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Analysis of Thick Skew Laminate with Elliptical Cutout Subjected to Non-Linear Temperature Distribution: Major Axis of Ellipse Collinear with Longer Diagonal

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Abstract

The thermoelastic behaviour of a cross-ply laminated composite skew plate with elliptical cutout subjected to non-linearly varying temperature loading has been investigated in the present analysis. Major axis of the ellipse collinear with longer diagonal of the skew plate has been taken for the present analysis. A finite element method which works on the basis of three-dimensional theory of elasticity is employed to evaluate the transverse deflection, in-plane stresses and interlaminar stresses. The results obtained by varying the skew angle and size of the cutout are discussed. The magnitudes of the transverse deflection and in-plane stresses for temperature loading are observed to be less at higher skew angles. It is observed that the values of inter-laminar stresses are observed to be minimum at lower d/l ratios (d/l = 0.1). The solutions of skew structures considered in the present analysis will be useful for the construction of safe and efficient structures like skew bridges and swept wings of aircraft structures.

Keywords: FEM, Skew Laminate, Cutout, Interlaminar stresses.

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Nomenclature

E_1 = Young's modulus of the lamina in the fibre direction
$E_2 = E_3$ = Young's modulus of the lamina in the transverse direction of the fibre
$G_{12} = G_{13}$ = Shear modulus in the longitudinal plane of the fibre
G_{23} = Shear modulus in the transverse plane of the fibre
$\nu_{12} = \nu_{13} =$ Poisons ratio in the longitudinal plane of the fibre
ν_{23} = Poisons ratio in the transverse plane of the fibre
α_1 = Coefficient of thermal expansion in the fiber direction
$\alpha_2 = \alpha_3$ = Coefficient of thermal expansion in the transverse direction of the fiber
EL = Exact elasticity solution
FE = Finite element solution

$$\text{Normalized } \boldsymbol{\sigma} = \frac{\sigma}{p_0 s^2}, \text{ Normalized } \boldsymbol{\tau} = \frac{\tau}{p_0 s}, \text{ Normalized } \mathbf{w} = \frac{100 E_2 w}{p_0 h s^4}$$

s = Length of the plate (l) / thickness of the plate (h)
$p_0 =$ The maximum intensity of sinusoidal load
a = Length and width of the square plate
$p_2 =$ Major axis of the ellipse collinear with longer diagonal of the skew plate.
1/2 and 1/3 are the normalized positions along the thickness direction

(Normalized $z = 2z / h$, z coordinate measured from middle plane of the plate and h=total thickness of the plate)

Introduction

The increasing use of fibre reinforced laminates in space vehicles, aircrafts, automobiles, ships and chemical vessels have necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy and non-homogeneity and larger ratio of longitudinal to transverse modulii of these new materials demand improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like (i) for the passage of various cables, (ii) for undertaking maintenance work and (iii) for fitting auxiliary equipment. Depending upon the nature of application, these structural elements are acted upon by mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending, and shear deformation modes. To capture the full mechanical behavior, it must be described by three dimensional elasticity theories.

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches have been proposed. Srinivas and Rao [1] and Srinivas *et al*. [2] presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. Zhang and Zhang [3] presented a new concise procedure for obtaining the static exact solution of composite laminates with piezo-thermo-elastic layers under cylindrical bending using the basic coupled thermo-electro-elastic differential equations. Setoodeh and Karami [4] employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Pagano *et al.*[5] has given exact solutions for the deflections and stresses of a cross- ply laminated rectangular composites without holes using elasticity theory. Kong and Cheung[6] proposed a displacement-based, three-dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a three-dimensional inhomogeneous anisotropic elastic body. Prasad and Shuart [7] presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Ukadgaonker *et al.*[8] gave a general solution for bending of symmetric laminates with holes. Morley *et al.*[9] developed an elementary bending theory for the small displacements of initially flat isotropic skew plates without hole. Karami *et al.*[10] has applied Differential Quadrature Method (DQM) for static, free vibration, and stability analysis of skewed and trapezoidal composite thin plates without hole. From the review of available literature it is observed that the static analysis of skew plates with cutouts using elasticity theory has not been studied. The behavior of a laminate with skew edges and having various types of cutouts is different from the one without skew edges and/or cut outs. So it is necessary to analyse this kind of problem using elasticity theory based finite element method to evaluate for the most accurate behaviour of thick laminated skew plates with cutouts.

