 3{,}412{,}500$$

According to the sharing scheme, the retailer and the wholesaler can obtain their respective profit as follows:

$$AE_c(W(q_T)) = \frac{E(R(q^*))}{T_N(q^*)} \times E_c(W(q_T)) = 1{,}570{,}400 \quad (26)$$

and

$$BE_c(W(q_T)) = \frac{E(W(q^*))}{T_N(q^*)} \times E_c(W(q_T)) = 1{,}842{,}100 \quad (27)$$

Table 1

Optimal Solutions.

	Order quantity	Retailer's profit	Wholesaler's profit	Total profit
Non-coordination	61000	1560000	1830000	3390000
Coordination	62500	1570400	1842100	3412500

Table 2

The Effects of Demand Fuzziness.

	Order quantity	Retailer's profit	Wholesaler's profit	Total profit
Non-coordination	60400	1640000	1830000	3390000
Coordination	61000	1655200	1869800	3525000

It is obvious that the channel's profit is increased by coordination and both players get more profit in the coordination situation than in the non-coordination situation. The results are summarized and reported in Table 1

Now we decrease the fuzziness of demand and observe its effects. Suppose that the fuzzy demand is $D = (45000, 50000, 60000, 62000)$ and other parameters remain as before. The results obtained by using our fuzzy models are given in Table 2

Comparing the results presented in Table 1 and 2, we can see that a retailer's profit will increase when the demand fuzziness decreases. The reason is that the less demand uncertainty, the less holding and shortage cost the retailer has to pay, and the more profit he obtains. We also note that when demand fuzziness decreases the wholesaler's profit drops slightly in the non-coordination situation. This is because the wholesaler cannot adjust the wholesale price quickly in response to the change of the demand uncertainty. However the wholesaler still obtains more profit once he shares more market information and cooperates with the retailer.

CONCLUSION

This study takes into account the subjective estimation of both the wholesaler and retailer's decisions and judgements in which their demand is fuzzy rather than stochastic. General fuzzy number is applied to exhibit the estimate for the external demand and evolve for the decision models to determine the optimal profit for both retailer and integrated supply chain. The closed form solutions for both models are got and can be specified to meet non- fuzzy situation when the fuzzy demand in our models is crisp real number. Based on the closed form solutions, both non-coordination and co-ordination situations are evaluated and it is exhibited that co-ordination enhances the anticipated profit. The merit of the closed form solutions is that they remove the need for enumeration over alternative values and clarify the relations among the model parameters.

In comparison to that of traditional probabilistic approach, the proposed model requires limited data to predict the uncertain demand and can make use of the subjective assessment on the basis of decision maker's perception, experience and judgement. In view of this the proposed methodology provides an alternative for supply chain planning under uncertain situations. It is apt when the scenario is volatile and there is absence of past records.

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