

Water Wave Radiation by a Submerged Circular Cylinder in Uniform Finite Depth Water with Ice-Cover

Manomita Sahu and Dilip Das*

*Diamond Harbour Women's University, Department of Mathematics,
Diamond Harbour Road, Sarisha, Pin- 743368, India.*

**(Corresponding author)*

ABSTRACT

We formulate the problems of water wave radiation (both heave and surge) by a submerged circular cylinder in water of uniform finite depth with an ice-cover by using the method of multipoles, the ice-cover being modelled as an elastic plate of very small thickness. The added mass and damping coefficients for a heaving and surge problems are obtained and depicted graphically against the wave number for various values of flexural rigidity of the ice-cover to show the effect of the presence of ice-cover on these quantities. When the flexural rigidity and surface density of ice-cover are taken to be zero, then the curves for added-mass and damping coefficients almost coincide with the curves for the case of uniform finite depth water with free surface. Also, it is observed that the added-mass and damping coefficients for the heave modes of motion are different from those for surge modes but for the case of radiation by cylinder submerged in deep water with ice-cover, the forces of heave problem are equal to the forces for the surge problem.

Key words- Water wave radiation, uniform finite depth, circular cylinder, ice-cover, added-mass and damping coefficients.

1. INTRODUCTION

Problems concerning the radiation of water waves by spherical objects have received extensive study, beginning with Havelock [1] who solved the heave radiation problem for a half-immersed sphere in deep water. Hulme [2] improved and extended it to the case of sway. Ursell [3] solved the problem of surface waves on deep water in the presence of a submerged circular cylinder by using the method of multipoles. This method has been used in various fields of theoretical physics (cf. Jackson [4], Morse and Feshbach [5]). Problems concerning the radiation of waves by spherical objects

fully submerged in deep water with a free surface were investigated by Srokosz [6]. Wang [7] used the method of Havelock [1] to examine the radiation and diffraction problems for a submerged sphere in deep water. Evans and Linton [8] used the multipole method to solve the two-dimensional problems of radiation and scattering of water waves by a submerged horizontal circular cylinder in finite depth water as part of a need to determine accurately the natural frequencies of oscillation of a highly buoyant tethered cylinder. Linton [9] considered the problems of radiation (both heave and sway) and scattering of water waves by a sphere submerged in finite depth water using the same method. Also, one of the attractive features of multipole expansion methods for this type of problem is that the evaluation of added-mass and damping coefficients on a submerged circular cylinder in infinite depth water (both heave and surge) is investigated (cf. Linton and McIver [10] and Eatock Taylor and Hu [11]).

Recently, Das and Mandal [12] studied the wave radiation by a sphere submerged in water with an ice-cover to obtain the added-mass and damping coefficients. Sturova [13] also considered the problem of hydrodynamic loads acting on an oscillating cylinder submerged in a stratified fluid with ice-cover. Thakur and Das[14] investigated the problem of hydrodynamic forces on a submerged horizontal circular cylinder in water with ice-cover. Li *et.all* [15] considered the wave radiation and diffraction by a circular cylinder submerged below an ice-sheet with a crack. They used the multipole expansion method and the solution was obtained for a fluid of both finite and infinite depth. Das and Sahu [16] recently studied the wave radiation by a horizontal circular cylinder submerged in deep water with ice-cover. Thus we extend the problem of Das and Sahu [16] to examine the radiation for a submerged circular cylinder in uniform finite depth water with an ice-cover. The added-mass and damping coefficients for a heaving and surge mode circular cylinder are obtained and depicted graphically against the wave number for various values of flexural rigidity of the ice-cover and depth of water to show the effect of the presence of ice-cover. When the flexural rigidity and surface density of ice-cover are taken to be zero, so that the ice-cover tends to a free-surface. Then the curves for added-mass and damping coefficients almost coincide with the curves for the case of water with free surface (cf. Eatock Taylor and Hu [11]). Also it is observed here that the added-mass and damping coefficients for the heave modes of motion are different from those for surge modes but for the case of radiation by cylinder submerged in deep water with ice-cover, the forces of heave problem are equal to the forces for the surge problem (cf. Das and Sahu [16]).