Skew Laminate

The term ‘skew’ in skew laminate refers to oblique, swept or parallelogram. In case of skew plate the angle between the adjacent sides of the plate is not equal to 90^0. If opposite sides of the plate are parallel, it becomes a parallelogram and when their lengths are equal, the plate is called a rhombic plate. In the present analysis a rhombic laminated plate is considered by varying the skew angle from 0^0 to 50^0 as shown in Fig.1.

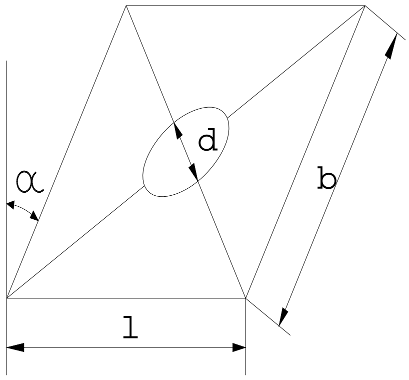


Figure 1 : Skew laminated composite plate with major axis of elliptical cutout collinear with longer diagonal

Problem Statement

The research problem deals with the thermoelastic analysis of thick skew laminated plate with elliptical cutout by elasticity theory based on finite element method.

Problem Modeling

Geometric modeling

The Figure.1 shows the in-plane dimensions of the laminate considered for the present analysis. The dimensions for ‘l’ and ‘b’ are taken as 20mm. d is the length of the minor axis of the ellipse. Major axis of the ellipse is taken as twice the length of the minor axis.

The value of *d* is determined from the ratio of *d/l* which is varied from 0.1 to 0.4, and the skew angle α is varied from 0^0 to 50^0, the thickness of the plate is fixed from the length to thickness ratio *l/h* (*s* =10). The individual layers are arranged so that the total thickness of the layers oriented in x- direction ($\theta = 0^0$) is equal to the total thickness of the layers oriented in y- direction ($\theta = 90^0$).

Finite Element Modeling

The finite element mesh is generated using a three dimensional brick elements ‘SOLID90’and ‘SOLID 95’of ANSYS [11]. The 20 node thermal element is applicable to a steady state or transient thermal analysis. The elements consists of has 20 nodes and temperature degree of freedom for ‘SOLID90’and X, Y, and Z directional displacement for ‘SOLID 95’. ‘SOLID90’ and ‘SOLID 95’ have compatibility to transfer the temperatures from thermal analysis to structural analysis This element (Fig.2) is a structural solid element designed based on three dimensional elasticity theory and is used to model thick orthotropic solids.

