International Journal of Mechanics and Solids.
ISSN 0973-1881 Volume 13, Number 1 (2018), pp. 1-8
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http://www.ripublication.com/ijms.htm

Free Vibration Analysis of Laminated Composite Plate using FSDT with Finite Element Approach

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Abstract

Free vibration theory of laminated composite plates using First-order Shear Deformation Theory (FSDT) is presented in this paper. The theoretical plate formulation is done using FSDT. A four-noded isoparametric quadrilateral element with 5 degrees of freedom at each node is considered. Finite element approach is employed for finding the shape functions. Hamilton's principle is used for deriving the equations of motion. The natural frequency of the laminated composite plate is computed using MATLAB programming. The natural frequency for different number of layers and a/h ratios is found out. The obtained values are compared with the values available in the literature.

Keywords: Free vibration, First-order theory, Laminated composite plate, Finite Element Method, MATLAB programming.

1. INTRODUCTION

Composite materials are those in which no less than two materials are combined on a naturally visible scale to shape a useful material. The individual materials are easily identifiable. The properties of composite material are far better when compared to the properties of its constituents, if designed properly, which is one of the major advantages. They have a major application in many modern fields of engineering. The Classical Laminate Plate Theory (CLPT) is inadequate for the analysis of laminated composite plates as it ignores the effect of transverse shear deformation. Hence, FSDT comes into picture. The first-order theories which are based on Mindlin and Reissner assume that the displacements and in-plane stresses through the thickness of the laminate are linear. [1] presented a simple first-order shear deformation theory for laminated composite plates. They obtained the analytical solutions for antisymmetric angle-ply and cross-ply and compared the results with 3D elasticity model. [2] presented the analytical solutions for free vibration of laminated composite plate. They presented a theoretical model that included laminate deformations such as transverse shear deformations. [3] proposed a theory on laminated composite plates with a new higher-order shear deformable concept. It is generated by inverse method with 3D elasticity bending solutions. FEM is normally used for the analysis of composite structures. In this method, there are numerous sub-domains that form a physical domain. These sub-domains are called finite elements.

2. FORMULATION FOR COMPOSITE PLATE

The displacement model assumed for FSDT is,

$$u(x,y,z) = u_o(x,y) + z\theta_x(x,y)$$
$$v(x,y,z) = v_o(x,y) + z\theta_y(x,y)$$
$$w(x,y,z) = w_o(x,y)$$

Where *u, v* and *w* are the displacements in the directions of *x, y and z* respectively of a point on the mid-plane (i.e., z=0)

The stress-strain constitutive relations with references to laminate axes are obtained in the following form,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} & 0 & 0 \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} & 0 & 0 \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} * \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

Finally,

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} D_M & D_C & 0 \\ D_C^T & D_B & 0 \\ 0 & 0 & D_s \end{bmatrix}$$

Where,

D_M – *D* matrix due to membrane

D_B – *D* matrix due to bending

D_C – *D* matrix due to combined membrane-bending

D_S – D matrix due to shear

3. FINITE ELEMENT FORMULATION

Element in natural coordinates (ξ, η) :

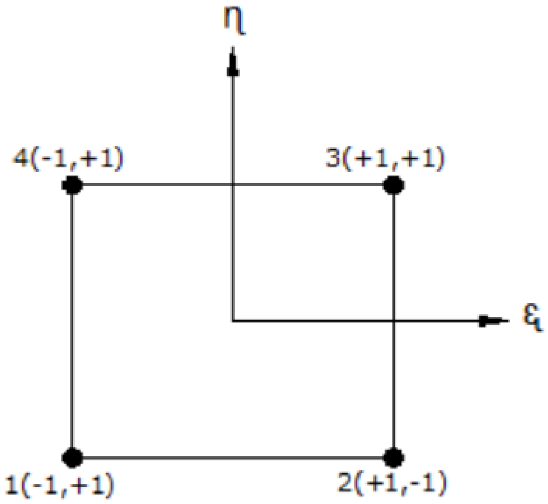

Figure - 1: Four-noded element in natural coordinate

Expressions for geometry of the element:

$$x = \sum_{i=1}^{n} N_i x_i = N_1 x_1 + N_2 x_2 + N_3 x_3 + N_4 x_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = [N]\{x\}$$

$$y = \sum_{i=1}^{n} N_i y_i = N_1 y_1 + N_2 y_2 + N_3 y_3 + N_4 y_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = [N]\{y\}$$

Where,

x_i & y_i - element nodal coordinates

N – number of nodes per element

N_i – shape functions

Shape functions for a four-noded element:

$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$, $N_2 = \frac{1}{4}(1+\xi)(1-\eta)$, $N_3 = \frac{1}{4}(1+\xi)(1+\eta)$,

$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$

There are 5 degrees of freedom out of which *u*, *v* and *w* are displacements and Θ_x and Θ_y are rotations.