2. MATHEMATICAL FORMULATION

A rectangular Cartesian co-ordinate systems are chosen such that $y = 0$ is the undisturbed position of the ice-cover, y being measured vertically downwards. The central axis of the cylinder with radius a is taken to be $x = 0, y = f (f > a)$. Assuming linear theory, the velocity potential function describing the resulting motion can be represented by $Re \{ \phi(x, y) e^{-i\sigma t} \}$, where the time-

independent complex valued potential function $\phi(x, y)$ satisfies

$$\nabla^2 \phi = 0 \text{ in the fluid region,} \quad (1)$$

the linearized ice-cover condition (cf. Fox and Squire[17])

$$\left(D \frac{\partial^4}{\partial x^4} + 1 - \epsilon K \right) \phi_y + K \phi = 0 \quad \text{on } y = 0. \quad (2)$$

Where $K = \frac{\sigma^2}{g}$, g being the gravity, $D = \frac{E h_0^3}{12(1-\nu^2)\rho_1 g}$ and $\epsilon = \frac{\rho_0}{\rho_1} h_0$, ρ_0 is the density of ice, ρ_1 is the density of water, h_0 is the small thickness of the ice-cover, and E, ν are the Young's modulus, and Poisson's ratio of the ice. The boundary conditions on the cylinder surface are

$$\frac{\partial \phi_s}{\partial r} = \sin \theta \quad \text{and} \quad \frac{\partial \phi_h}{\partial r} = \cos \theta \quad \text{at } r = a$$

The bottom condition is given by

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{as } y = h. \quad (3)$$

Where h is the depth of the water.

3. METHOD OF SOLUTION

Here r and θ are polar coordinates defined by $x = r \sin \theta, y = f + r \cos \theta$ ($-\pi \leq \theta \leq \pi$).

Let ϕ_n^s and ϕ_n^a denote the symmetric (heave potential) and antisymmetric (surge potential) multipoles satisfy (1) except at $(0, f)$ with boundary conditions (3),(2).

The multipoles are (cf. Thorne [18])

$$\phi_n^s = \frac{\cos n\theta}{r^n} + \oint_0^\infty [A_1(k) \sinh ky + B_1(k) \cosh k(h-y)] k^{n-1} \cos kx \, dk, \quad (4)$$

$$\phi_n^a = \frac{\sin n\theta}{r^n} + \oint_0^\infty [A_2(k) \sinh ky + B_2(k) \cosh k(h-y)] k^{n-1} \sin kx \, dk, \quad (5)$$

$$A_1(k) = A_2(k) = \frac{1}{(n-1)!} \frac{e^{-k(h-f)}}{\cosh kh},$$

$$B_1(k) = \frac{(-1)^n}{(n-1)!} e^{-kf} \frac{k(Dk^4 + 1 - \epsilon K) + K}{k(Dk^4 + 1 - \epsilon K) \sinh kh - K \cosh kh} + \frac{1}{(n-1)!} e^{-k(h-f)} \frac{k(Dk^4 + 1 - \epsilon K)}{\cosh kh [k(Dk^4 + 1 - \epsilon K) \sinh kh - K \cosh kh]}$$

$$B_2(k) = \frac{(-1)^{n+1}}{(n-1)!} e^{-kf} \frac{k(Dk^4+1-\epsilon K)+K}{k(Dk^4+1-\epsilon K) \sinh kh - K \cosh kh} +$$

$$\frac{1}{(n-1)!} e^{-k(h-f)} \frac{k(Dk^4+1-\epsilon K)}{\cosh kh [k(Dk^4+1-\epsilon K) \sinh kh - K \cosh kh]}.$$

Where the contour is intended below the pole $k = \lambda$ on the real k -axis, λ being the only real positive root of the dispersion equation

$$k(Dk^4 + 1 - \epsilon k) \sinh kh = K \cosh kh. \quad (6)$$

This accounts for outgoing behaviour of ϕ_n^s and ϕ_n^a at infinity.

