International Journal of Mechanics and Solids
ISSN 0973-1881 Volume 3 Number 1 (2008) pp. 109–118
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http://www.ripublication.com/ijms.htm

Simplified 3-D Fem for the Prediction of Stiffness Elements of Unidirectional Specially and Generally Orthotropic T300-Epoxy Lamina

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Abstract

In the present paper simplified 3-D Finite Element models are developed with governing boundary conditions and solved using the FEA software ANSYS version 11 for the prediction of stiffness elements of unidirectional continuous fiber reinforced specially and generally orthotropic laminae. The results of the present work are compared with the analytical solutions and found very close agreement. It is observed that there exist 13 independent elastic constants for generally orthotropic lamina and behaves like a monoclinic material. A special case of monoclinic material with 9 independent constants is seen at fiber angle of 45 degrees. The specially orthotropic lamina with 5 independent constants is behaving like a transversely isotropic. The present analysis is useful to study the mechanical behavior of an angle ply lamina which is required for selection of stacking sequence of laminates. The analysis can be extended for the laminates where the analytical solutions are not available.

Keywords: Stiffness elements, Angle-ply lamina, Finite Element Method

Notation

σ : Normal Stress
τ : Shear Stress
ε : Normal Strain
γ : Shear Strain
$\bar{Q}_{ij}$: Elastic Constants (Elements of stiffness matrix) (i,j = 1,2,……6)

θ : Fiber angle in degrees

Introduction

There has been a tremendous advancement in the science and technology of fiber-reinforced composite materials due to applications of this material in the key fields like aerospace. The low density, high strength, high stiffness to weight ratio, excellent durability and design flexibility of fiber-reinforced composite materials are the primary reasons for their extended use.

The layer moduli are the basic inputs for stress analysis of composite laminates. As a consequence of the layer anisotropy, experimental measurement requires significant testing effort, and hence alternate methods of predicting the elastic constants of a composite lamina are of considerable interest.

A great number of micromechanical models have been proposed in the literature [1-4] for predicting various mechanical properties of composite materials. Several other models have been proposed such as numerical homogenization [5] and FEM [6,7] for the micro level prediction of properties. Engineering properties of angle-ply lamina have been predicted [8,9] using constrained finite element models where these models can not exhibit the natural behavior of FRP lamina during deformation.

The Rule of Mixtures expressions are given in [10] for predicting the elastic constants of unidirectional specially orthotropic continuous fiber reinforced lamina. For generally orthotropic lamina the equations of [10] are transformed for a given fiber angle θ in the plane of lamina [11]. The analytical expressions for the stiffness matrix of specially and generally orthotropic fiber reinforced lamina are given in [12].

The analytical solutions given by [10-12] fails in predicting the mechanical properties and elastic constants of stiffness matrix for FRP laminates with and without imperfections such as cracks and delaminations. The models proposed don’t insist any restriction on the actual behavior of the lamina and are capable of predicting the elastic constants accurately for lamina as well as laminates. These models can also accommodate imperfections in laminates if any.

Modeling the Problem

In the present analysis 6 different 3-D finite element models are developed to predict all the 36 elastic constants at 60% fiber volume fraction of T300-epoxy lamina by treating it as an anisotropic material. The material properties are given as

- Young’s moduli: E_1 = 134.483 GPa, $E_2 = E_3$ = 9.918 GPa
- Poisson’s ratios: $\nu_{12} = \nu_{13}$ = 0.2570, ν_{23} = 0.3627
- Shear moduli: $G_{12} = G_{13}$ = 4.420 GPa, G_{23} = 3.639 GPa

The stress-strain relations of anisotropic material are given as

$$\sigma_i = [\bar{Q}_{ij}]\ \varepsilon_i \quad \text{where i, j = 1 to 6}$$

For evaluation of $\bar{Q}$ matrix, the finite element models are created with a geometry of 10x10x10 units using SOLID45 element of ANSYS software [13]. The stresses σ_x, σ_y, σ_z, τ_{xy}, τ_{yz} and τ_{zx} are given as σ_1, σ_2, σ_3, σ_4, σ_5 and σ_6 respectively for convenience. The details of the finite element models are given below.

