A Short Note on Fuzzy $\sigma$-Baire Spaces

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Abstract

In this paper several characterizations of fuzzy $\sigma$-Baire spaces are studied and the conditions under which a fuzzy topological space becomes a fuzzy $\sigma$-Baire space are investigated.

KEY WORDS: Fuzzy $G_\delta$-set, Fuzzy $F_\sigma$-set, fuzzy dense, fuzzy nowhere dense, fuzzy $\sigma$-nowhere dense, fuzzy $\sigma$-P-space, fuzzy hyperconnected space, fuzzy Baire space.

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INTRODUCTION

The fundamental concept of a fuzzy set introduced by Zadeh [11] in 1965, provides a natural foundation for building new branches of fuzzy mathematics. After the discovery of the fuzzy sets, many attempts have been made to extend various branches of mathematics to the fuzzy setting. Fuzzy topological spaces as a very natural generalization of topological spaces were first put forward in the literature by Chang [3] in 1968. Many authors used Chang’s definition in many direction to obtain some results which are compatible with results in general topology.

The concept of $\sigma$-nowhere dense set was introduced and studied by Jiling Cao and Sina Greenwood [4]. The concept of $\sigma$-nowhere dense set in fuzzy setting was introduced by the authors and by means of fuzzy $\sigma$-nowhere dense sets, the concept of fuzzy $\sigma$-Baire spaces was introduced and studied in [9]. In this paper, we investigate several characterizations of fuzzy $\sigma$-Baire spaces and the characterizations of fuzzy $\sigma$-first category sets in fuzzy $\sigma$-P-spaces are studied.
Preliminaries

By a fuzzy topological space we shall mean a non-empty set X together with a topology T (in the sense of Chang) and denote it by (X,T).

Definition 2.1: Let λ and μ be any two fuzzy sets in a fuzzy topological space (X,T). Then we define

1. \( \lambda \lor \mu : X \to [0,1] \) as follows: \( (\lambda \lor \mu)(x) = \text{Max} \{ \lambda(x), \mu(x) \} \) where \( x \in X \),
2. \( \lambda \land \mu : X \to [0,1] \) as follows: \( (\lambda \land \mu)(x) = \text{Min} \{ \lambda(x), \mu(x) \} \) where \( x \in X \),
3. \( \mu = \lambda^c \iff \mu(x) = 1 - \lambda(x) \) where \( x \in X \).

More generally, for a family \( \{ \lambda_i / i \in I \} \) of fuzzy sets in (X,T), the union \( \psi = \lor_i \lambda_i \) and intersection \( \delta = \land_i \lambda_i \) are defined respectively as \( \psi(x) = \text{Sup}_i \{ \lambda_i(x), x \in X \} \) and \( \delta(x) = \text{inf}_i \{ \lambda_i(x), x \in X \} \).

Definition 2.2 [1]: Let (X,T) be a fuzzy topological space. For a fuzzy set λ of X, the interior and the closure of \( \lambda \) are defined respectively as \( \text{int}(\lambda) = \lor \{ \mu / \mu \leq \lambda, \mu \in T \} \) and \( \text{cl}(\lambda) = \land \{ \mu / \mu \leq \lambda, 1 - \mu \in T \} \).

Lemma 2.1 [1]: Let \( \lambda \) be any fuzzy set in a fuzzy topological space (X,T). Then \( 1 - \text{cl}(\lambda) = \text{int}(1 - \lambda) \) and \( 1 - \text{int}(\lambda) = \text{cl}(1 - \lambda) \).

Definition 2.3 [2]: Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then λ is called a fuzzy G_δ-set if \( \lambda = \land_{i=1}^{\infty} (\lambda_i) \) for each \( \lambda_i \in T \).

Definition 2.4 [2]: Let (X,T) be a fuzzy topological space and λ a fuzzy set in X. Then λ is called a fuzzy F_σ-set if \( \lambda = \lor_{i=1}^{\infty} (\lambda_i) \) for each \( 1 - \lambda_i \in T \).