Strain energy expression for mechanical strain:

$$U_M = \int_v \{\varepsilon\}^T \{\sigma\}\, dv \qquad \text{where, } \{\varepsilon\} = [B]\{d_e\}$$

$$\{\varepsilon\}^T = \{d_e\}^T [B]^T, \qquad \{\sigma\} = [D]\{\varepsilon\} = [D][B]\{d_e\}$$

$$U_M = \frac{1}{2}\int_0^a\int_0^b \{d_e\}^T [B]^T [D][B]\{d_e\}\, dxdy$$

The elastic stiffness matrix is given by,

$$K_{UU} = \frac{1}{2}\int_0^a\int_0^b [B]^T [D][B]\, dxdy$$

Elemental area conversion from Cartesian to natural coordinates

$$dxdy = |J|\, d\varepsilon d\eta$$

where, $|J|$ - determinant of Jacobian matrix

$$|J| = \begin{vmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{vmatrix}$$

Finally,

$$K_{UU} = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T [D][B]|J|\, d\xi d\eta$$

4. FREE VIBRATION THEORY

Equations of motion using Hamilton's principle

$$M\ddot{u} + KU = f$$

Natural frequencies and vibration modes,

$$(K - \omega^2 M)X = 0$$

Mass matrix can be computed as [4]

$$M^e = \int_A \rho N^T \begin{bmatrix} h & 0 & 0 \\ 0 & \frac{h^3}{12} & 0 \\ 0 & 0 & \frac{h^3}{12} \end{bmatrix} N dA$$

The dimensionless natural frequency is given by,

$$\varpi = \omega_{mn} a \sqrt{\frac{\rho}{G}}$$

Where, ϖ - natural frequency

ρ - density of material

G – shear modulus

5. NUMERICAL RESULTS

Variation of dimensionless natural frequencies with different a/h ratios for a simply supported symmetric cross-ply square laminated plate:

Material 1:

E_1 = 25 Gpa, E_2 = 1 Gpa, E_3 = 1 Gpa, G_{12} = 0.5 Gpa, G_{13} = 0.5 Gpa, G_{23} = 0.2 Gpa, μ_{12} = 0.25, μ_{13} = 0.25, μ_{23} = 0.25, q_o = 1 N/sqm

Table 1: Variation of dimensionless natural frequencies from J. N. Reddy [5]

a/h	Source	0°	Three-ply	Five-ply	Seven-ply	Nine-ply
5	**Present** **Reddy** [5]	8.955(0.5) 8.909	8.195(-6.4) 8.766	8.270(-10.2) 9.215	7.953(-14) 9.301	7.579(-18) 9.333
10	**Present** **Reddy** [5]	12.576(0.9) 12.452	11.893(-2.7) 12.227	12.003(-4.9) 12.633	11.801(-7.2) 12.720	11.491(-9.9) 12.754
20	**Present** **Reddy** [5]	14.547(1.3) 14.355	14.293(0.3) 14.244	14.292(-0.8) 14.415	14.092(-2.5) 14.453	13.895(-3.9) 14.468
25	**Present** **Reddy** [5]	14.855(1.3) 14.651	14.600(0.2) 14.573	14.678(-0.08) 14.690	14.603(-0.7) 14.717	14.512 (-1.4) 14.727
50	**Present** **Reddy** [5]	15.300(1.4) 15.077	15.356(2) 15.055	15.250(1) 15.087	14.787(-2) 15.095	14.986 (-0.7) 15.098
100	**Present** **Reddy** [5]	15.418(1.5) 15.190	15.174(-0.06) 15.184	15.405(1.4) 15.192	14.943 (-1.6) 15.194	15.166 (-0.2) 15.195

*Numbers in parentheses indicate percentage error

Variation of dimensionless natural frequencies with varying a/h ratios for a simply supported cross-ply square laminated plate:

Material 2:

E_1 = 40 Gpa, E_2 = 1, Gpa, E_3 = 1 Gpa, G_{12} = 0.6 Gpa, G_{13} = 0.6 Gpa, G_{23} = 0.5 Gpa, μ_{12} = 0.25, μ_{13} = 0.25, μ_{23} = 0.25, q_o = 1 N/sqm