i) ε_x-model: In this model the lamina with the given dimensions in x, y and z direction respectively, is subjected to a uni-axial strain in the x-direction.. The stresses σ_x, σ_y, σ_z, τ_{xy}, τ_{yz} and τ_{zx} are calculated from the finite element solution and the following equations are used to determine the first column elements of stiffness matrix.

$\sigma_i = [\bar{Q}_{i1}]\ \varepsilon_x$ where i = 1 to 6.

ii) ε_y -model: The uni-axial strain is applied in the y-direction and all the stresses are calculated from the finite element solution and the following equations are used to determine the second column elements of the stiffness matrix.

$\sigma_i = [\bar{Q}_{i2}]\ \varepsilon_y$ where i = 1 to 6.

iii) ε_z -model: The uni-axial strain is applied in the z-direction and all the stresses are calculated from the finite element solution and the following equations are used to determine the third column elements of the stiffness matrix.

$\sigma_i = [\bar{Q}_{i3}]\ \varepsilon_z$ where j = 1 to 6.

iv) γ_{xy} -model: In this case a pure shear strain is applied in the x-y plane and all the stresses are calculated from the finite element solution and the following equations are used to determine the fourth column elements of the stiffness matrix.

$\sigma_i = [\bar{Q}_{i4}]\ \gamma_{xy}$ where i = 1 to 6.

v) γ_{yz} -model: In this case a pure shear strain is applied in the y-z plane and all the stresses are calculated from the finite element solution and the following equations are used to determine the fifth column elements of the stiffness matrix.

$\sigma_i = [\bar{Q}_{i5}]\ \gamma_{yz}$ where i = 1 to 6.

vi) γ_{zx} -model: In this case a pure shear strain is applied in the x-z plane and all the strains are calculated from the finite element solution and the following equations are used to determine the sixth column elements of the stiffness matrix.

$\sigma_i = [\bar{Q}_{i6}]\ \gamma_{zx}$ where i = 1 to 6.

Analysis of Results

The elastic constants (Elements of stiffness matrix) of T300-epoxy lamina for $\theta = 0^0$, 30^0, 45^0, 60^0 and 90^0 are presented in Tables 1 to 5.

In Table1, the elastic constant $\bar{Q}_{55} = (\bar{Q}_{22} - \bar{Q}_{23})/2$ and therefore only 5 independent elastic constants exists and hence can be considered as transversely isotropic material. Tables 2 and 4 show the behavior of a monoclinic material as there exits 13 independent elastic constants due to one plane of material symmetry i.e. z-plane. The number of independent elastic constants are 9 in Table 3, but can not be considered as an orthotropic material since there exits coupling between shear and extensions hence it is a special case of monoclinic material as the fibers make equal angles with x and y axes. In Table 5 there exists 5 independent elastic constants similar to Table 1 as $\bar{Q}_{66} = (\bar{Q}_{11} - \bar{Q}_{13})/2$.

Figures 1-5 show the variation of transformed stiffnesses with respect to fiber angle. The results of present analysis are in very close agreement with the analytical results of [12], hence the proposed models are suitable for the analysis of FRP lamina and can be extended to laminates where the analytical solutions do not exist for the prediction of stiffness coefficients.

For convenience, the first three diagonal elements drawn against transformation angles between -90 to 90 degrees are shown in Fig.1 and remaining three diagonal elements are shown in Fig. 2. Maximum value of Q_{11} is observed at angle of $\theta = 0$ degrees and reduced to a value of Q_{33} which is constant for all values of θ. It is also observed that Q_{11} drops steeply until 45 degrees to 25% of its initial value. Q_{22} is exhibiting reverse trend to Q_{11} where maximum value at ± 90 degrees and minimum at 0 degrees are observed. No significant variation is observed in Q_{55} and Q_{66} with respect to transformation angle, where as Q_{44} showing parabolic variation with maximum value at ± 45 degrees and minimum at 0 and ± 90 degrees.