Lemma 2.2 [1]: For a family A = \( \{ \lambda_a \} \) of fuzzy sets of a fuzzy space X, \( \lor (\text{cl}(\lambda_a)) \leq \text{cl}(\lor (\lambda_a)) \). In case A is a finite set, \( \lor (\text{int}(\lambda_a)) = \text{cl}(\lor (\lambda_a)) \). Also \( \lor (\text{int}(\lambda_a)) \leq \text{int}(\lor (\lambda_a)) \).

Definition 2.5 [6]: A fuzzy set \( \lambda \) in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that \( \lambda < \mu < 1 \). That is, \( \text{cl}(\lambda) = 1 \).

Definition 2.6 [7]: A fuzzy set \( \lambda \) in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that \( \lambda < \text{cl}(\lambda) \). That is, \( \text{int}\text{cl}(\lambda) = 0 \).

Definition 2.7 [9]: A fuzzy set \( \lambda \) in a fuzzy topological space (X,T) is called fuzzy σ-nowhere dense if \( \lambda \) is a fuzzy F_σ-set in (X,T) such that \( \text{int}(\lambda) = 0 \).

Definition 2.8 [9]: A fuzzy set \( \lambda \) in a fuzzy topological space (X,T) is called a fuzzy σ-first category set if \( \lambda = \lor_{i=1}^{\infty} (\lambda_i) \), where \( \lambda_i \)'s are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy σ-second category.

Definition 2.9 [9]: Let \( \lambda \) be a fuzzy σ-first category set in a fuzzy topological space (X,T). Then, the fuzzy set \( 1 - \lambda \) is called a fuzzy σ-residual set in (X,T).

Definition 2.10 [10]: A fuzzy topological space (X,T) is called an fuzzy P-space, if
countable intersection of fuzzy open sets in \((X,T)\) is fuzzy open. That is, every non-zero fuzzy \(G_δ\)-set in \((X,T)\) is fuzzy open in \((X,T)\).

**Definition 2.11 [5]:** A fuzzy topological space \((X,T)\) is called a fuzzy hyperconnected space if every non-null fuzzy open set is fuzzy dense in \((X,T)\).

**FUZZY \(σ\)-BAIRE SPACES**

**Definition 3.1 [9]:** Let \((X,T)\) be a fuzzy topological space. Then \((X,T)\) is called a fuzzy \(σ\)-Baire space if \(\text{int}(\bigvee_{i=1}^{∞} (\lambda_i)) = 0\), where the fuzzy sets \((\lambda_i)\)'s are fuzzy \(σ\)-nowhere dense sets in \((X,T)\).

**Theorem 3.1 [9]:** Let \((X,T)\) be a fuzzy topological space. Then the following are equivalent:

1. \((X,T)\) is a fuzzy \(σ\)-Baire space.
2. \(\text{int}(\lambda) = 0\) for every fuzzy \(σ\)-first category set \(\lambda\) in \((X,T)\).
3. \(\text{cl}(\mu) = 1\) for every fuzzy \(σ\)-residual set \(\mu\) in \((X,T)\).

The following proposition ensures the existence of fuzzy first category sets in a fuzzy topological space if there are fuzzy \(σ\)-nowhere dense sets in \((X,T)\).

**Proposition 3.1:** If \(\lambda\) is a fuzzy \(σ\)-nowhere dense set in a fuzzy topological space \((X,T)\), then \(\lambda\) is a fuzzy first category set in \((X,T)\).