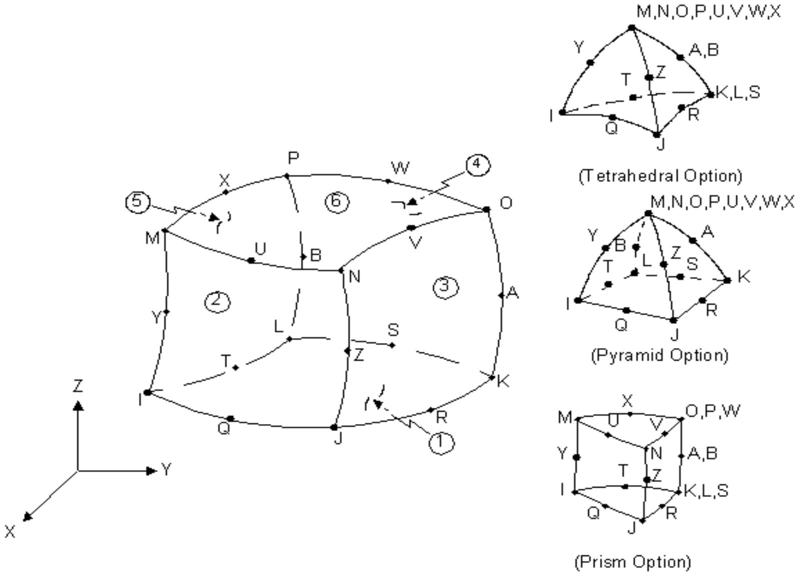


Figure 2 : SOLID95 Element

Boundary conditions

Thermal

A temperature of 100^0 C on the top face and 25^0 C on the bottom and side faces is applied. The surface of the hole is subjected to convection with film coefficient h = 5 and bulk temperature 25^0 C.

Structural

All the edges of the skew plate are clamped i.e. all the three degrees of freedom (Displacements in global x-, y- and z- directions) of the nodes attached to the side faces of the plate are constrained.

Loading

i) The output from the thermal analysis is applied as thermal loading.

Material Properties (Graphite-Epoxy)

$K_L = 36.42$ W/m K $K_T = 0.96$ W/m K
$E_1 = 172.72$ GPa, $E_2 = E_3 = 6.909$ GPa
$G_{12} = G_{13} = 3.45$ GPa, $G_{23} = 1.38$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$
$\alpha_1 = 0.57 \times 10^{-6}\ /^0C$ $\alpha_2 = \alpha_3 = 35.6 \times 10^{-6}\ /^0C$

Validity of the Present Analysis

To validate the finite element results, a square plate with simply supported edges and subjected to a sinusoidal load of $p = p_0 \sin(\pi x/a) \sin(\pi y/b)$, where a and b are the length and width of the plate, is modeled with SOLID95 element. The results obtained from this model are compared with the exact elasticity solution [5] for various lengths to thickness ratios of the plate (Table 1). It is observed that the finite element results are in close agreement with the exact elasticity solution.

Table 1 : Comparison of present work with exact elasticity theory [5]

S = l/h	**Normalized σ_x (a/2,a/2,±1/2)**	**Normalized σ_y (a/2,a/2,±1/3)**	**Normalized τ_{yz} (0,a/2,0)**	**Normalized τ_{zx} (a/2,0,0)**	**Normalized w (a/2,a/2,0)**
10	EL 0.545 -0.545 FE 0.537 -0.536	EL 0. 430 -0.432 FE 0.431 -0.431	EL 0.223 FE 0.209	EL 0.258 FE 0.212	EL 0.677 FE 0.692
20	EL 0.539 -0.539 FE 0.534 -0.535	EL 0.380 -0.380 FE 0.377 -0.378	EL 0.212 FE 0.218	EL 0.268 FE 0.271	EL 0.4938 FE 0.4838

Table 1. Presents the results of a square laminate (‘a’ = ‘b’). The location for maximum σ_x i.e. (a/2, a/2, ± 1/2) is at the centre and at top and bottom faces of the laminate. Maximum value of σ_y is observed at the centre and at the interface of outer and its adjacent layers (a/2, a/2, ±1/3) of the laminate. The value of the shear stress τ_{yz} is taken at the mid point of one of the vertical sides and at the neutral surface of the laminate (0, a/2, 0). The value of the shear stress τ_{zx} is taken at the mid point of one of the horizontal sides and at the neutral surface of the laminate (a/2,0,0) and the transverse deflection is obtained at geometric centre of the laminate (a/2,a/2,0).

In the present work the transverse deflection and stresses (including the inter-laminar stresses at the free edge of the elliptical cutout) of a clamped skew laminated plate with elliptical cutout at the centre of the plate and subjected to a non-linearly varying temperature loading is evaluated by varying the size of the elliptical cutout and skew angle.

