International Journal of Fuzzy Mathematics and Systems.
ISSN 2248-9940 Volume 5, Number 1 (2015), pp. 41-45

http://www.ripublication.com

Fuzzy Supra S-Open And S-Closed Mappings

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Abstract

In this paper we introduce the concepts of fuzzy supra S-open and fuzzy supra S-closed mappings and investigate some characterizations of these notions.

Keywords: Fuzzy topological spaces, fuzzy supra topological spaces, fuzzy supra S-open mappings, fuzzy supra S-closed mappings.

1. Introduction

M. E. Abd El-Monsef et al. [4] introduced Fuzzy supra topological spaces as a natural generalizations of classical supra topological spaces introduced by A.S. Mashhour et al. Due to the importance and need for the fuzzification of weaker forms of classical topological notions, several works has been found in this field among the various existing fuzzy topological spaces in recent years. Fuzzy s-continuous functions are investigated by Won Keun Min [5]. B. Ahmed et al. [6] worked on fuzzy S-open and fuzzy S-closed mappings and established number of characterizations.

In this paper, we introduce fuzzy supra S-open and fuzzy supra S-closed mappings as a natural generalizations of fuzzy S-open and fuzzy S-closed mappings and obtain important characterizations of these mapping.

2. Preliminaries

In order to make this paper self-contained, first we briefly recall certain definitions and results. Throughout the paper X, Y are non-empty sets.

A fuzzy set in X is a function from X into the closed unit interval I = [0, 1]. The collection of all fuzzy sets of X is denoted by I^X. The basic operations on fuzzy sets are taken as usual.

The fuzzy sets 0_x and 1_x in X are defined as $0_x(x) = 0$, $1_x(x) = 1$, for all $x \in X$.

Definition 2.1: Let f*:* X→Y be a mapping. Let Aϵ I^X and Bϵ I^Y, then f (A) is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup\{A(x) : x \in f^{-1}(y)\}, \text{ if } f^{-1}(y) \neq \phi \\ 0, \text{ if } f^{-1}(y) = \phi \end{cases}$$

for all $y \in Y$. $f^{-1}(B)$ is a fuzzy set in Y, defined by $f^{-1}(B)(x) = B(f(x))$ for all $x \in X$.

Theorem 2.2: (see [1])**:** Let f*:* X→Y be a mapping, then

(1) $f^{-1}(B^c) = (f^{-1}(B))^c$, for any fuzzy set B in Y.

(2) $f(f^{-1}(B)) \leq B$, for any fuzzy set B in Y.

(3) $A \leq f^{-1}(f(A))$ for any fuzzy set A in X.

(4) $f^{-1}(\vee A_i) = \vee f^{-1}(A_i)$ and $f^{-1}(\wedge A_i) = \wedge f^{-1}(A_i)$, where $A_i \in I^Y$.

Definition 2.3: A collection T^* of fuzzy sets in X is called a fuzzy supra topology on X if the following conditions are satisfied: (1) $0_x, 1_x \in T$ and (2) $A_i \in T \Rightarrow \vee A_i \in T$.

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp. fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS (X) (resp. FSCS (X)).

Definition 2.4: Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as

$Cl^*(A) = \wedge \{B : A \leq B$, B is a fuzzy supra closed set in X$\}$,

$Int^*(A) = \vee \{B : B \leq A$, B is a fuzzy supra open set in X$\}$.

Definition 2.5 (see [2])**:** The fuzzy supra boundary of a fuzzy set A in FSTS (X, T^*) is denoted and defined as $\partial^*(A) = Cl^*(A) \wedge Cl^*(A^c)$. Clearly $\partial^*(A)$ is a FSCS in X.

Definition 2.6 (see [2])**:** A fuzzy set A in a FSTS (X, T^*) is called a fuzzy supra semi open set (FSSOS) if there exist a fuzzy supra open set $\mu \in T^*$ such that $\mu \leq A \leq Cl^*(\mu)$.

The complement of fuzzy supra semi open set is called fuzzy supra semi closed set (FSSCS). The collection of all FSSOS (resp. FSSCS) of the FSTS (X, T^*) is denoted by FSSOS (X) (resp. FSSCS (X)).

Definition 2.7 (see [2])**:** Let (X, T^*) is a FSTS and A be fuzzy set in X, then the fuzzy supra semi closure and fuzzy supra semi interior are denoted and defined respectively as

$sCl^*(A) = \wedge\{B : A \leq B$, B is a fuzzy supra semi closed set in X$\}$,

$sInt^*(A) = \vee\{B : B \leq A$, B is a fuzzy supra semi open set in X$\}$.