**Proof:** Let \(\lambda\) be a fuzzy \(σ\)-nowhere dense set in a fuzzy topological space \((X,T)\). Then there is a fuzzy fuzzy \(F_σ\)-set such that \(\text{int} (\lambda) = 0\). Since \(\lambda\) is a fuzzy \(F_σ\)-set, \(\lambda = \bigvee_{i=1}^{∞} (\lambda_i)\), where \(\lambda_i\)'s are fuzzy closed sets in \((X,T)\). Now \(\text{int} (\lambda) = 0\) implies that \(\text{int}(\bigvee_{i=1}^{∞} (\lambda_i)) = 0\). But, from Lemma 2.2, we have \(\bigvee_{i=1}^{∞} \text{int}(\lambda_i) \leq \text{int}(\bigvee_{i=1}^{∞} (\lambda_i))\). This implies that \(\bigvee_{i=1}^{∞} \text{int}(\lambda_i) = 0\). That is, \(\bigvee_{i=1}^{∞} \text{int}(\lambda_i) = 0\). Hence we have \(\text{int}(\lambda_i) = 0\) for each \(i\). Since \((\lambda_i)\)'s are fuzzy closed, \(\text{cl}(\lambda_i) = \lambda_i\). Then \(\text{intcl}(\lambda_i) = \text{int}(\lambda_i) = 0\). That is, \(\text{intcl}(\lambda_i) = 0\) for each \(i\). Hence \((\lambda_i)\)'s are fuzzy nowhere dense sets in \((X,T)\). Therefore \(\lambda = \bigvee_{i=1}^{∞} (\lambda_i)\), where \((\lambda_i)\)'s are fuzzy nowhere dense sets, implies that \(\lambda\) is a fuzzy first category set in \((X,T)\).

The existence of fuzzy nowhere dense sets which are fuzzy \(F_σ\), ensures the existence of fuzzy \(σ\)-nowhere dense sets in a fuzzy topological space.

**Proposition 3.2:** If the fuzzy nowhere dense set \(\lambda\) is a fuzzy \(F_σ\)-set in a fuzzy topological space \((X,T)\), then \(\lambda\) is a fuzzy \(σ\)-nowhere dense set in \((X,T)\).

**Proof:** Let the fuzzy nowhere dense set \(\lambda\) be a fuzzy \(F_σ\)-set in \((X,T)\). Then \(\lambda = \bigvee_{i=1}^{∞} (\lambda_i)\), where \(1 - \lambda_i \in T\). Since \(\lambda\) is a fuzzy nowhere dense set, we have \(\text{intcl}(\lambda) = 0\) in \((X,T)\). Now \(\text{int}(\lambda) \leq \text{intcl}(\lambda)\) implies that \(\text{int}(\lambda) = 0\). Hence \(\lambda\) is fuzzy \(F_σ\)-set such that \(\text{int}(\lambda) = 0\). Then \(\lambda\) is a fuzzy \(σ\)-nowhere dense set in \((X,T)\).

**Proposition 3.3:** If each fuzzy nowhere dense set \(\lambda\) is a fuzzy \(F_σ\)-set in a fuzzy topological space \((X,T)\), then \(\lambda\) is a fuzzy first category set in \((X,T)\).
Proof: Let each fuzzy nowhere dense set $\lambda$ be a fuzzy $F_\sigma$-set in $(X,T)$. Since $\lambda$ is a fuzzy nowhere dense set in $(X,T)$, we have $intcl(\lambda) = 0$. Now $\text{int} (\lambda) \leq \text{intcl}(\lambda)$ implies that $\text{int} (\lambda) \leq 0$. That is, $\text{int} (\lambda) = 0$. Hence $\lambda$ is a fuzzy $F_\sigma$-set such that $\text{int} (\lambda) = 0$. Then $\lambda$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$. Then, by proposition 3.1, $\lambda$ is a fuzzy first category set in $(X,T)$.

Proposition 3.4: If a fuzzy $\sigma$-first category set $\lambda$ is a fuzzy closed set in a fuzzy $\sigma$-Baire space $(X,T)$, then $\lambda$ is a fuzzy nowhere dense set in $(X,T)$.

Proof: Let $\lambda$ be a fuzzy $\sigma$-first category set in $(X,T)$. Since $(X,T)$ is a fuzzy $\sigma$-Baire space by theorem 3.1, $\text{int} (\lambda) = 0$. By hypothesis, $\lambda$ is fuzzy closed in $(X,T)$. This implies that $\text{cl}(\lambda) = \lambda$. Then $\text{int cl}(\lambda) = \text{int} (\lambda) = 0$. That is, $\text{int cl}(\lambda) = 0$ in $(X,T)$. Hence $(X,T)$ is a fuzzy nowhere dense set in $(X,T)$.

Proposition 3.5: If a fuzzy $\sigma$-first category set in fuzzy topological space $(X,T)$ is a fuzzy nowhere dense set, then $(X,T)$ is a fuzzy $\sigma$-Baire space.