Results and Discussion

Numerical results are obtained for temperature loading as mentioned above. Variation of the stresses and deflection with respect to the skew angle (α) for different d/l ratio’s is shown in Figs. 3-9. The following observations are made.

Effect of Skew Angle

The in-plane normal stress, σ_x (Fig.) decreases with increase in skew angle. The increase in skew angle increases the length of the longer diagonal and decreases the length of the shorter diagonal of the skew plate. The first factor (increase in the length of the longer diagonal) increases the flexibility of the plate where as the second factor (decrease in the length of the shorter diagonal) increases the stiffness of the plate. The reduction in the stress σ_x is due to the domination of stiffness effect. (Fig).

The In plane normal stress σ_y increases up to $\alpha = 20^0$ and then decreases for all d/l ratios except for d/l = 0.4. Here the flexibility effect is dominating up to $\alpha = 20^0$, from $\alpha = 20^0$ onwards the stiffness factor is dominating For d/l = 0.4 this stress increases up to $\alpha = 10^0$ and then decreases. (Fig).

The in-plane shear stress, τ_{xy} decreases with increases in skew angle for all the d/l ratios (Fig).

The inter-laminar normal stress, σ_z at the free edge of the cutout decreases with increase in skew angle up to $\alpha = 20^0$ and then increases for d/l = 0.1. For d/l = 0.2 this stress decreases with increase in skew angle α up to $\alpha = 40^0$ and then increases. For other two d/l ratios this stress decreases with increase in skew angle.

The inter laminar shear stress, τ_{yz} at the free edge of the cutout slightly increases with increase in skew angle α for d/l = 0.1. For d/l = 0.2 this stress increases up to $\alpha = 10^0$ and then decreases up to $\alpha = 20^0$ and again increases up to $\alpha = 40^0$ and then decreases. For d/l = 0.3 this stress increases up to $\alpha = 40^0$ and then decreases. For d/l = 0.4 this stress increases up to $\alpha = 30^0$ and then decreases (Fig.).

The inter laminar shear stress, τ_{zx} at the free edge of the cutout decreases with increase in skew angle up to $\alpha = 30^0$ and then increases for d/l = 0.1 in case of thermal loading. For d/l = 0.2 there is no significant variation in this stress with respect to α. For d/l = 0.3 this stress increases with increase in skew angle. For this stress increases up to $\alpha = 40^0$ and then decreases. However the variation is very small when compared to the magnitude of the in-plane stresses. (Fig)

The transverse deflection 'w' decreases due to variation in skew angle for all d/l ratios in case of thermal loading.). The reduction in '*w*' with respect to α may be due to the domination of the stiffness factor (Fig.).

Effect of d/l ratio

When the size of the ellipse increases, the area of the cutout boundary increases, providing more scope for free expansion of the plate. Due to this factor, the stresses will decrease. At the same time, the resisting volume of the material decreases and as a result the induced stresses will increase. The resultant effect of these factors is discussed below.

The in-plane normal stress σ_x increases with increase in d/l ratio up to d/l = 0.3 and then decreases for all values of α except for $\alpha = 10^0$ and 50^0.the second factor (decrease in resisting volume) is dominating up to d/l = 0.3 and later on the first factor (increase in area of hole boundary) is dominating. For $\alpha = 10^0$ and 50^0 this stress increases with increase in d/l ratio up to d/l = 0.2 and then decreases (Fig.).

The in-plane normal stress σ_y increases with increase in d/l ratio up to d/l = 0.2 and then decreases for all values of α except for $\alpha = 50^0$. For $\alpha = 50^0$ this stress decreases with increase in d/l ratio (Fig.).

The in-plane shear stress τ_{xy} increases with increase in d/l ratio up to d/l = 0.2 and then decreases for all values of α except for $\alpha = 40^0$ and 50^0 . For $\alpha = 40^0$ and 50^0 this stress decreases with increase in d/l ratio (Fig.).