We discussed the properties of Cl^*, Int^*, sCl^* and $sInt^*$ in [2,3].

Definition 2.8 (see [2])**:** The fuzzy supra semi boundary of a fuzzy set A in FSTS (X, T^*) is denoted and defined as $s\partial^*$ (A) = sCl^* (A) $\wedge$ sCl^* (A^c). Clearly $s\partial^*$ (A) is a FSSCS in X and $s\partial^*$ (A)) $\leq \partial^*$ (A).

Properties of fuzzy supra boundary and fuzzy supra semi boundary are discussed in [2].

Definition 2.9 (see [2])**:** Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A mapping f*:* X→Y is called fuzzy supra semi continuous if f^{-1} (A) ϵ FSSOS (X) for each A ϵ T_2^*.

3. Fuzzy supra S-open and S-closed maps

Definition 3.1: Let (X, T_1^*) and (Y, T_2^*) be two FSTS's. A mapping f*:* X→Y is called

(1) fuzzy supra S-open if f (A) is fuzzy supra open in Y for each A ϵ FSSOS (X),

(2) fuzzy supra S-closed if f (A) is fuzzy supra closed in Y for each A ϵ FSSCS (X).

Theorem 3.2: Let f*:* X→Y be a mapping between two FSTS's. Then the following are equivalent:

(1) f is fuzzy supra S-open;

(2) f ($sInt^*$ (A)) $\leq$ Int^* (f (A)) for each fuzzy set A of X;

(3) $sInt^*$ (f^{-1} (B)) $\leq$ f^{-1} (Int^* (B)) for each fuzzy set B of Y;

(4) f^{-1} (Cl^* (B)) $\leq$ sCl^* (f^{-1} (B)) for each fuzzy set B of Y.

Proof:

(1) ⇒ (2): Let f be fuzzy supra S-open, then f ($sInt^*$ (A)) ϵ FSOS (Y) for each A ϵ I^X.

∴ f ($sInt^*$ (A)) = Int^* (f ($sInt^*$ (A))) $\leq$ Int^* (f (A))

(2) ⇒ (3): Let B ϵ I^Y and A = f^{-1} (B). By (2), f ($sInt^*$ (A)) $\leq$ Int^* (f (f^{-1} (B))) $\leq$ Int^* (B).

∴ $sInt^*$ (f^{-1} (B)) = $sInt^*$ (A) $\leq$ f^{-1} (f ($sInt^*$ (A))) $\leq$ f^{-1} (Int^* (B)).

(3) ⇒ (4): For B ϵ I^Y. By (3), $sInt^*$ (f^{-1} (B^c)) $\leq$ f^{-1} (Int^* (B^c)).

∴ (f^{-1} (Int^* (B^c)))^c $\leq$ ($sInt^*$ (f^{-1} (B^c)))^c ⇒ f^{-1} (Cl^* (B)) $\leq$ sCl^* (f^{-1} (B)).

(4) ⇒ (3): Similar to above.

(3) ⇒ (2): Let A ϵ I^X and B = f (A). By (3), $sInt^*$ (A) $\leq$ $sInt^*$ (f^{-1} (f (A))) $\leq$ $sInt^*$ (f^{-1} (B)) $\leq$ f^{-1} (Int^* (B)).

∴ f ($sInt^*$ (A)) $\leq$ f (f^{-1} (Int^* (B))) = Int^* (B) = Int^* (f (A)).

(2) ⇒ (1): Let A ϵ FSSOS (X). By (2), f (A) = f ($sInt^*$ (A)) $\leq$ Int^* (f (A)) $\leq$ f (A).

∴ Int^* (f (A)) = f (A) ⇒ f (A) ϵ FSOS (Y).

Theorem 3.3: Let f*:* X→Y be a fuzzy supra S-open map. Then f^{-1} (∂^* (B)) $\leq$ $s\partial^*$ (f^{-1} (B)) for each B ϵ I^Y.

Proof: Let B ϵ I^Y, then by (4) of the above theorem, f^{-1} (∂^* (B)) = f^{-1} (Cl^* (B) $\wedge$ Cl^* (B^c)) = f^{-1} (Cl^* (B)) $\wedge$ f^{-1} (Cl^* (B^c)) $\leq$ sCl^* (f^{-1} (B)) $\wedge$ sCl^* (f^{-1} (B^c)) = sCl^* (f^{-1} (B))

$\wedge$ $sCl^*(f^{-1}(B))^c = s\partial^*(f^{-1}(B))$.