Proof: Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy topological space $(X,T)$ such that $\text{int cl}(\lambda) = 0$. Then $\text{int} (\lambda) \leq \text{intcl}(\lambda)$ implies that $\text{int} (\lambda) = 0$. Hence, for a fuzzy $\sigma$-first category set $\lambda$ in $(X,T)$, we have $\text{int} (\lambda) = 0$. Hence by theorem 3.1, $(X,T)$ is a fuzzy $\sigma$-Baire space.

Theorem 3.2 [8]: If $\lambda$ is a fuzzy first category set in $(X,T)$, then there is a fuzzy $F_\sigma$-set in $(X,T)$ such that $\lambda \leq \eta$.

Proposition 3.6: If $\lambda$ is a fuzzy $\sigma$-nowhere dense set in a fuzzy topological space $(X,T)$, then there exists a fuzzy $F_\sigma$-set $\eta$ in $(X,T)$ such that $\lambda \leq \eta$.

Proof: Let $\lambda$ be a fuzzy $\sigma$-nowhere dense set in a fuzzy topological space $(X,T)$. Then by proposition 3.1, $\lambda$ is a fuzzy first category set in $(X,T)$. Then by theorem 3.2, there is a fuzzy $F_\sigma$-set in $(X,T)$ such that $\lambda \leq \eta$.

Proposition 3.7: If $(X,T)$ is a fuzzy $\sigma$-first category space, then $\nu_{k=1}^\omega (\alpha_k) = 1$, where $(\alpha_k)$'s are fuzzy closed sets in $(X,T)$.

Proof: Let $(X,T)$ be a fuzzy $\sigma$-first category space. Then $\nu_{i=1}^\omega (\lambda_i) = 1$, where $(\lambda_i)$'s are fuzzy $\sigma$-nowhere dense sets in $(X,T)$. Since $(\lambda_i)$'s ( $i = 1$ to $\infty$) are fuzzy $\sigma$-nowhere dense sets in $(X,T)$, by proposition 3.6, there exists $F_\sigma$-sets $\eta_i$ in $(X,T)$ such that $\lambda_i \leq \eta_i$. Since $(\eta_i)$'s ( $i = 1$ to $\infty$) are fuzzy $F_\sigma$-sets, $\eta_i = \nu_{j=1}^\omega (\delta_{ij})$, where $(\delta_{ij})$'s are fuzzy closed sets in $(X,T)$. Now $\lambda_i \leq \eta_i$ implies that $\nu_{i=1}^\omega (\lambda_i) \leq \nu_{i=1}^\omega (\eta_i) = \nu_{i=1}^\omega (\nu_{j=1}^\omega (\delta_{ij}))$. Let $\nu_{i=1}^\omega (\nu_{j=1}^\omega (\delta_{ij}) = \nu_{k=1}^\omega (\alpha_k)$, where $k = ij$ and $(\alpha_k)$'s are fuzzy closed sets in $(X,T)$. Hence $\nu_{i=1}^\omega (\lambda_i) \leq \nu_{k=1}^\omega (\alpha_k)$. Then from (1), $1 \leq \nu_{k=1}^\omega (\alpha_k)$. Hence we have $\nu_{k=1}^\omega (\alpha_k) = 1$, where $(\alpha_k)$'s are fuzzy closed sets in $(X,T)$.

Proposition 3.8: If $(\lambda_i)$'s ( $i = 1$ to $\infty$) are fuzzy nowhere dense sets such that $\mu_i \leq \lambda_i$, where $(\mu_i)$'s are fuzzy $F_\sigma$-sets in a fuzzy Baire space $(X,T)$, then $(X,T)$ is a fuzzy $\sigma$-Baire space.