The inter-laminar normal stress σ_z at the free edge of the cut out increases with increase in d/l ratio for all values of skew angle except for $\alpha = 40^0$. For $\alpha = 40^0$ this stress decreases up to d/l = 0.2 and then increases (Fig.).

The inter-laminar shear stresses τ_{yz} and τ_{zx} at the free edge of the cut out increase with increase in d/l ratio for all values of skew angle. The forces causing the inter-laminar stresses form in couples to balance the forces for equilibrium. When the size of the cutout increases, the moment arm of these forces decreases and this may be the reason for increase in inter-laminar stresses (Figs.).

The transverse deflection 'w' decreases due to increase in d/l ratio for all values of skew angle (Fig.).

Conclusions

Thermoelastic analysis of a thick laminated composite skew plate with an elliptical cutout at the centre of the plate has been carried out in the present work. The transverse deflection, maximum in-plane stresses and maximum interlaminar stresses at the free edge of the cutout have been evaluated using three-dimensional theory of elasticity based finite element analysis. The results obtained for non-linearly varying temperature loading are analyzed for the variation of skew angle of the plate, size of the ellipse. The magnitudes of the in-plane normal stresses and the transverse deflection due to temperature loading are greatly affected by the skew angle variation and their magnitudes are observed to be minimum at higher value of the skew angle.

