International Journal of Mechanics and Solids.
ISSN 0973-1881 Volume 14, Number 1 (2019), pp. 1-6

http://www.ripublication.com/ijms.htm

Stress Analysis of Laminated Composite Plates by Higher Order Theory Using Finite Element Method

Praveen S Mugali
PG Scholar, Department of Civil Engineering,
SDM College of Engineering and Technology, Dharwad, Karnataka, India.

Dr. R. J. Fernandes
Assistant Professor, Department of Civil Engineering,
SDM College of Engineering and Technology, Dharwad, Karnataka, India.

Abstract

Finite Element Analysis is used to perform analysis on a laminated composite plate based on the Higher Order Shear Deformation Theory (HSDT). The theory accounts for variation of displacement &stresses, across the thickness of the laminate. The element formulated is a 9-noded iso-parametric quadrilateral element having 9 degrees of freedom at each node (HSDT9). In this analysis, the square plate is analyzed for uniformly distributed load and sinusoidal loads under simply supported boundary conditions. A program is written in MATLAB to obtain the finite element solutions for displacements & stresses. Solutions are obtained for laminate with cross-ply having different values of side to thickness ratios. The model is validated with the analytical results available in the literature.

Keywords: Stresses, higher order shear deformation theory, laminated composite plates, finite element method, MATLAB programming.

1. INTRODUCTION

In recent years, increased composite materials are being used in many engineering and civil applications, from tennis racket frame to fuselage of airplane. The main reason for extensive use of composite plates is that the high stiffness to weight ratio of composite plates. Due to this increased use of laminated composite plates it is necessary to analyze for stresses that develop in the lamina. Composite plates are used basically in the fields of application where light weighted materials are required with high strength. Earlier classical plate theory (CLPT) was used for analyzing the stresses of the composite plates but it neglected the transverse effect of deformation of shear. But this was taken into account in first order shear deformation theory (FSDT) but it considered uniform shear deformation across thickness of plate. Higher order theories accounted nonlinear shear deformations in thickness direction without correction factor. In recent years, finite element method has become established and powerful numerical technique and calculation tool for analyzing the complex behavior of composite materials. A displacement finite element model of theory is developed and applications to bending, vibrations and stability of laminated plates are discussed and concluded that for the plates with simply supported boundary conditions, exact solutions were obtained when compared with the classical plate theory[1].Finite element formulation of a higher-order theory for flexure of thick arbitrary laminated composite plates under transverse loads is presented with the element having nine nodes and nine degrees of freedom at each nodes. They concluded that shear correction factors are not required in higher order theories [2]. The main aim of the work was to calculate and compare the response of composite plate using first order and higher order shear deformation theories. Mathematical formulation and Matlab coding using First Order Shear Deformation Theory (FSDT) and Higher Order Shear Deformation Theory (HSDT) was done [3]. The proposed element is developed where Reddy's plate theory is successfully implemented. It has four nodes and each node contains 7 degrees of freedom. Element is tested numerically in a wide range of problems covering different boundary conditions, loading, material property and concluded that the element is free from shear locking problem and it does not possess any spurious modes [4]. In this paper stress analysis of laminated composite plates is carried out by finite element analysis of nine noded element having nine degrees of freedom at each node (HSDT9).

2. FORMULATION OF LAMINATED COMPOSITE PLATE

The displacement model developed for higher order shear deformation theory is

$$u(x, y, z) = u_0(x,y)+z\theta_x(x,y)+z^2u_0^*(x,y)+z^3\theta_x^*(x,y)$$

$$v(x, y, z) = v_0(x,y)+z\theta_y(x,y)+z^2v_0^*(x,y)+z^3\theta_y^*(x,y)$$

$$w(x, y, z) = w_0(x,y)$$

where, u_0, v_0, w_0 are the displacements, θ_x, θ_y are the rotations, u_0^*, v_0^*, θ_x^*, θ_y^* are the corresponding higher order shear deformation terms.

The strain displacement relations that are obtained from the above displacement equation are

$$\varepsilon_X = \varepsilon_{x0}+zl_x+z^2\varepsilon_{x0}^*+z^3l_x^*$$

$$\varepsilon_Y = \varepsilon_{y0}+zl_y+z^2\varepsilon_{y0}^*+z^3l_y^*$$

$$\varepsilon_z = 0$$

$$\gamma_{xy} = \varepsilon_{xy0}+zl_{xy}+z^2\varepsilon_{xy0}^*+z^3l_{xy}^*$$

$$\gamma_{yz} = \varphi_y+z\varepsilon_{yz0}+z^2\varphi_y^*$$

$$\gamma_{xz} = \varphi_x+z\varepsilon_{xzo}+z^2\varphi_x^*$$

For the lamina (Lth) in the co-ordinate axes (x, y, z) the stress strain relations can be given by using the usual transformation by the rule of stresses and strains between laminate co-ordinate axes and laminates is as follows [5]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^L = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^L$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{pmatrix} C_{44} & C_{45} \\ C_{45} & C_{55} \end{pmatrix}^L \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L$$

Where

$\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}$ are the stresses

$\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$ are the strain vectors

And *Cij's* are stress resultant elastic constants

After the simplification finally the laminate stress strain relation is given by [5]

$$\begin{Bmatrix} N \\ N^* \\ M \\ M^* \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B^t & D_b & 0 \\ 0 & 0 & D_s \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ \mu \\ \mu^* \\ \phi \\ \phi^* \end{Bmatrix}$$

By using finite element formulation, the strains, transverse shear strains and bending curvatures can be written in the form of displacements in terms of extension (L_E), bending (L_B) and shear (L_S) [5].