A power series expansion can be obtained from (4) and (5) as

$$\phi_n^s = \frac{\cos n\theta}{r^n} + \sum_{m=0}^{\infty} A_{mn} r^m \cos m\theta, \quad (7)$$

and

$$\phi_n^a = \frac{\sin n\theta}{r^n} + \sum_{m=0}^{\infty} B_{mn} r^m \sin m\theta. \quad (8)$$

Where

$$A_{mn} = \frac{1}{2} \frac{1}{m!} \oint_0^{\infty} k^{m+n-1} [A_1(k)C_1(k) + B_1(k)C_2(k)] dk,$$

$$B_{mn} = \frac{1}{2} \frac{1}{m!} \oint_0^{\infty} k^{m+n-1} [A_2(k)C_3(k) + B_2(k)C_4(k)] dk,$$

Where

$$C_{1,3}(k) = e^{kf} \mp (-1)^m e^{-kf},$$

$$C_{2,4}(k) = e^{-k(h-f)} \pm (-1)^m e^{k(h-f)}.$$

Let us now consider the heave and surge radiation problems for the cylinder. The surge problem is anti-symmetric about $x = 0$, whereas the heave problem is symmetric and so we can write (cf. Das and Sahu [16])

$$\phi_s = \sum_{n=1}^{\infty} a^{n+1} \alpha_n \phi_n^a \quad (9)$$

$$\phi_h = \sum_{n=1}^{\infty} a^{n+1} \beta_n \phi_n^s \quad (10)$$

for some set of unknowns α_n, β_n and here, ϕ_s and ϕ_h denotes the potential functions for the surge and heave problems respectively.

Now using (7) and (8) in (9) and (10) respectively and using structural boundary

conditions

$$\frac{\partial \phi_s}{\partial r} = \sin \theta \quad \text{and} \quad \frac{\partial \phi_h}{\partial r} = \cos \theta \quad \text{at} \quad r = a$$

and using the orthogonal property of trigonometry, we obtained two infinite system of equations

$$\alpha_m - \sum_{n=1}^{\infty} a^{m+n} B_{mn} \alpha_n = -\delta_{1m}, \quad m = 1, 2, 3, \dots, \quad (11)$$

$$\beta_m - \sum_{n=1}^{\infty} a^{m+n} A_{mn} \beta_n = -\delta_{1m}, \quad m = 1, 2, 3, \dots, \quad (12)$$

Where δ_{nm} is the kronecker delta function.

These systems can be solved numerically by truncating the infinite series and solving an $N \times N$ system of equations.

The hydrodynamic forces on the cylinder in the direction of motion is $F^m = \text{Re}\{f^m e^{-i\omega t}\}$, ($m = 0$ surge and $m = 1$ for heave problems), where the time-independent part f^m is given by

$$f^0 = f_s = \omega \rho \int_0^a \int_{-\pi}^{\pi} \phi_s \sin \theta r d\theta dr,$$

for surge problem and

$$f^1 = f_h = \omega \rho \int_0^a \int_{-\pi}^{\pi} \phi_h \cos \theta r d\theta dr,$$

for heave problem.