Fig. 3 shows the variation of non-diagonal elements of the first row of stiffness matrix. Q_{12} varies in parabolic manner with positive maximum value maximum value at ±45 degrees and minimum at 0 and ±90 degrees. No variation is found in Q_{13} with respect to fiber angle. A parabolic trend is also observed in Q_{14} with positive and negative maximum values at ± 30 degrees respectively.

Variation of non-diagonal elements of second row of the stiffness matrix with respect to the fiber angle is shown in Fig. 4. No variation of Q_{23} is observed with respect to θ. The value of Q_{24} increases up to 60 degrees of θ and later decreases and similar trend is observed in negative direction with negative values.

Variation of non-diagonal elements of third and fifth rows of the stiffness matrix with respect to the fiber angle is shown in Fig. 5. A parabolic trend is observed for Q_{34} with a maximum value at 45 degrees. Q_{56} is varying in a similar fashion but with reverse trend and higher magnitude.

Table 1 : (θ = 0°)

$\bar{Q}_{ij} / 10^3$	1	2	3	4	5	6
1	136.57	4.0633	4.0633	0.0	0.0	0.0
2	4.0633	11.541	4.2631	0.0	0.0	0.0
3	4.0633	4.2631	11.541	0.0	0.0	0.0
4	0.0	0.0	0.0	4.4200	0.0	0.0
5	0.0	0.0	0.0	0.0	3.6390	0.0
6	0.0	0.0	0.0	0.0	0.0	4.4200

Table 2 : (θ =30°)

$Q_{ij} / 10^3$	1	2	3	4	5	6
1	82.382	26.996	4.1132	40.310	0.0	0.0
2	26.996	19.867	4.2131	13.830	0.0	0.0
3	4.1132	4.2131	11.541	-0.0865	0.0	0.0
4	40.310	13.830	-0.0865	27.353	0.0	0.0
5	0.0	0.0	0.0	0.0	3.8343	0.33818
6	0.0	0.0	0.0	0.0	0.33818	4.2248

Table 3 : (θ =45°)

$Q_{ij} / 10^3$	1	2	3	4	5	6
1	43.480	34.640	4.1632	31.258	0.0	0.0
2	34.640	43.480	4.1632	31.258	0.0	0.0
3	4.1632	4.1632	11.541	-0.09988	0.0	0.0
4	31.258	31.258	-0.09988	34.997	0.0	0.0
5	0.0	0.0	0.0	0.0	4.0295	0.3905
6	0.0	0.0	0.0	0.0	0.3905	4.0295

Table 4 : (θ =60°)

$Q_{ij} / 10^3$	1	2	3	4	5	6
1	19.867	26.996	4.2131	13.830	0.0	0.0
2	26.996	82.382	4.1132	40.310	0.0	0.0
3	4.2131	4.1132	11.541	-0.0865	0.0	0.0
4	13.830	40.310	-0.0865	27.353	0.0	0.0
5	0.0	0.0	0.0	0.0	4.2248	0.33818
6	0.0	0.0	0.0	0.0	0.33818	3.8343

Table 5 : (θ =90^o)

Q_{ij} / 10^3	1	2	3	4	5	6
1	11.541	4.0633	4.2631	0.0	0.0	0.0
2	4.0633	136.57	4.0633	0.0	0.0	0.0
3	4.2631	4.0633	11.541	0.0	0.0	0.0
4	0.0	0.0	0.0	4.420	0.0	0.0
5	0.0	0.0	0.0	0.0	4.420	0.0
6	0.0	0.0	0.0	0.0	0.0	3.639