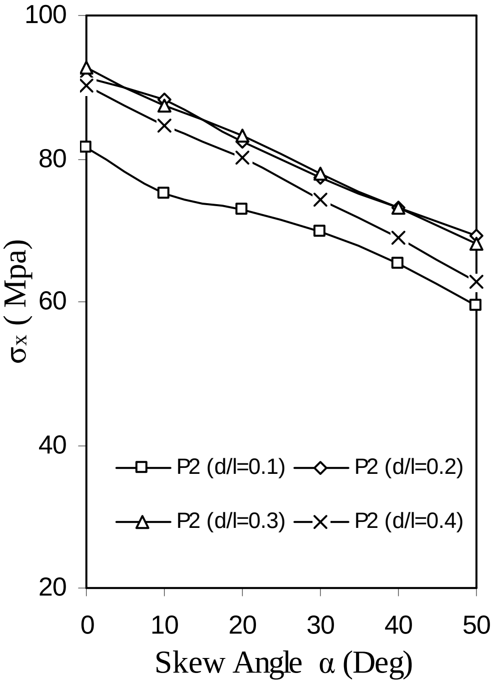


Figure 3 : Variation of σ_x with respect to α

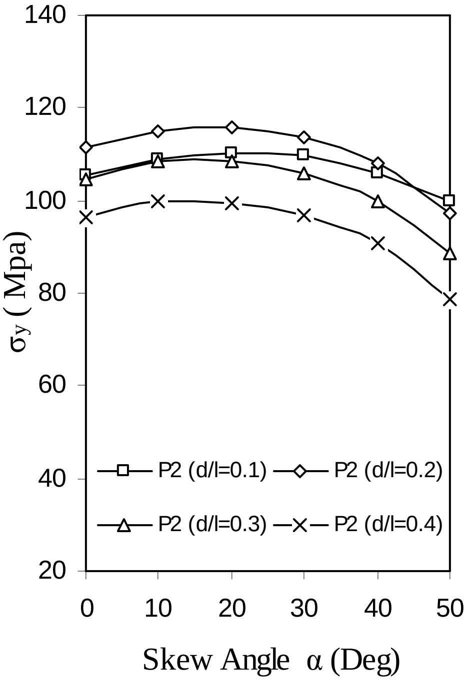


Figure 4 : Variation of σ_y with respect to α

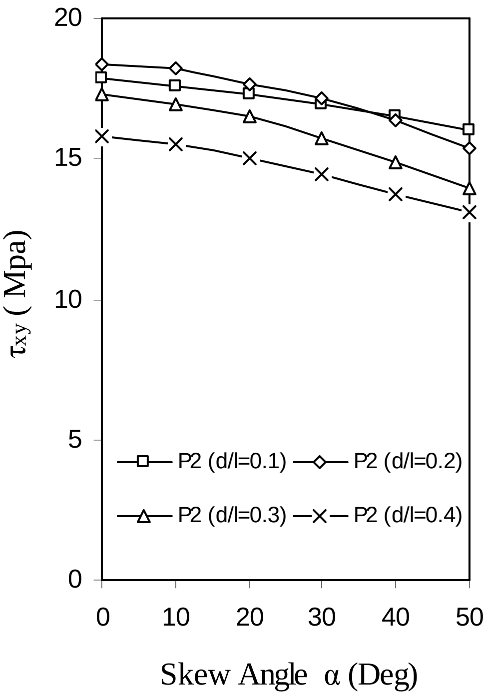


Figure 5 : Variation of τ_{xy} with respect to α

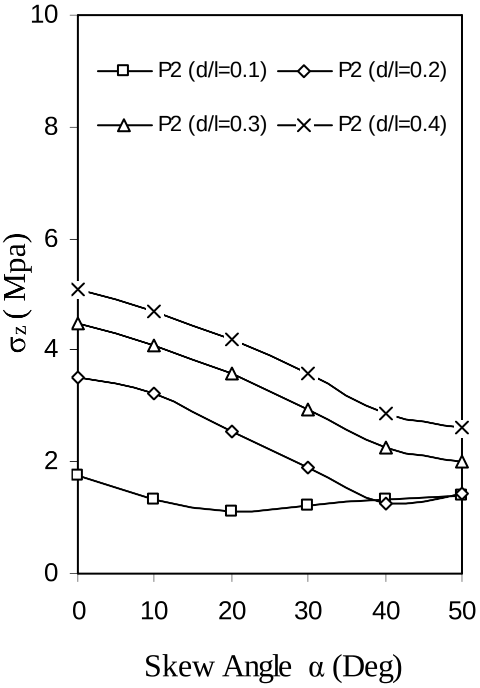
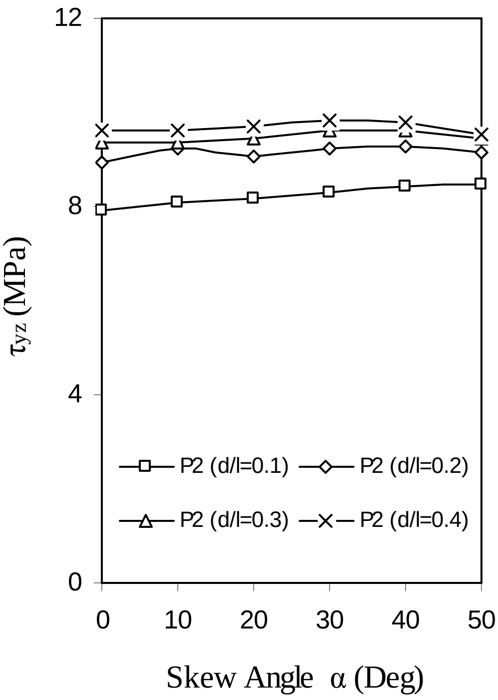


