Soft pre $T_1$ Space in the Soft Topological Spaces

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Abstract

This paper introduces soft pre $T_1$ space in the soft topological spaces. The notations of soft pre interior and soft pre closure are generalized using these sets. In a soft topological space, a soft set $F_A$ is said to be soft pre-open set (soft P-open) if there exists a soft open set $F_O$ such that $F_A \subseteq F_O \subseteq \overline{F_A}$. Its complement is known as soft p-closed set of $F_A$ denoted by $F_{sp}(F_E, \tau)$. A detail study is carried out on properties of soft $PT_1$ Space.

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1. Introduction.

Soft set theory was first introduced by Molodtsov [3] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. Modern topology depends strongly on the ideas of set theory. But in 2010 Muhammad shabir, Munazza Naz [4] used soft sets to define a topology namely Soft topology and defined soft separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In 2013 J. Subhashini and C. Sekar defined soft pre-open sets [7] in a soft topological spaces. In this paper we introduce soft pre separation axioms, soft $PT_0$ – Space and some of its properties.
2. Preliminaries For basic notations and definitions not given here, the reader can refer [1-8].

2. 1. Definition. [4] A soft set $F_A$ on the universe $U$ is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$, where $E$ is a set of parameters, $A \subseteq E$, $P(U)$ is the power set of $U$, and $f_A: A \rightarrow P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here, $f_A$ is called an approximate function of the soft set $F_A$. The value of $f_A(e)$ may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over $U$ is denoted by $S(U)$.

2. 2. Example. Suppose that there are five cars in the universe. Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ under consideration, and that $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ is a set of decision parameters. Then $e_i (i = 1, 2, 3, 4, 5, 6, 7, 8)$ stand for the parameters “expensive”, “beautiful”, “manual gear”, “cheap”, “automatic gear”, “in good repair”, “in bad repair” and “costly” respectively. In this case, to define a soft set means to point out expensive cars, beautiful cars and so on. Let $A \subseteq E$, the soft set $F_A$ that describes the “attractiveness of the cars” in the opinion of a buyer say Ram, may be defined like $A = \{e_2, e_3, e_4, e_5, e_7\}$, $f_A(e_2) = \{c_2, c_3, c_5\}$, $f_A(e_3) = \{c_2, c_4\}$, $f_A(e_4) = \{c_1\}$, $f_A(e_5) = \{U\}$ and $f_A(e_7) = \{c_3, c_5\}$. We can view this soft set $F_A$ as consisting of the following collection of approximations:

$F_A = \{(e_2, \{c_2, c_3, c_5\}), (e_3, \{c_2, c_4\}), (e_4, \{c_1\}), (e_5, \{U\}), (e_7, \{c_3, c_5\})\}$.

2. 3. Definition. [1] Let $F_A$ be a soft set over $U$ and for the point $x \in U$, we say that $x \in F_A$ read as $x$ belongs to the soft set $F_A$ whenever $x \in f_A(e)$ for all $e \in A$. Not that for any $x \in U$, $x \in U$ if and only if $x \in F_A$ for some $e \in A$.

2. 4. Definition. [1] The soft set $F_A$ over $U$ such that $f_A(e) = \{x\}$ for all $e \in A$ is called soft singleton and is denoted by $x_A$. It’s compliment is denoted by $x_A^c$.

2. 5. Definition. [7] Let $F_A \in S(U)$. The soft power set of $F_A$ is defined by $\hat{P}(F_A) = \{F_{A_i} : F_{A_i} \subseteq F_A, i \in I \subseteq N\}$ and its cardinality is defined by $|\hat{P}(F_A)| = 2^{\sum_{e \in A} f_A(e)}$, where $|f_A(e)|$ is the cardinality of $f_A(e)$.

2. 6. Example. [7] Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$, $A \subseteq E$, $A = \{e_1, e_2\}$ and $F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}$. Then $F_{A_1} = F_A$, $F_{A_2} = F_{\emptyset}$, $F_{A_3} = \{(e_1, \{h_1, h_2\})\}$, $F_{A_4} = \{(e_1, \{h_1\})\}$, $F_{A_5} = \{(e_2, \{h_1, h_2\})\}$, $F_{A_6} = \{(e_2, \{h_1\})\}$, $F_{A_7} = \{(e_1, \{h_2\})\}$, $F_{A_8} = \{(e_2, \{h_2\})\}$, $F_{A_9} = \{(e_1, \{h_2\})\}$, $F_{A_{10}} = \{(e_2, \{h_2\})\}$, $F_{A_{11}} = \{(e_2, \{h_1\})\}$, $F_{A_{12}} = \{(e_2, \{h_2\})\}$, $F_{A_{13}} = \{(e_1, \{h_1, h_2\})\}$, $F_{A_{14}} = \{(e_1, \{h_1, h_2\})\}$, $F_{A_{15}} = \{(e_2, \{h_2\})\}$, $F_{A_{16}} = \{(e_1, \{h_2\})\}$, $F_{A_{17}} = \{(e_2, \{h_2\})\}$ are all soft subset of $F_A$. So $|\hat{P}(F_A)| = 2^4 = 16$.