In [2] we established for a fuzzy supra semi continuous function f*:* X→Y that $s\partial^*(f^{-1}(B)) \leq f^{-1}(\partial^*(B))$ for each fuzzy set B of Y. Hence we have the following theorem

Theorem 3.4: If f*:* X→Y is a fuzzy supra semi continuous and S-open map. Then $s\partial^*(f^{-1}(B)) = f^{-1}(\partial^*(B))$ for each $B \in I^Y$.

Theorem 3.5: Let f*:* X→Y be a mapping between two FSTS's. Then the following are equivalent:

(1) f is fuzzy supra S-closed;

(2) $Cl^*(f(A)) \leq f(sCl^*(A))$ for each fuzzy set A of X.

Proof:

(1) ⇒ (2): Let f be fuzzy supra S-closed, then $f(sCl^*(A)) \in FSCS(Y)$ for each $A \in I^X$.

$\therefore Cl^*(f(A)) \leq Cl^*(f(sCl^*(A))) = f(sCl^*(A))$.

(2) ⇒ (1): Let $A \in FSSCS(X)$. By (2), $f(A) = f(sCl^*(A)) \geq Cl^*(f(A)) \Rightarrow f(A) = Cl^*(f(A)) \Rightarrow f(A) \in FSCS(Y)$.

Theorem 3.6: Let f*:* X→Y be a fuzzy supra S-closed map. Then for each $B \in I^Y$ and $\mu \in FSSOS(X)$ with $f^{-1}(B) \leq \mu$, there exists $\nu \in FSOS(Y)$ such that $B \leq \nu$ and $f^{-1}(\nu) \leq \mu$.

Proof: Since f is fuzzy supra S-closed and $\mu \in FSSOS(X)$, so $\nu = (f(\mu^c))^c \in FSOS(Y)$.

Again $f^{-1}(B) \leq \mu \Rightarrow f^{-1}(B^c) \geq \mu^c \Rightarrow B^c \geq f(f^{-1}(B^c)) \geq f(\mu^c) \Rightarrow B \leq (f(\mu^c))^c = \nu$ and $f^{-1}(\nu) = f^{-1}((f(\mu^c))^c) = (f^{-1}(f(\mu^c)))^c \leq (\mu^c)^c = \mu$.

Theorem 3.7: Let f*:* X→Y be a mapping between two FSTS's and let for each $B \in I^Y$ and $\mu \in FSSOS(X)$ with $f^{-1}(B) \leq \mu$, there exists $\nu \in FSOS(Y)$ such that $B \leq \nu$ and $f^{-1}(\nu) \leq \mu$, then f is fuzzy supra S-closed.

Proof: Let $A \in FSSCS(X)$, then $\mu = A^c \in FSSOS(X)$. Let $B_i \leq (f(A))^c$ be any fuzzy set of Y. Then $f^{-1}(B_i) \leq f^{-1}((f(A))^c) \leq (f^{-1}(f(A)))^c \leq A^c = \mu$. So, there exists $\nu_i \in FSOS(Y)$ such that $B_i \leq \nu_i$ and $f^{-1}(\nu_i) \leq \mu = A^c$.

$\therefore A \leq (f^{-1}(\nu_i))^c = f^{-1}(\nu_i^c) \Rightarrow f(A) \leq f(f^{-1}(\nu_i^c)) \leq \nu_i^c \Rightarrow \nu_i \leq (f(A))^c$. Thus for each fuzzy set $B_i \leq (f(A))^c$, there exists a fuzzy supra open set ν_i in Y such that $B_i \leq \nu_i \leq (f(A))^c$.

$\therefore (f(A))^c = \vee\{\nu_i: B_i \leq (f(A))^c, B_i \in I^Y\}$ is fuzzy supra open in Y ⇒ $f(A) \in FSCS(Y)$.

Theorem 3.8: Let f*:* X→Y be a mapping between two FSTS's. Then the following are equivalent:

(1) f is fuzzy supra S-closed;

(2) $Cl^*(f(A)) \leq f(sCl^*(A))$ for each fuzzy set A of X;

(3) for each $B \in I^Y$ and $\mu \in$ FSSOS (X) with $f^{-1}(B) \leq \mu$, there exists $\nu \in$ FSOS (Y) such that $B \leq \nu$ and $f^{-1}(\nu) \leq \mu$.

Theorem 3.9: Let f: X→Y be a fuzzy supra S-open map. Then for each $B \in I^Y$ and $\mu \in$ FSSCS (X) with $f^{-1}(B) \leq \mu$, there exists $\in$ FSCS (Y) such that $B \leq \nu$ and $f^{-1}(\nu) \leq \mu$.

Acknowledgement

The first author is thankful to the University Grant Commission, New Delhi, India for its financial support in the form of Minor Research Project (No: F.5-135/2012-13/MRP/NERO).

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