Finally, from the potential energy equation the element stiffness matrix is given by

$$K^e = \int_A (B_E^t A B_E + B_B^t B B_E + B_E^t B B_B + B_B^t D_B B_B + B_S^t D_S B_S) dA$$

The element stiffness matrix is evaluated by using the Gauss quadrature rule. In an element for the flexural analysis, the total work done by the applied external loads (e) is evaluated and solving the total work done based on the Gauss quadrature rule as follows

$$P_i = W_a W_b |J| N_i^t \{0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\}^t \left(q + P_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \right)$$

Where *a* and *b* are plate dimensions, *x* and *y* are Gauss point coordinates and *m* and *n* are the usual harmonic numbers

3. NUMERICAL EXAMPLE

In the present work, by using finite element method analysis of laminated composite plates is completed based on higher order shear deformation theory (HSDT). The boundary condition used is simply supported. The plate is square plate for a/b ratio= 1. To complete this work a MATLAB code is developed and analysis is done then the obtained values are validated using standard values available in the literature. In the examples given below the material properties are:

Material properties:

$\frac{E_1}{E_2} = 25$, $\frac{G_{12}}{E_2} = 0.5$, $\frac{G_{23}}{E_2} = 0.2$, $E_2=E_3=10^6$, $G_{13}=G_{12}$, $V_{12}=V_{23}=V_{13}=0.25$

By using the following multipliers deflection, internal stress resultants and stresses are obtained in non-dimensional form:

$$m_1 = \frac{10E_2h^3}{qa^4}\ ,\quad m_2 = \frac{h^2}{qa^2}\ ,\quad m_3 = \frac{h}{qa}$$

Displacement w is obtained by multiplying m_1, $\sigma_x, \sigma_y, \tau_{xy}$ are obtained by multiplying

m_2 and τ_{xz}, τ_{yz} are obtained by multiplying m_3

Example:

A simply supported cross ply (0/90/0) plate under uniformly distributed loads and sinusoidal distributed loads is considered for the comparison of the displacement and stresses for different a/h ratios and the properties of the material used are the same shown in Material 1 as above the obtained results are then compared with the standard values available in the literature

Table 1

Deflection and stresses for a simply supported rectangular plate (0/90/0) under uniformly distributed loading

Source	a/h	W (a/2,b/2,0)	σ x (a/2,b/2,h/2)	σ y (a/2,b/2,h/4)	τ xy (a,b,-h/2)	τ xz (0,b/2,0)	τ yz (a/2,0,0)
Present	10	1.0979	0.8348	0.3228	0.0496	0.5722	0.3036
J N REDDY[5]		1.0219	0.7719	0.3072	0.0514	0.7548*	0.3107#
Present	20	0.7794	0.7950	0.2221	0.0418	0.6057	0.3023
J N REDDY[5]		0.7719	0.7983	0.2227	0.0453	0.7697*	0.2902#
Present	100	0.6691	0.7858	0.1895	0.0390	0.6993	0.5270
J N REDDY[5]		0.6697	0.8072	0.1925	0.0426	0.7744*	0.2842#

Table 2

Deflection and stresses for a simply supported rectangular plate (0/90/0) under sinusoidal distributed loading

Source	a/h	W (a/2,b/2,0)	σ x (a/2,b/2,h/2)	σ y (a/2,b/2,h/4)	τ xy (a,b,-h/2)	τ xz (0,b/2,0)	τ yz (a/2,0,0)
Present	10	0.7106	0.5598	0.2544	0.0279	0.4738	0.1895
J N REDDY[5]		0.6693	0.5134	0.2536	0.0252	0.4089*	0.0914#
Present	20	0.5022	0.5269	0.1917	0.0264	0.2858	0.1775
J N REDDY[5]		0.4921	0.5318	0.1997	0.0223	0.4205*	0.0759#
Present	100	0.4307	0.5175	0.1691	0.0242	0.3811	0.2954
J N REDDY[5]		0.4337	0.5384	0.1804	0.0213	0.4247*	0.0703#

* (0, b/2) in layers 1 & 3

(a/2, 0) in layer 2

4. CONCLUSION

In the present work, by using finite element method, analysis of laminated composite plates is computed based on higher order shear deformation theory (HSDT9). To complete this work a MATLAB code is developed and analysis is done for different layers, orientations and a/h ratios for uniformly distributed loads and sinusoidal loads. The obtained values are then validated using standard values from J N REDDY reference book. The variation of transverse stresses is due to constitutive matrix theory.

REFERENCES

[1] N.D. Phan and J, N, Reddy *"Analysis of Laminated Composite Plates Using a Higher-Order Shear Deformation Theory",* International journal for numerical methods in engineering, vol-21,2201-2219(1985)

[2] B. N. Pandya & T Kant*, "Finite element analysis of laminated composite plates using a higher order displacement model",* Composite Science and Technology 32 (1988) 137-155

[3] V M Srihari, *"A detailed comparison study of first order and higher order shear deformation theories in the analysis of laminated composite plates",* IOP conference series: Material science and Engineering, IOP conf. Ser : Mater. Sci. Eng. 225 012018.

[4] Tran Ich Thinh, Ngo Nhu Khoa, Do Tien Dung, *" A C^1 bending element for composite plates based on higher order shear deformation theory",* Vietnam Journal of Mechanics, VAST, Vol. 30, No. 4 (2008), pp. 337 – 346

[5] J.N. Reddy*, "Mechanics of laminated composite plates and shells, theory and analysis",* second edition, Special Indian Edition(2013), CRC press (India) private limited.