2. 7. Definition. [7] Let $F_E \in S(U)$. A soft topology on $E$ denoted by $\tau$ is a collection of soft subsets of $F_E$ having the following properties:

(i). $F_{\emptyset}, F_{E} \in \tau$

(ii). $\{F_{E_i} \subseteq F_E : i \in I \subseteq N\} \subseteq \tau \Rightarrow \bigcup_{i \in I} F_{E_i} \in \tau$

(iii). $\{F_{E_i} \subseteq F_E : 1 \leq i \leq n, n \in N\} \subseteq \tau \Rightarrow \bigcap_{i=1}^{n} F_{E_i} \in \tau$. 
The pair \((F_E, \tilde{\tau})\) or \((\widetilde{U}, \tilde{\tau}, E)\) is called a soft topological space.

2. 8. Example. Let us consider the soft subsets of \(F_A\) that are given in Example 2. 6. Then \(\tilde{\tau}_1 = \{F_\emptyset, F_A, F_{A^3}, F_{A^4}\}\), \(\tilde{\tau}_2 = \{F_\emptyset, F_A, F_{A^5}\}\), \(\tilde{\tau}_3 = \{\widetilde{P}(F_A)\}\) are soft topologies on \(F_A\).

2. 9. Definition [7] Let \((F_E, \tilde{\tau})\) be a soft topological space, a soft set \(F_A\) is said to be soft pre-open set (soft P-open) if there exists a soft open set \(F_O\) such that \(F_A \subseteq F_O \subseteq F_A\). The set of all soft P-open set of \(F_E\) is denoted by \(\mathcal{C}_{SP}(F_E, \tilde{\tau})\) or \(\mathcal{C}_{SP}(F_E)\). Its complement is known as soft p-closed set of \(F_A\) denoted by \(\mathcal{F}_{SP}(F_E, \tilde{\tau})\).

2. 10. Definition [7] Let \((F_E, \tilde{\tau})\) be a soft topological space and \(F_A \subseteq F_E\). Then the soft pre closure (soft P-closure) of \(F_A\) denoted by \(p(F_A)\) is defined as the soft intersection of all soft P-closed supersets of \(F_A\).

2. 11. Definition. [8] Let \((F_E, \tilde{\tau})\) be a soft topological space over \(U\), \(F_E \subseteq S(U)\) and Let \(V\) be a non null subset of \(U\). Then the soft subset of \(F_E\) over \(V\) denoted by \(vF_E\), is defined as follows:\(vF_E(e) = V \cap \tilde{\tau}_E(e)\), for all \(e \in E\). In other words \(vF_E = \widetilde{V} \cap F_E\).

2. 12. Theorem. [7] (i) Arbitrary soft union(intersection) of soft P-open(soft p-closed)sets is a soft P-open(soft p-closed) set. (ii) The soft intersection(union) of any two soft P-open(soft p-closed) set need not be a soft P-open(closed)set.

3. Soft Pre \(T_1\)-space. 

The soft pre separation axioms are various conditions that are sometimes imposed upon topological spaces which can be described in terms of various types of soft separated sets. We have introduced soft pre \(T_1\) (soft PT\(_1\)) spaces in a soft Topological spaces.

3. 1. Definition Let \((F_E, \tilde{\tau})\) be a soft topological space over \(U\), and \(x, y \in U\) such that \(x \neq y\). If there exist soft P-open sets \(F_O_1\) and \(F_O_2\) such that \(x \in F_O_1\) but \(y \notin F_O_1\) and \(y \in F_O_2\) but \(x \notin F_O_2\), then \((F_E, \tilde{\tau})\) is called a soft pre \(T_1\) space (soft PT\(_1\)-space).

3. 2. Example Consider a soft topological space \((F_A, \tilde{\tau})\) where \(\tilde{\tau} = \{F_\emptyset, F_A, F_{A^3}, F_{A^10}\}\) and consider the soft subset that is given in Example 2. 6. Where \(U = \{h_1, h_2\}\), \(A = \{e_1, e_2\}\), \(F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}\) and \(\tilde{\tau} = \{F_\emptyset, F_A, F_{A^3}, F_{A^10}\}\). The soft sets are defined as follows\(F_{A^3} = \{(e_1, \{U\})\}, F_{A^10} = \{(e_2, \{U\})\}\). The soft topological space \((F_A, \tilde{\tau})\) is also a soft PT\(_1\)-space. Since \(h_1, h_2 \in U\) and \(h_1 \neq h_2\) such that there exist a soft P-open sets \(F_{A^6}\) and \(F_{A^7}\) such that \(h_1 \notin F_{A^6}\) but \(h_2 \notin F_{A^6}\) and \(h_2 \notin F_{A^7}\) but \(h_1 \notin F_{A^7}\).