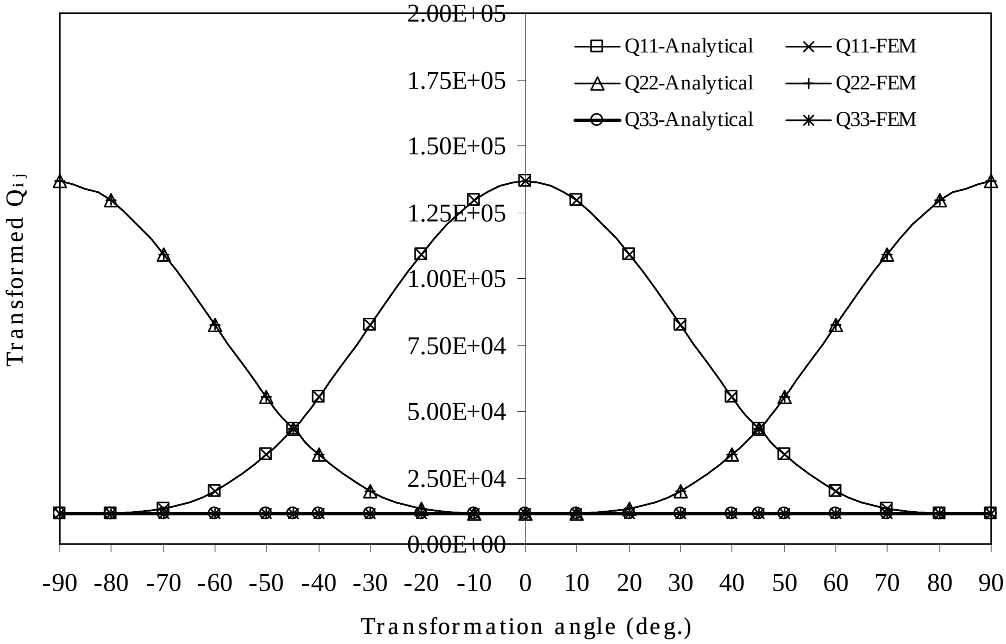

Figure 1 : Variation of first three diagonal elastic constants with respect to θ

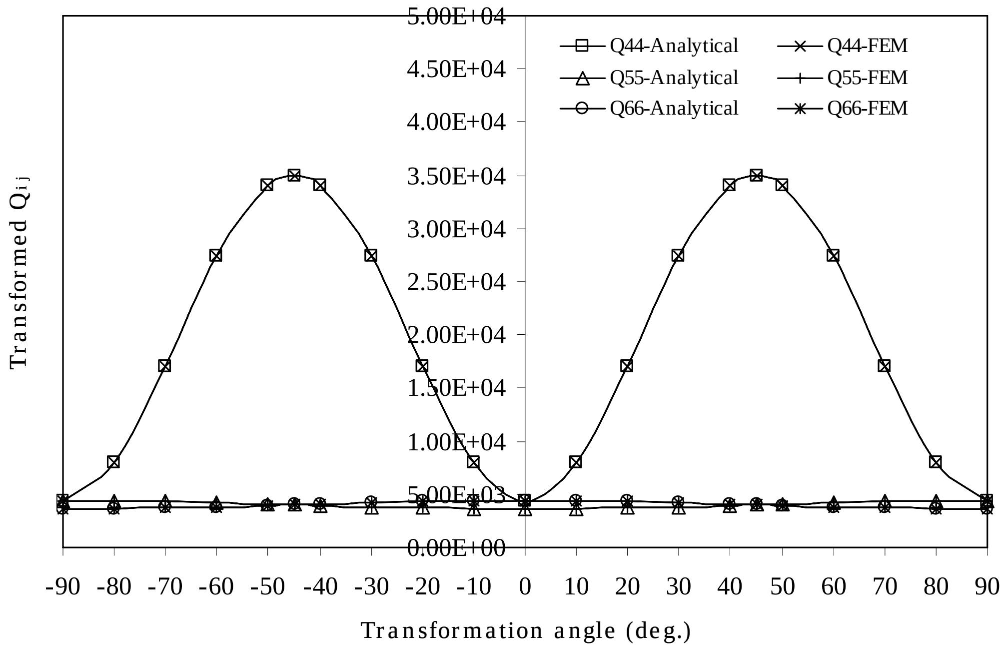