Using (9) and (10) and orthogonal properties of trigonometry, one can reduces it to

$$f_s = \pi \rho \omega a^2 [\alpha_1 + \sum_{n=1}^{\infty} B_{1n} \alpha_n], \quad (13)$$

$$f_h = \pi \rho \omega a^2 [\beta_1 + \sum_{n=1}^{\infty} A_{1n} \beta_n], \quad (14)$$

The forces are non-dimesionalized with respect to the mass of fluid displaced by the cylinder, we get the simplified expansion for the non-dimensional added-mass and damping coefficients $\mu^{s,h}$ and $\nu^{s,h}$ respectively, by using (11) and (12) respectively with $m = 1$. These are given by

$$\mu^s + i\nu^s = 1 + 2\alpha_1, \quad (15)$$

$$\mu^h + i\nu^h = 1 + 2\beta_1. \quad (16)$$

The constants α_1 and β_1 appearing in (15) and (16) respectively can be obtained numerically by solving the linear system (11) and (12) after truncation. Here (11) and (12) are truncated up to five terms and added-mass and damping coefficients for the surge and heave problems are depicted graphically against the wave number for

various values of flexural rigidity of the ice-cover to show the effect of the presence of ice-cover on these quantities.

4. NUMERICAL RESULTS

Curves for the added-mass μ and damping coefficients ν for both the cases (surge and heave) are shown in figures 1 to 12.

Figs. 1-6 show μ^s, ν^s (surge mode) and Figs. 7-12 for μ^h, ν^h (heave mode) plotted against Ka for different values of f/a , viz. $f/a = 1.5, 1.2, 1.05$ and different values of D/a^4 . In the Figs. 1, 7 and Figs. 2, 8 for added-mass and damping coefficient respectively, $D/a^4 = 0, \epsilon/a = 0$. Here we observed that the curves are almost coincides with the curves for the case of water with free surface (cf. Eatock Taylor and Hu, [11]).

In the Figs. 3, 4 and 9, 10, $D/a^4 = 1, \epsilon/a = 0.01, h/a = 3$, Figs. 5, 6 and 11, 12, $f/a^4 = 1.5, \epsilon/a = 0.01, h/a = 4$ and different values of D/a^4 . It is observed that added-mass and damping coefficients first increase as Ka increases; each attains a maximum value and then decrease as Ka further increases for all cases except added-mass of the heave problem. Here added-mass gradually decrease as increases of Ka . Also we noticed that due to the presence of ice-cover, the non-dimensional μ and ν get reduced.

The damping coefficients here are always non-negative while the added-mass can be negative. This occurs for a submerged cylinder, while it is very closed to the ice-cover surface. Here it is also observed that the forces of the heave problem have significant change, where as those for surge rather smoothly but for the case of radiation by submerged circular cylinder in deep water (cf. Das and Sahu [16]), the forces of heave problem are equal to the forces for the surge problem.