Proof: Let $\lambda_i$ ( $i = 1$ to $\infty$) be a fuzzy nowhere dense set in a fuzzy Baire space $(X,T)$.
such that $\mu_i \leq \lambda_i$, where $\mu_i$ is a fuzzy $F_\sigma$-set in $(X,T)$. Then $\text{int}(\mu_i) \leq \text{int}(\lambda_i)$......(1). Since $\lambda_i$ is a fuzzy nowhere dense set, $\text{int cl}(\lambda_i) = 0$. Now $\text{int}(\lambda_i) \leq \text{int cl}(\lambda_i)$ implies that $\text{int}(\lambda_i) = 0$. Hence we have from (1) $\text{int}(\mu_i) \leq 0$. That is, $\text{int}(\mu_i) = 0$ and hence $(\mu_i)$ is a fuzzy $F_\sigma$-set such that $\text{int}(\mu_i) = 0$. This implies that $\mu_i$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$. Now $\mu_i \leq \lambda_i$ implies that $\nu_{i=1}^\infty(\mu_i) \leq \nu_{i=1}^\infty(\lambda_i)$ and hence $\text{int}(\nu_{i=1}^\infty(\mu_i)) \leq \text{int}(\nu_{i=1}^\infty(\lambda_i))$. Since $(X,T)$ is a fuzzy Baire space $\text{int}(\nu_{i=1}^\infty(\lambda_i)) = 0$. This implies that $\text{int}(\nu_{i=1}^\infty(\mu_i)) = 0$. That is, $\text{int}(\nu_{i=1}^\infty(\mu_i)) = 0$, where $(\mu_i)$’s are fuzzy-$\sigma$-nowhere dense sets in $(X,T)$. Therefore, $(X,T)$ is a fuzzy $\sigma$-Baire space.

**Fuzzy $\sigma$-First Category Sets and Fuzzy P-Spaces**

**Proposition 4.1:** If $\lambda$ is a fuzzy $\sigma$–first category set in a fuzzy P-space $(X,T)$, then $\lambda$ is not a fuzzy dense set in $(X,T)$.

**Proof:** Let us assume the contrary. Suppose that $\lambda$ is a fuzzy $\sigma$-first category set in $(X,T)$ such that $cl(\lambda) = 1$. Then $\lambda = \nu_{i=1}^\infty(\lambda_i)$, where $(\lambda_i)$’s are fuzzy $\sigma$-nowhere dense sets in $(X,T)$. Since $(\lambda_i)$’s are fuzzy-$\sigma$-nowhere dense sets, $(\lambda_i)$’s are fuzzy-$F_\sigma$-sets and $\text{int}(\lambda_i) = 0$ ($i = 1$ to $\infty$). Then $(1 - \lambda_i)$’s are fuzzy $G_\delta$-sets in $(X,T)$. Since $(X,T)$ is a fuzzy P-space, the fuzzy $G_\delta$-sets $(1 - \lambda_i)$’s are fuzzy open sets in $(X,T)$. Let $\mu = \lambda_{i=1}^\infty(1 - \lambda_i)$. Then $\mu$ is a non-zero fuzzy $G_\delta$-set in $(X,T)$. Since $(X,T)$ is a fuzzy P-space, $\mu$ is a fuzzy open set in $(X,T)$ and hence $\text{int}(\mu) = \mu$. Now $\mu = 1 - \nu_{i=1}^\infty(\lambda_i) = 1 - \lambda$. Then we have $\text{int}(\mu) = \text{int}(1 - \lambda) = 1 - \text{cl}(\lambda) = 1 - 1 = 0$, a contradiction. Hence, in a fuzzy P-space $(X,T)$, fuzzy $\sigma$-first category sets are not fuzzy dense sets in $(X,T)$.

**Proposition 4.2:** If $\lambda$ is a fuzzy $\sigma$-first category set in a fuzzy P-space then $\text{int}(1 - \lambda) \neq 0$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy P-space $(X,T)$. Then by proposition 4.1, $\lambda$ is not a fuzzy dense set. That is, $cl(\lambda) \neq 1$. Then, $1 - cl(\lambda) \neq 0$. This implies that $\text{int}(1 - \lambda) \neq 0$.

**Remark:** In view of the above proposition 4.2, we have the following: In a fuzzy P-space $(X,T)$, the interior of a fuzzy $\sigma$-residual set is non-zero in $(X,T)$.