Figure 2 : Variation of last three diagonal elastic constants with respect to θ

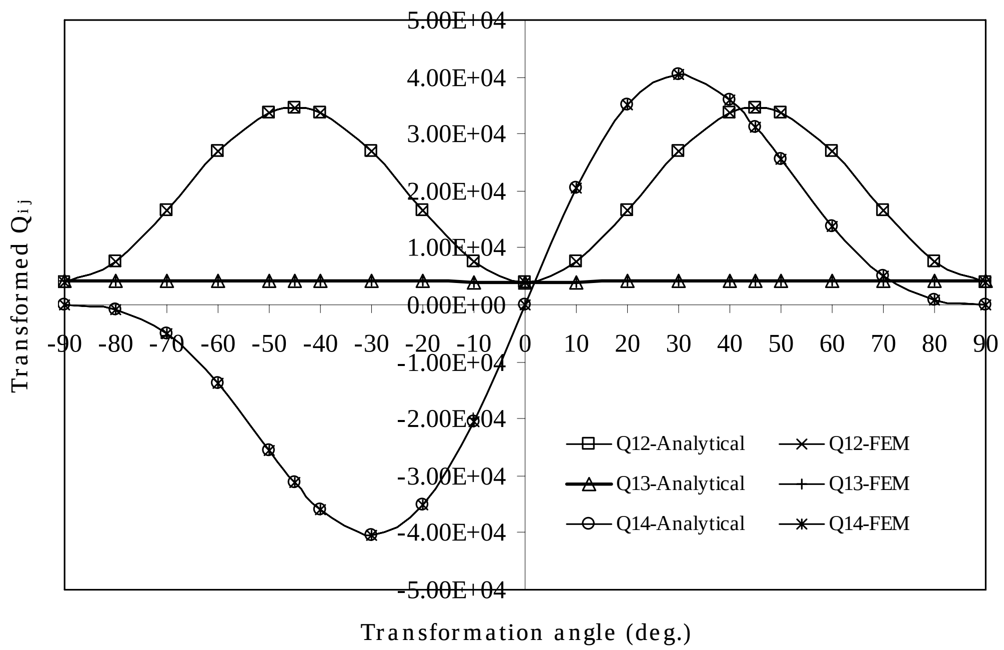

Figure 3 : Variation of non-diagonal elastic constants of first row with respect to θ

