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Mathematical Characterization of the Signals that Determine the Erosion by Cavitation[1]

Alina Barbulescu
Ovidius University of Constanţa, Romania
E-mail: alinadumitriu@yahoo.com

Camelia Ciobanu
Mircea cel Batran Naval Academy, Constanţa, Romania

Abstract

The results presented in this paper are a part of our study on the relation between the character of the electrical signals produced by cavitation in different liquids and their erosion effect produced on the copper alloys. In order to try to make a classification of the electrical signals induced at the boundary of a cavitation zone, we study the long time dependence and the Box dimension of the graph of these signals, collected in water.

AMS Subject Classification:
Keywords:

1. Introduction

Cavitation is a dynamical process of apparition, development and collapse of some bubbles or cavities that contain vapours or gases in the liquid mass. The collapse/rebound cycle results in such effects as: pressure oscillations with frequency different from that of the stimulating ultrasonic field, sonoluminescence and rectified diffusion of gas dissolved in the liquid, emulsification of multiphase media and chemical reactions in cavitating liquids.

The erosion and unpassivation of solid boundary surfaces are phenomena of undisputable technical significance [1].

The phenomena related to the mechanical erosion produced by cavitation are of major importance point of view of the machine components working in cavitation conditions, in different media. Some of them are made of copper alloys.

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In our studies, the cavitation was produced using an ultrasound generator of I.U.S. - 150 type working at: 80 W, 120 W and 180 W [2].

The signals analyzed in this paper were collected by the measurement electrodes introduced in the water put in the generator bulk.

2. Mathematical Background

Definition 2.1. [3] Let β be a positive number and E a nonempty subset of a metric space (X, d). Let N_β be the smallest number of sets of diameter at most β that cover E. The *upper and lower Box dimensions* of E are defined respectively by:

$$\overline{\dim}_B E = \overline{\lim_{\beta \to 0}} \frac{\log N_\beta (E)}{-\log \beta}; \quad \underline{\dim}_B E = \underline{\lim_{\beta \to 0}} \frac{\log N_\beta (E)}{-\log \beta}.$$

If these limits are equal, the common value is called *Box dimension* of E and is denoted by $\dim_B E$.

In what follows, we shall work in the hypotheses: $X = \mathbb{R}^2$, d - the Euclidean metric, E - the graph of the captured signal (Ox is the time axis (in μs) and Oy - the amplitude (in volts) of the signal axis).

Definition 2.2. [4] A time series $(X_t)_{t\in\mathbb{N}^*}$ is called *weakly stationary* if it has a finite mean and the covariance depends only on the lag between two points in the series.

Definition 2.3. [5] The *autocorrelation function* of the weakly stationary time series $(X_t)_{t\in\mathbb{N}^*}$ is defined by:

$$\rho(k) = \frac{E[(X_t - \mu)(X_{t+k} - \mu)]}{\sigma^2}$$

where: $E(X_t)$ is the expectation, μ - the mean and σ^2 - the variance of $(X_t)_{t\in\mathbb{N}^*}$.

Definition 2.4. [4] A time series has the *long range dependence* (LRD) property if the series $\sum_{k=-\infty}^{+\infty} \rho(k)$ diverges (*i.e.*, it has correlations that persist over all time scales).

The rescaled range (R/S) method allows the calculation of self-similarity parameter H which measures the intensity of LRD in a time series. To study the LRD in a time series, the following algorithm is used [6].

A time series of L data $(X_i)_{i\in\overline{1,n}}$ is divided into d sub-series of length n. For each sub-series $m = \overline{1,n}$:

– Find the mean (E_m) and the standard deviation (S_m);

– Normalize the data (Z_m) by subtracting the sub-series mean, *i.e*,

$$Z_{im} = X_{im} - E_m, \quad i = \overline{1,n};$$

– Create a cumulative time series:

$$Z_{im} = \sum_{j=1}^{i} X_{jm}, \quad i = \overline{1, n};$$

– Find the range

$$R_m = \max_{j=\overline{1,n}} Y_{jm} - \min_{j=\overline{1,n}} Y_{jm};$$

– Rescale the range, *i.e.*, calculate R_m/S_m;

– Calculate the mean value $(R/S)_n$ of the rescaled range for all sub-series of length n, *i.e.*,

$$(R/S)_n = \frac{1}{d} \sum_{m=1}^{d} R_m/S_m.$$

Remark 2.5. It can be shown that R/S statistic asymptotically follows the relation:

$$(R/S)_n \sim cn^H. \tag{2.1}$$

H is called *Hurst coefficient* and its value can be determined by running a simple linear regression over a sample of increasing time horizons:

$$\log(R/S)_n = \log c + H \log n.$$

Equivalently, we can plot the statistics against n on a double logarithmic paper. If the process is a white noise, then the plot is roughly a straight line with slope 0.5. If the process is persistent, the slope is greater than 0.5 [4].