Table - 2: Data validation of ϖ from Kant and Swaminathan [2] for (0/90/90/0)

a/h	Present	Kant and Swaminathan [2]
2	4.885(-9.5)	5.403
4	10.093(8.6)	9.287
10	14.607(-3.2)	15.105
20	17.585(-0.3)	17.647
50	18.890(1.1)	18.672
100	19.105(1.4)	18.836

*Numbers in parentheses indicate percentage error

Comparison of dimensionless frequencies for four layered cross-ply (0/90/90/0) with varying a/h and E_1/E_2 ratios:

Material 3:

G_{12} = 0.6 Gpa, G_{13} = 0.6 Gpa, G_{23} = 0.5 Gpa, μ_{12} = 0.25, μ_{13} = 0.25, μ_{23} = 0.25, q_o = 1 N/sqm

Table - 3: Data validation of ϖ from Metin Aydogdu [3] for different a/h andE_1/E_2 ratios

a/h	Source	E_1/E_2				
		3	**10**	**20**	**30**	**40**
5	**Present**	6.462 (-1.2)	7.984 (-3.1)	9.044 (-4.6)	9.665 (-5.4)	10.094 (-5.9)
	Metin Aydodgu [3]	6.546	8.243	9.484	10.220	10.728
10	**Present**	7.278 (0.5)	9.794 (-0.3)	12.014 (-1.4)	13.505 (-2.3)	14.608 (-3)
	Metin Aydogdu [3]	7.238	9.828	12.193	13.828	15.062
20	**Present**	7.545 (1.1)	10.520 (0.9)	13.500 (0.5)	15.755 (0.3)	17.585 (-0.2)
	Metin Aydogdu [3]	7.458	10.422	13.429	15.734	17.627
50	**Present**	7.626 (1.9)	10.759 (1.6)	14.045 (1.3)	16.662 (1.2)	18.890 (1.2)
	Metin Aydogdu [3]	7.483	10.583	13.860	16.454	18.654
100	**Present**	7.638 (1.4)	10.795 (1.4)	14.129 (1.4)	16.807 (1.4)	19.105 (2.5)
	Metin Aydogdu [3]	7.532	10.640	13.926	16.566	18.637

*Numbers in parentheses indicate percentage error

Comparison of dimensionless natural frequency of 10-layer antisymmetric angle-ply $(\theta/-\theta)_5$ square laminates:

Material 4:

E_1 = 15 Gpa, E_2 = 1 Gpa, E_3 = 1 Gpa, G_{12} = 0.5 Gpa, G_{13} = 0.5 Gpa, G_{23} = 0.35Gpa, μ_{12} = 0.3, μ_{13} = 0.3, μ_{23} = 0.3, q_o = 1 N/sqm

Table - 4: Data validation of ϖ from Huu-Tai Thai [1] for angle-ply $(\theta/-\theta)_5$

a/h	**Source**	θ		
		15^0	**30^0**	**45^0**
5	**Present** **Huu-Tai Thai** [1]	7.389(-17) 8.981	7.929(-19) 9.827	8.105(-19) 10.128
10	**Present** **Huu-Tai Thai** [1]	10.659(-8) 11.633	11.811(-9) 13.031	12.256(-9) 13.614
100	**Present** **Huu-Tai Thai** [1]	13.482(1.5) 13.276	15.352 (1.6) 15.103	16.213 (1.6) 15.948

*Numbers in parentheses indicate percentage error

6. CONCLUSION

1. The First-order Shear Deformation Theory (FSDT) gives accurate results for thin plates and lower E_1/E_2 ratios.
2. When the natural frequencies obtained from present work are compared with J. N. Reddy [5] values, it can be seen that the percentage error goes on increasing as a/h ratio reduces and number of plies increase.
3. Further, when compared with Kant and Swaminathan [2] for two-layered and four-layered plates, it shows higher percentage error for lower a/h ratio of four-layered plate as compared to two-layered.
4. The same goes with angle-plies of different orientations. There is high error for thick plates compared to thin plates.
5. When the natural frequencies are compared in terms of E_1/E_2 ratios, the percentage error increases with increase in E_1/E_2 ratios.
6. The reason for increase in percentage error is that, in FSDT, the shear deformation factor is not accounted for thick plates.
7. FSDT can be used for thin plates and for thick plates, it is better to go for Higher-order Shear Deformation Theory (HSDT).

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