Figure 6 : Variation of σ_z with respect to α



Fiugure 7 : Variation of τ_{yz} with respect to α

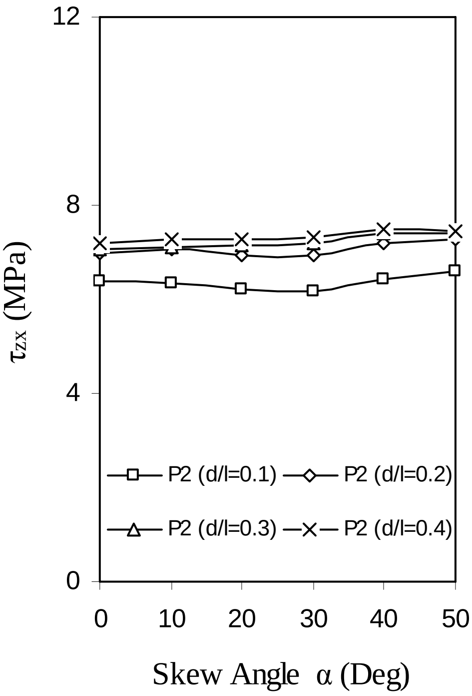


Figure 8 : Variation of τ_{zx} with respect to α

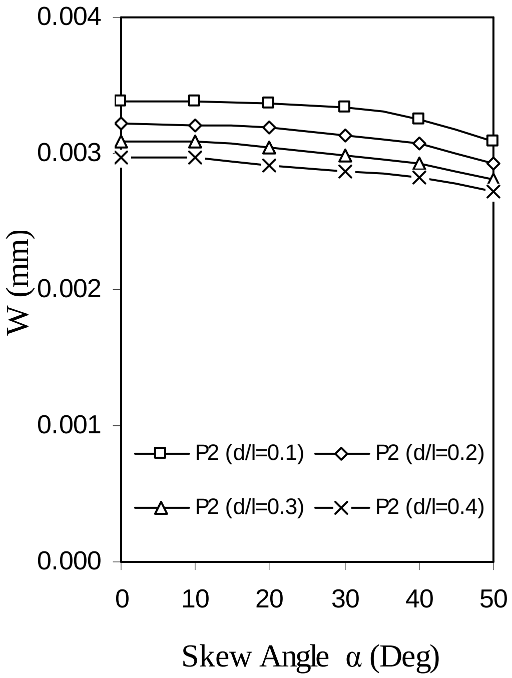


Figure 9 : Variation of '*w*' with respect to α

References

[1] Srinivas, S. and Rao, A.K. "Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates". *Int. J. Solids Structures*, vol.6, 1970, pp.1463-81.

[2] Srinivas, S., Rao, C.V. and Rao, A.K. "An exact analysis for vibration of simply supported homogeneous and laminated thick rectangular plates". *J. Sound Vibrations*, vol.12, 1970, pp.187-99.

[3] Zhang, C., Di, S. and Zhang, N. "A new procedure for static analysis of thermo-electric laminated composite plates under cylindrical bending". *Composite Structures*, vol.56, 2002, pp.131-40.

[4] Setoodeh, A. R. and Karami, G. "Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3-D layer-wise FEM". *Engineering Structures*, vol. 26, 2004, pp. 211-20.

[5] Pagano, N.J. and Hatfield, S.J. "Elastic behavior of multilayered bidirectional composites". *AIAA Journal*, vol.10, 1972, pp.931 – 33.

[6] Kong J. and Cheung, Y. K. "Three-dimensional finite element analysis of thick laminated plates". *Computers and Structures*, vol. 57, 1995, pp.1051-62.

[7] Prasad, C.B. and Shuart, M.J. “Moment distributions around holes in symmetric composite laminates subjected to bending moments”. *AIAA Journal*, vol.28, 1990, pp.877-82.

[8] Ukadgaonker, V.G. and Rao, D.K.N. “A general solution for moments around holes in symmetric laminates”. *Comp. Structures*, vol.49, 2000, pp.41-54.

[9] Morley, L. S. D. “Skew Plates and Structures. Division I: Solid and Structural Mechanics. *Int. Series of Monographs on Aeronautics and Astronautics*”, Pergamon Press, NewYork, 1963.

[10] Karami, G, Shahpari, S.A and Malekzadeh, P. “DQM analysis of skewed and trapezoidal laminated plates”. *Comp. Structures*., vol.59, 2003, pp.393-402.

[11] ANSYS Reference Manual 2006.