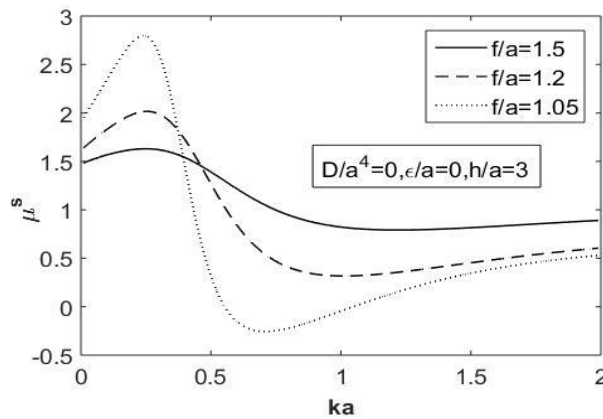


Fig.1 Added mass against the wave number ka

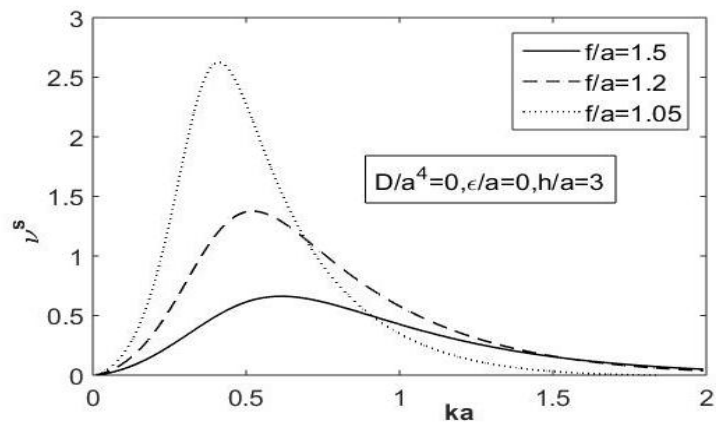


Fig.2 Damping coefficient against the wave number ka

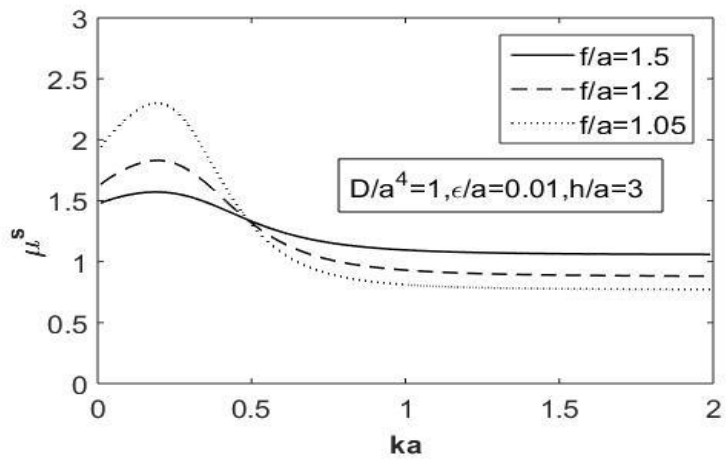


Fig.3 Added mass against the wave number ka

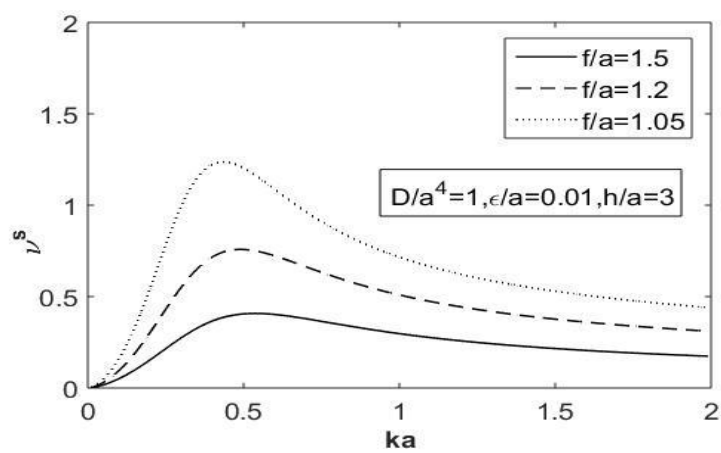


Fig.4 Damping coefficient against the wave number ka

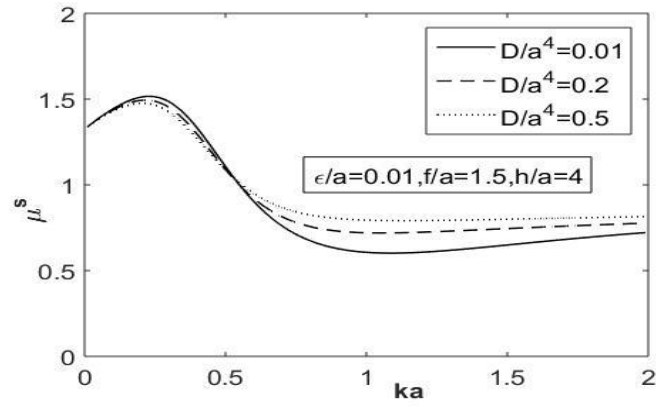


Fig.5 Added mass against the wave number ka

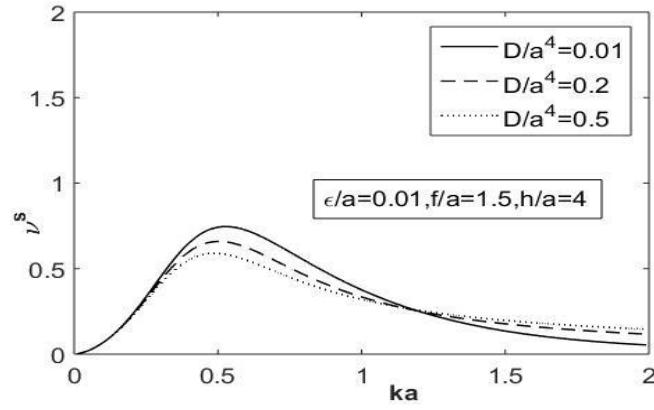


Fig.6 Damping coefficient against the wave number ka

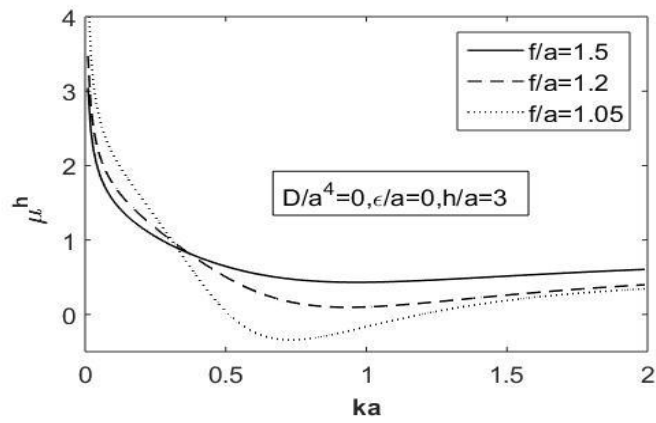


Fig.7 Added mass against the wave number ka

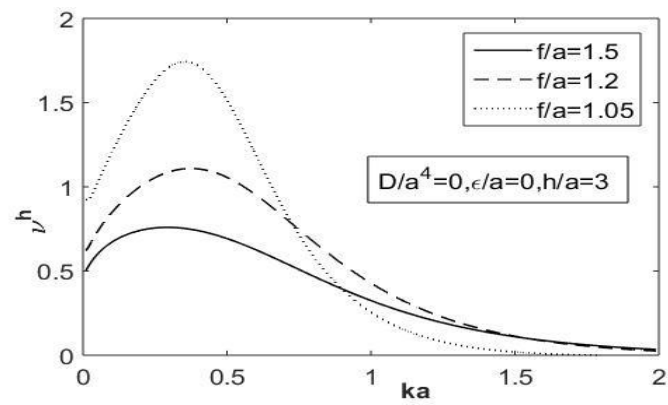