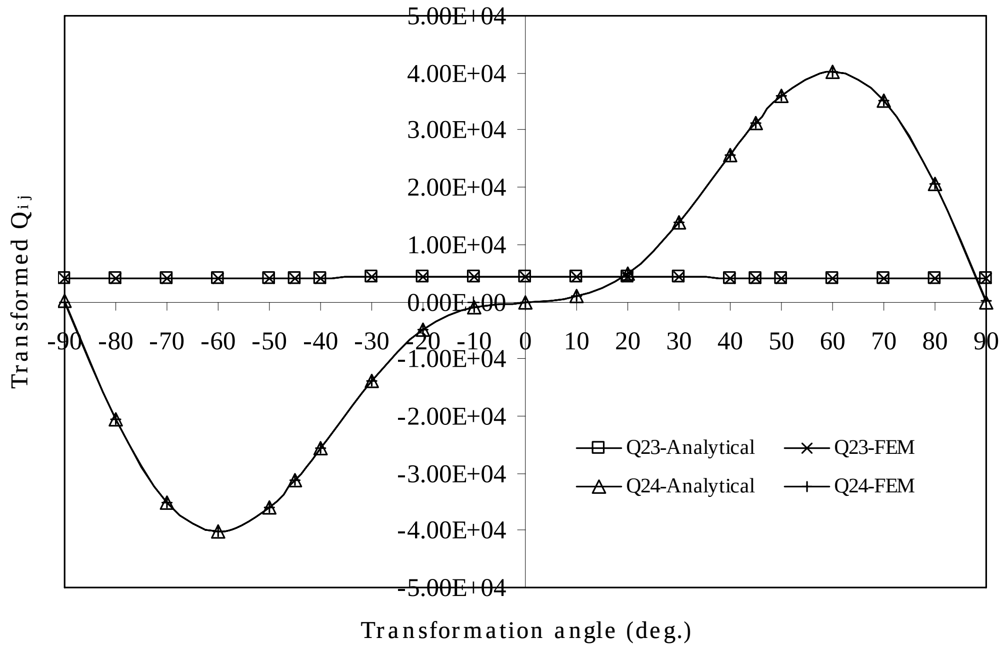

Figure 4 : Variation of non-diagonal elastic constants of second row with respect to θ

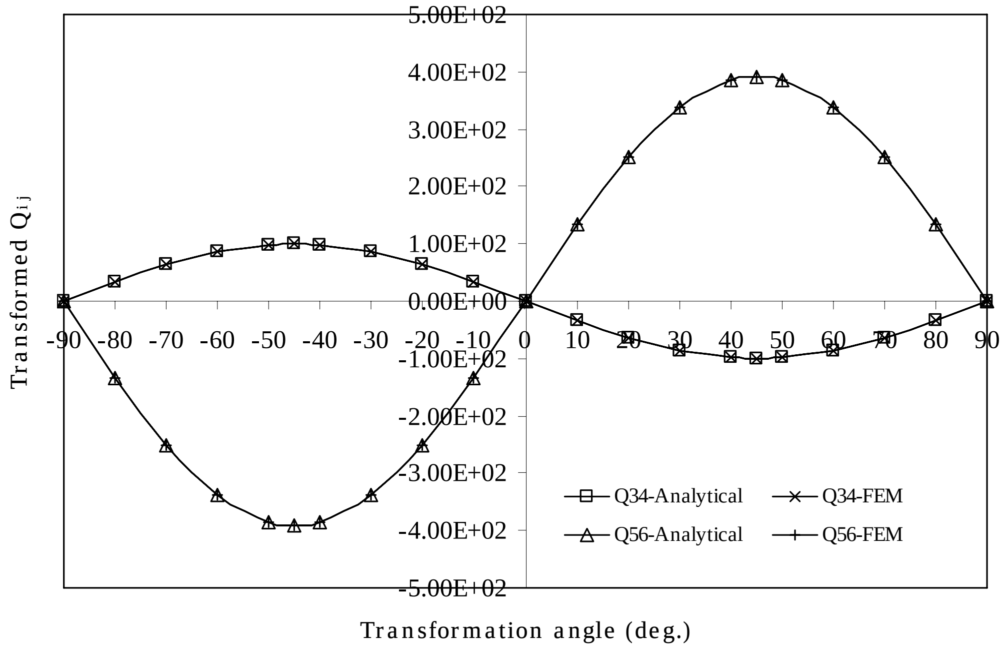

Figure 5 : Variation of non-diagonal elastic constants of third and fifth rows with respect to θ

Conclusions

Six simplified 3-D finite element models are developed for the prediction of elastic constants (elements of stiffness matrix) for unidirectional continuous fiber reinforced specially and generally orthotropic T300-epoxy lamina. The following conclusions are drawn from the present analysis.

- The results of the present work are in very close agreement with the analytical results.
- The specially orthotropic lamina behaves as transversely isotropic material.
- The generally orthotropic lamina behaves as a monoclinic material.
- The generally orthotropic lamina with equal fiber angle with respect to x- and y- axes behaves as a special case of monoclinic material with 9 independent elastic constants.

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