**Proposition 4.3:** If $\lambda$ is a fuzzy $\sigma$-first category set in a fuzzy P-space $(X,T)$, then $\lambda$ is a fuzzy $F_\sigma$-set in $(X,T)$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in $(X,T)$. Then $\lambda = \nu_{i=1}^\infty(\lambda_i)$, where $(\lambda_i)$’s are fuzzy $\sigma$-nowhere dense sets in $(X,T)$. Since $(\lambda_i)$’s are fuzzy $\sigma$-nowhere dense sets, $(\lambda_i)$’s are fuzzy $F_\sigma$-sets and $\text{int}(\lambda_i) = 0$ ($i = 1$ to $\infty$). Then $(1 - \lambda_i)$’s are fuzzy $G_\delta$-sets in $(X,T)$. Since $(X,T)$ is a fuzzy P-space, the fuzzy $G_\delta$-sets $(1 - \lambda_i)$’s are fuzzy open sets in $(X,T)$. Let $\mu = \lambda_{i=1}^\infty(1 - \lambda_i)$. Then $\mu$ is a non-zero fuzzy $G_\delta$-set in $(X,T)$. Then, from $\mu = 1 - \nu_{i=1}^\infty(\lambda_i)$, we have $\mu = 1 - \lambda$ and hence $\lambda = 1 - \mu$. Since $\mu$ is a fuzzy $G_\delta$-set in $(X,T)$, $\lambda$ is a fuzzy $F_\sigma$-set in $(X,T)$. Hence in a fuzzy P-space $(X,T)$ fuzzy $\sigma$-first category sets are fuzzy $F_\sigma$-sets in $(X,T)$. 


If $\lambda$ and $\mu$ are any two fuzzy $\sigma$-nowhere dense sets in a fuzzy topological space $(X,T)$, then $\lambda \cup \mu$ need not be a fuzzy $\sigma$-nowhere dense set in $(X,T)$. If $(\lambda_i)$’s ($i = 1$ to $\infty$) are fuzzy $\sigma$-nowhere dense sets in a fuzzy topological space $(X,T)$, when $\bigcup_{i=1}^{\infty} (\lambda_i)$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$? The answer for this question is obtained in the following proposition.

**Proposition 4.4:** If $(X,T)$ is a fuzzy $\sigma$-Baire and fuzzy P-space, then each fuzzy $\sigma$-first category set is a fuzzy $\sigma$-nowhere dense set in $(X,T)$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy $\sigma$-Baire and fuzzy P-space $(X,T)$. Then $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)$’s are fuzzy $\sigma$-nowhere dense sets in $(X,T)$. Since $(X,T)$ is a fuzzy $\sigma$-Baire space, by theorem 3.1, $\text{int} (\lambda) = 0$. Since $(X,T)$ is a fuzzy P-space, by proposition 4.3, the fuzzy $\sigma$-first category set $\lambda$ is a fuzzy $F_\sigma$-set in $(X,T)$. Hence $\lambda$ is a fuzzy $F_\sigma$-set such that $\text{int} (\lambda) = 0$. Then $\lambda$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$. Hence, if $(X,T)$ is a fuzzy $\sigma$-Baire and fuzzy P-space, then each fuzzy $\sigma$-first category set is a fuzzy $\sigma$-nowhere dense set in $(X,T)$.

**Proposition 4.5:** If $\lambda$ is a fuzzy $\sigma$-first category set in a fuzzy hyperconnected and fuzzy P-space $(X,T)$, then $\lambda$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy hyperconnected and fuzzy P-space $(X,T)$. Since $(X,T)$ is a fuzzy P-space, by proposition 4.3, $\lambda$ is a fuzzy $F_\sigma$-set and hence $1 - \lambda$ is a fuzzy $G_\sigma$-set. Since $(X,T)$ is a fuzzy P-space, $1 - \lambda$ is fuzzy open in $(X,T)$. Again since $(X,T)$ is a fuzzy hyperconnected space the fuzzy open set $1 - \lambda$ is fuzzy dense in $(X,T)$. That is $\text{cl}(1 - \lambda) = 1$. Then we have $\text{int} (\lambda) = 0$. Hence $\lambda$ is a fuzzy $F_\sigma$-set such that $\text{int} (\lambda) = 0$. Therefore, in a fuzzy hyperconnected and fuzzy P-space $(X,T)$, the fuzzy $\sigma$-first category set $\lambda$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$.