3. Results and Discussions

The signals were collected in water, when the generator works at 80 W, 120 W and 180 W. The analysis was made on 100 signals, captured on different time intervals. The forms of some signals are presented in Figs. 1–3.

3.1. In order to determine the Box dimension value of a graph of a signal, a program was built. It requires, as entrance data, the time interval and the voltage values. The Box - dimension estimation for graphs of six signals captured on different time periods and at different power levels of the ultrasound generator (1 and 4 — at 80 W, 2 and 5 — at 120 W, 3 and 6 — at 180 W) are presented in Table 1.

The following remarks can be made:

i. For a given signal, the estimated Box dimension does not depend on the length of the time interval.

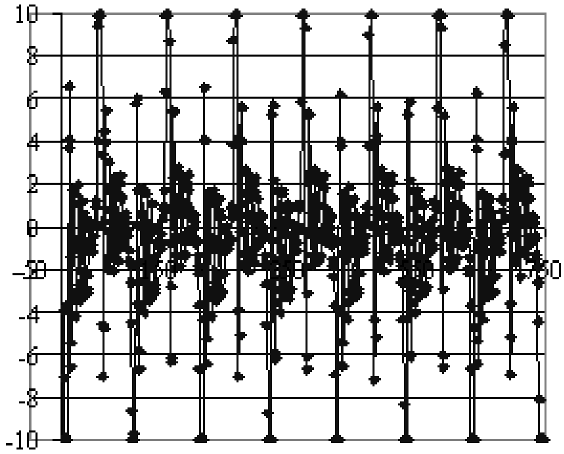

Figure 1: The signal 1 in water, at 80 W.

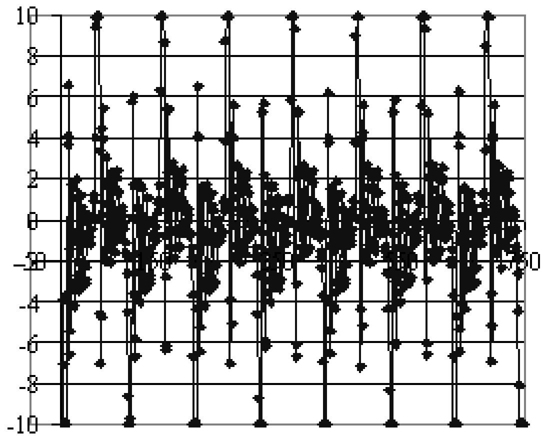

Figure 2: The signal 2 in water, at 120 W.

ii. The estimated Box dimension does not vary too much function on the power of the generator.

iii. The estimated Box dimension does not vary too much as a function of the temperature.

3.2. The study of the long range dependence was made on the same signals. The R/S method was used. The calculus of Hurst's coefficient is exemplified in Figs. 4–6, for the signals 1–3.

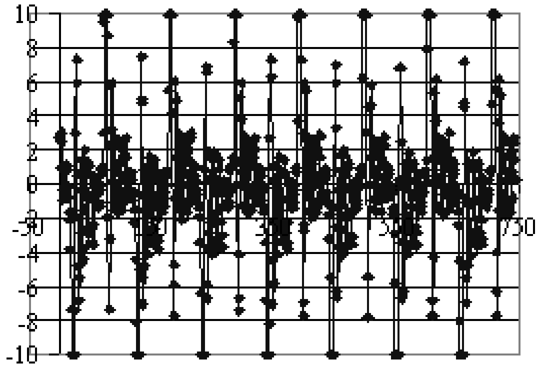

Figure 3: The signal 3 in water, at 180 W.

Table 1: The Box dimension estimation.

signal	temperature	750 μs	784 μs	1500 μs	2250 μs	3000 μs	3750 μs
1	20	1.010015	1.012728	1.013738	1.013892	1.014021	1.014035
5	20	1.010118	1.010181	1.011111	1.011682	1.012080	1.012383
3	20	1.010182	1.010259	1.011181	1.011776	1.012192	1.012509
4	24	1.005594	1.005652	1.006612	1.007175	1.007585	1.007907
5	24	1.005005	1.005080	1.006035	1.006601	1.007002	1.007323
6	24	1.005016	1.005095	1.006045	1.006608	1.007012	1.007336

The regression equations are respectively:

$$\begin{aligned}\log(R/S)_n &= 0.0151 + 0.612 \log n,\\ \log(R/S)_n &= -0.0216 + 0.6487 \log n,\\ \log(R/S)_n &= -0.0151 + 0.6421 \log n.\end{aligned}$$

The values of Hurst's coefficients, H (see (1)) and those of the determination coefficients, R^2 are given in Table 2.

By definition, the determination coefficient is between zero and one. The model is better if the determination coefficient is close to 1.

