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Second Order Zuckerman Number

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Abstract

Zuckerman numbers are well known in Number Theory. Here a new definition is given for second order Zuckerman numbers.

INTRODUCTION

In mathematics, a Zuckerman number is a positive integer that is divisible by the product of its digits when written in base 10 [1]. A sequence of Zuckerman numbers is listed in the Encyclopedia of Integer Sequences [2]. The list is 1,2, 3,4,5,6,7,8,9,11,12,15,24,36,111,112,115, etc. These numbers have some analogy with the Harshad numbers [3], which are defined as the number that are divisible by the sum of its digits. Later second order Harshad numbers were defined by Pal and Gopalan [4] as integers which are divisible by the square of the sum of squares of its digits.

Obviously, in the above sequence of Zuckerman numbers, the numbers with 0 as one of the digits are not cosidered, as the product of digits in that case will be zero. There is some sequence of numbers listed in [3] which satisfies both criteria of Harshad numbers and Zuckerman numbers.

The second order Harshad numbers [4] gives the idea, that there are many such numbers, which are divisible by the square of the product of its digits. Let us call those numbers as second order Zuckerman numbers. Here we shall discuss about the numbers only in base 10.

DEFINITION

Second order Zuckerman number is a positive integer in which none of its digits is zero and the number is divisible by the square of the product of its digits.

DISCUSSION

All single digit numbers except 1 will not be second order Zuckerman number. The Zuckerman numbers 11 and 12 will also satisfy the definition of second order Zuckerman numbers. Some Zuckerman numbers may also fall in the category of second order Zuckerman numbers, but not all Zuckerman numbers will satisfy the criterion for second order Zuckerman number. Obviously, all second order Zuckerman numbers will satisfy the definition for Zuckerman number.

In fact, there are five Zuckerman numbers between 11 and 99, namely, 11,12,15,24,36 among which only two numbers 11 and 12 are second order Zuckerman numbers. From 1 to 10000, there are 74 Zuckerman numbers, but there are only nine second order Zuckerman numbers which are

1, 11, 12, 111, 112, 1111, 1112, 1116, 2112

It is interesting to note that there are no second order Zuckerman number between 2113 and 9999.

All the above nine numbers are also Zuckerman numbers. Among these numbers, only 1 and 111 are second order Harshad numbers and 1, 12, 111, 112, 1116 and 2112 are Harshad numbers. It may be noted that up to 10000, there are no second order Zuckerman number with a digit more than six in the number. The first second order Zuckerman number with 7 as one of its digits is 11172 and the first such number with 8 as one of its digits is 218112. The first second order Zuckerman number with 9 as one of its digits is a seven digit number 1113912 and the second such number is 9716112.

There are many pairs of consecutive second order Zuckerman numbers in the form (11, 12), (111, 112) etc. The first three consecutive second order Zuckerman numbers are 1111111, 1111112 and 1111113.

It is obvious that there are many large second order Zuckerman numbers.

CONCLUSION

From the above discussion, it may be concluded that there are infinite numbers of second order Zuckerman numbers. The set of second order Zuckerman numbers is a subset of the set of Zuckerman numbers, but not all Zuckerman numbers satisfy the definition of second order Zuckerman number. This sequence of second order Zuckerman numbers is a new sequence which is not listed in the Encyclopedia of Integer sequences [3].

REFERENCE

[1] https://proofwiki.org/wiki/Definition:Zuckerman_Number

[2] N. J. A. Sloane and P. Simon, The Encyclopedia of Integer Sequences,

Academic Press, 1995

[3] https://en.wikipedia.org/wiki/Harshad_number

[4] P. K. Pal, Second Order Harshad Number, International Journal of mathematical Education, V. 13, No. 1, p. 25-26