**Proposition 4.6:** If $\lambda$ is a fuzzy $\sigma$-first category set in a fuzzy hyperconnected and fuzzy P-space $(X,T)$, then $\lambda$ is a fuzzy first category set in $(X,T)$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy hyperconnected and fuzzy P-space $(X,T)$. Then by proposition 4.5, $\lambda$ is a fuzzy $\sigma$-nowhere dense set in $(X,T)$. Then by proposition 3.1, $\lambda$ is a fuzzy first category set in $(X,T)$.

**Proposition 4.7:** If $(X,T)$ is a fuzzy Baire, fuzzy P-space and fuzzy hyperconnected space, then $(X,T)$ is a fuzzy $\sigma$-Baire space.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy Baire, fuzzy P-space and fuzzy hyperconnected space $(X,T)$. Then by proposition 4.6, $\lambda$ is a fuzzy first category set in $(X,T)$. Since $(X,T)$ is a fuzzy Baire space, by theorem 3.1, $\text{int} (\lambda) = 0$. Hence, for the fuzzy $\sigma$-first category set $\lambda$ in $(X,T)$, we have $\text{int} (\lambda) = 0$. Then, by theorem 3.1, $(X,T)$ is a fuzzy $\sigma$-Baire space.

**Proposition 4.8:** If each fuzzy open set is fuzzy dense in a fuzzy Baire and fuzzy P-space $(X,T)$, then $(X,T)$ is a fuzzy $\sigma$-Baire space.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-first category set in a fuzzy Baire and fuzzy P-space $(X,T)$. Since $(X,T)$ is a fuzzy P-space by proposition 4.3, the fuzzy $\sigma$-first category set $\lambda$ is a
fuzzy $F_\sigma$-set in $(X,T)$. Then $1 - \lambda$ is a fuzzy $G_\delta$-set in $(X,T)$. Again, since $(X,T)$ is a fuzzy P-space, $1 - \lambda$ is a fuzzy open in $(X,T)$. By hypothesis, $1 - \lambda$ is fuzzy dense in $(X,T)$. That is, $cl(1 - \lambda) = 1$. Then $1 - int(\lambda) = 1$ and hence $int(\lambda) = 0$. Hence, for fuzzy $\sigma$-first category set $\lambda$ in $(X,T)$, we have $int(\lambda) = 0$. Then, by theorem 3.1, $(X,T)$ is a fuzzy $\sigma$-Baire space.

**Proposition 4.9:** If $\lambda$ is a fuzzy $\sigma$-nowhere dense set in a fuzzy P-space $(X,T)$, then $\lambda$ is not a fuzzy dense set in $(X,T)$.

**Proof:** Let $\lambda$ be a fuzzy $\sigma$-nowhere dense set in a fuzzy P-space $(X,T)$. Then by proposition 3.6, there exists a fuzzy $F_\sigma$-set $\eta$ in $(X,T)$ such that $\lambda \leq \eta$. Then $1 - \lambda \geq 1 - \eta$. Since $\eta$ is a fuzzy $F_\sigma$-set, $1 - \eta$ is a fuzzy $G_\delta$-set in $(X,T)$. Since $(X,T)$ is a fuzzy P-space, $1 - \eta$ is a fuzzy open set in $(X,T)$. That is, $int(1 - \eta) = 1 - \eta$......(1). Now $\eta \leq \eta$ implies that $1 - \lambda \geq 1 - \eta$. Then $int(1 - \lambda) \geq int(1 - \eta)$ and hence $int(1 - \lambda) \geq 1 - \eta$ (from (1)). Since $1 - \eta \neq 0$, $int(1 - \lambda) \neq 0$. Then $1 - cl(\lambda) \neq 0$. This implies that $cl(\lambda) \neq 1$. Hence, if $\lambda$ is a fuzzy $\sigma$-nowhere dense set in a fuzzy P-space, then $cl(\lambda) \neq 1$. Therefore, $\lambda$ is not a fuzzy dense set in $(X,T)$.

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