For the signals 1–3 Hurst's coefficients are around 0.6 and the determination coefficients are big, so the slopes in the regression models are well determined. In the case of the signals 4–6, the determination coefficients are around 0.8, but we still can say that the slopes in the regression equations are well determined.

Since in all the cases Hurst's coefficients are greater than 0.5, the signals have the LRD property. At each power stage of the ultrasound generator, the values of Hurst's coefficients are different, function of the temperature. For example, at 80 W: at 20^0C, $H = 0.612$ and at 24^0C, $H = 1.075$.

3.3. In order to determine the common characteristics of the signals, normality and break tests (Buisard, Pettitt, Lee and Heginian, Hubert) were also done.

Table 2: Hurst's and determination coefficients.

signal	1	2	3	4	5	6
H	0.612	0.6487	0.6421	1.075	1.150	1.200
R^2	0.9316	0.9444	0.9366	0.7839	0.7882	0.7889

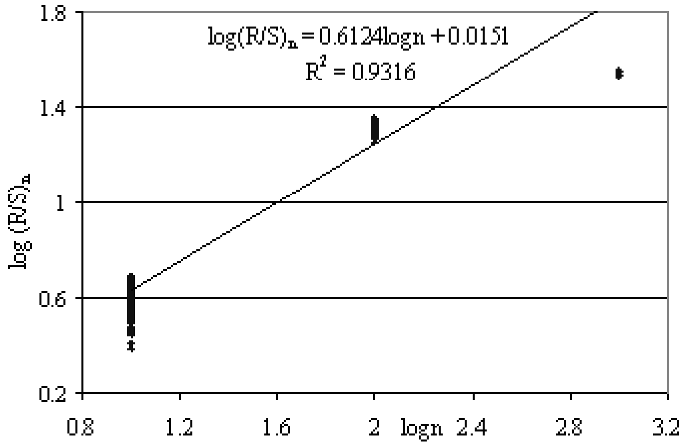

Figure 4: Determination of Hurst's coefficient for the signal 1.

Let us consider the Box–Cox transformation [5]:

$$y_i = \begin{cases} \dfrac{x_i{}^\lambda - 1}{\lambda}, & \lambda > 0, \\ \log(y), & \lambda = 0. \end{cases}$$

Applying it to the time series attached to the signals 2, 3, 6, with the parameter λ respectively -0.37, -0.4, -0.37, then the data are normal. For the other series, the normality could not be reached by transformations.

The results of the break tests were different, from a signal to another. They will be discussed in another article.

4. Conclusions

In this study it was proved that Box dimension of the graph of the signals induced by cavitation in ultrasonic field does not vary too much as a function of the power of the ultrasound generator or of the temperature.

It has been seen that the series of the data collected (the voltage values captured) has the LRD dependence and the value of Hurst's coefficient does not vary too much as a function of the generator power, but it is strongly dependent on the temperature at which the signals were collected.

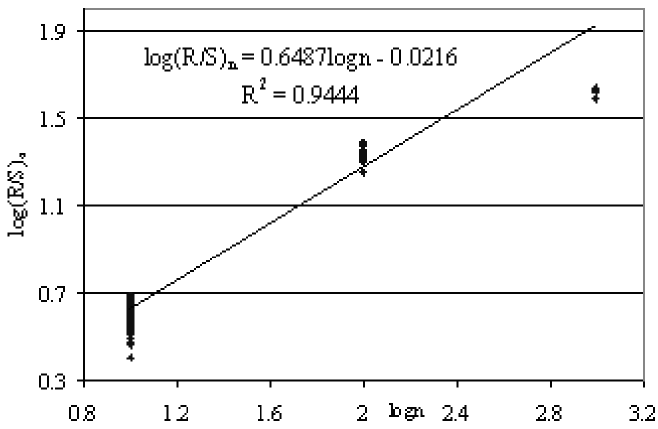

Figure 5: Determination of Hurst's coefficient for the signal 2.

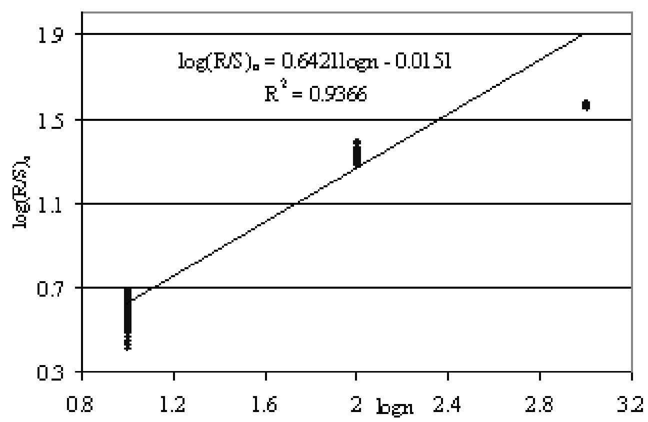

Figure 6: Determination of Hurst's coefficient for the signal 3.

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