3. 3. Example Soft discrete topology is a soft PT\(_1\)-space.

3. 4. Example The soft indiscrete topological space is not a soft PT\(_1\) space.

3. 5. Example Consider the soft topological space \((F_A, \tilde{\tau})\) that is given in Example 2. 6. Where \(U = \{h_1, h_2\}\), \(E = \{e_1, e_2, e_3\}\), \(A \subseteq E\), \(A = \{e_1, e_2\}\) and \(F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}\). Then \(\tilde{\tau} = \{F_\emptyset, F_A, F_{A^1}\}\) where \(F_{A^1} = \{(e_1, \{U\}), (e_2, \{h_1, h_2\})\}\). Then \(\tilde{\tau}\) defines a soft topology on \(U\). But this is not a soft PT\(_1\)-space because \(h_1, h_2 \in U\) but there do not exist soft P-open sets \(F_O_1\) and \(F_O_2\) such that
h₁ ∈ F₀₁ but h₂ ̸∈ F₀₁ and h₂ ̸∈ F₀₂ but h₁ ̸∈ F₀₂.

3. 6. Theorem A soft subspace of a soft PT₁-space is soft PT₁-space.

Proof: Let (F_E, ℓ) be a soft PT₁-space over U, and (vF_E, vℓ) be a soft subspace of (F_E, ℓ) over V. Let x, y ∈ V such that x ̸= y. Since V ⊆ U, then x, y ∈ U such that x ̸= y. Since (F_E, ℓ) is a soft PT₀-space over U. Hence there exist soft P-open sets F₀₁ and F₀₂ in U such that x ̸∈ F₀₁ but y ̸∈ F₀₁ and y ̸∈ F₀₂ but x ̸∈ F₀₂. Since x ∈ V. Then x ∈ ̸V. Hence x ∈ ̸V ∩ F₀₁ = vF₀₁, F₀₁ is soft P-open set. Consider y ̸∈ F₀₁, this implies that y ̸∈ f₀₁(e) for some e ∈ E. Therefore, y ̸∈ ̸V ∩ F₀₁ = vF₀₁. Similarly, if y ̸∈ F₀₂ but x ̸∈ F₀₂, then y ̸∈ vF₀₂ but x ̸∈ vF₀₂. Thus (vF_E, vℓ) is also a soft PT₁-space. Therefore the property of being a soft PT₁-space is a hereditary property.

3. 7. Proposition Every soft PT₁-space is a soft topological space.

3. 8. Note Not all soft topological spaces are soft PT₁-spaces.

3. 9. Example Let us consider the soft topological space ℓ = {F₀₁,F₀₂,F₀₃}. But by Example 3. 5 it is not a soft PT₁-space.

3. 10. Theorem Let (F_E, ℓ) be a soft PT₁-space over U iff for each x ∈ U, every soft singleton xₑ of Fₑ is a soft P-closed set.

Proof: Necessity, Suppose that (F_E, ℓ) is soft PT₁-space over U and x ∈ U. We have to prove that the soft singleton xₑ is soft P-closed or alternatively xₑ is soft P-open set. Let y ∈ xₑ, then clearly y ̸= x. Now the soft space being soft PT₁ and y ̸= x so that there must exist a soft P-open set F₀₁ such that y ̸∈ F₀₁ but x ̸∈ F₀₁. Thus corresponding to each y ∈ xₑ there exist a soft p-open set F₀₁ such that y ̸∈ F₀₁ but x ̸∈ F₀₁. Since F₀₁ is soft p-closed and hence xₑ is soft p-closed. Therefore, (F_E, ℓ) is soft PT₁-space over U.

3. 11. Proposition A soft topological space (F_E, ℓ) is soft PT₁-space iff every soft subset of Fₑ is soft p-closed set.

Proof: Let (F_E, ℓ) be a soft PT₁-space and xₑ be the arbitrary soft subset of Fₑ. Since by Theorem 3. 10, the soft singleton xₑ is a soft p-closed. Now every soft subset of Fₑ is the soft union of finite number of soft singletons. Then by Theorem 2. 12 It is soft p-closed. Hence every soft subset of Fₑ is soft p-closed set. Conversely, if every soft subset is soft p-closed then as a particular case every soft singleton xₑ is also soft p-
closed set. Therefore by Theorem 3. 10 the soft space \((F_E, \tilde{t})\) is a soft \(PT_1\)-space.

4. Conclusion:
The initiation of notation of soft topological space was introduced by D. Molodtsov [5] in 1999. Many Mathematicians turned their attention to the various concepts of soft topological space. By this way, [6] Muhammad Shabir and Munazza Naz introduced the concept soft topological spaces. J. Subhashini and C. Sekar [11] introduced the concept of soft pre-open sets. In this paper, we continue this work and introduce soft \(PT_1\) spaces taking help of the soft preopen sets. Also we have studied some related properties. Finally we prove every soft topological space need not be a soft \(PT_1\)-Spaces.

Reference