Fig.8 Damping coefficient against the wave number ka

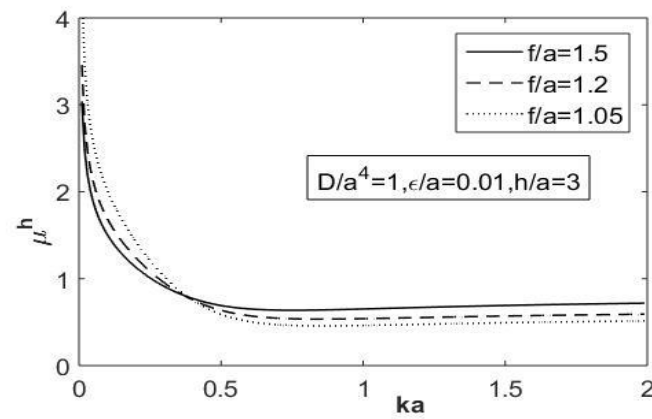


Fig.9 Added mass against the wave number ka

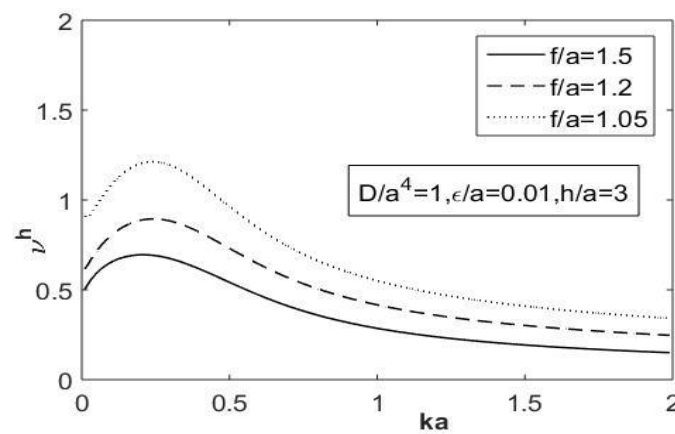


Fig.10 Damping coefficient against the wave number ka

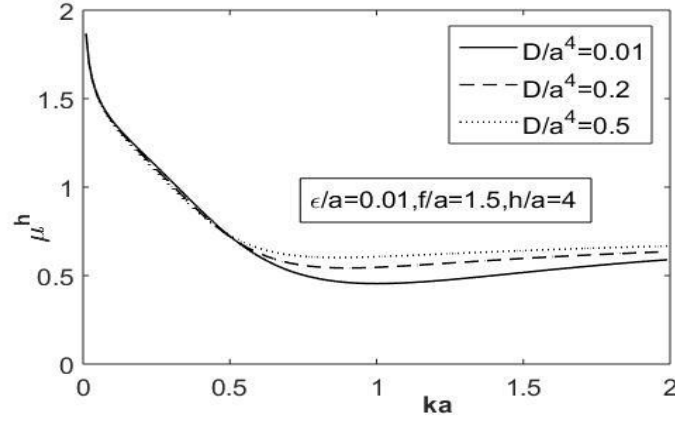


Fig.11 Added mass against the wave number ka

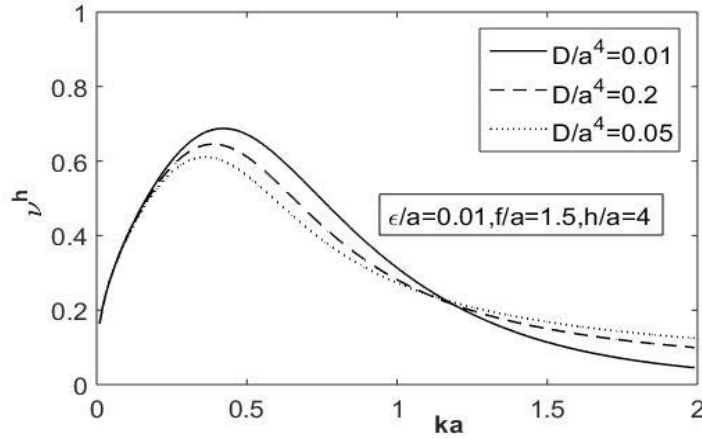


Fig.12 Damping coefficient against the wave number ka

5. CONCLUSION

Water wave radiation by a horizontal circular cylinder submerged in uniform finite depth water beneath the free surface is extended here when the free surface is replaced by a thin ice-cover modelled as a thin elastic plate. Numerical results for the added-mass and damping coefficients for surge and heaving cylinder are obtained. The added mass and damping coefficients are depicted graphically against the wave number in a number of figures. When the ice-cover is replaced by a free surface curves for added-mass and damping coefficients almost coincide with the curves for the cases of uniform finite depth water with free surface. Here it is also observed that the forces of the heave problem have significant change, where as those for surge rather smoothly but for the case of radiation by submerged circular cylinder in deep water, the forces of heave problem are equal to the forces for the surge problem.

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