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Common Fixed Point of Absorbing Map in Intuitionistic Fuzzy Metric Space using General Contractive Condition of Integral Type

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Abstract

Using a notion of absorbing map, compatibility, reciprocally continuous a common fixed point theorem in an Intuitionistic Fuzzy Metric Space with EA property using contractive condition of integral type is proved. Our paper extend the result of Abhay S. Ranadive and S. Muralishankar.

Mathematics Subject Classification: 47H10, 54H25

Keywords: Fuzzy set, Intuitionistic fuzzy metric space, compatibility, absorbing map, EA property, reciprocally continuous, Integral type.

Introduction

Zadeh[15] introduced the concept of fuzzy set as a new way to represent vagueness in our everyday life. Since then, many authors regarding the theory of fuzzy sets and its applications have developed a lot of literatures.

The concept of fuzzy set corresponds to the degree of nearness between two objects. Deng[17], Erceg[18], George and Veeramani[4], Kaleva and Seikkala, Kramosil and Michalek[9] have introduced the concept of fuzzy metric spaces in different ways. In the study of fixed points of metric spaces, Pant[20] has initiated work using the concept of non compatibility maps in fuzzy metric spaces.Aamri and Moutawakil[19] introduces the E A property that have generalized the concept of non compatibility. Recently Mishra[16], Ranadive and Gopal[11] introduced the notion of

absorbing maps. They explore the possibility of applying the notion of reciprocal continuity and absorbing maps for.finding common.fixed points of four mappings.

Atanassove[2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Intuitionistic fuzzy sets deals with both degree of nearness and non-nearness. Park defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani. Further, Alaca[1] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space, as Park[10] with the help of continuous t-norms and continuous t-conorms, as a generalization of fuzzy metric space due to Kramosil and Michalek[9].

Preliminaries

Definition 2.1 : T-norm / Fuzzy intersection

A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) $a * 1 = a$ for all $a \in [0, 1]$;

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Examples of t- norm are

1) Standard Intersection: $a*b = \min\{a, b\}$ 2) Algebraic product : $a*b = ab$

Definition 2.2 : T-conorm / Fuzzy Union

A binary operation $\diamondsuit: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-conorm if $\diamondsuit$ is satisfying the following conditions:

(a) $\diamondsuit$ is commutative and associative;

(b) $\diamondsuit$ is continuous;

(c) $a \diamondsuit 0 = a$ for all $a \in [0, 1]$;

(d) $a \diamondsuit b \leq c \diamondsuit d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Examples of t- conorm are

1) Standard union: $a \diamondsuit b = \max\{a, b\}$ 2) Algebraic sum : $a \diamondsuit b = a+b-ab$

Definition 2.3 : Intuitionistic Fuzzy Metric Space :

A 5-tuple $(X, M, N, *, \diamondsuit)$ is said to be a Intuitionistic Fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and $\diamondsuit$ is a continuous t-conorm, M, N is a Fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, $s, t > 0$,

(IFM1) $M(x, y, t) + N(x, y, t) \leq 1$

(IFM2) $M(x, y, 0) = 0$;

(IFM3) $M(x, y, t) = 1$ if and only if $x = y$;

(IFM4) $M(x, y, t) = M(y, x, t)$;

(IFM5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(IFM6) $M(x, y, .) : [0, \infty) \rightarrow (0, 1]$ is continuous.
(IFM7) $\lim_{t\rightarrow\infty} M(x, y, t) = 1$
(IFM8) $N(x, y, 0) = 1$;
(IFM9) $N(x, y, t) = 0$ if and only if $x = y$;
(IFM10) $N(x, y, t) = N(y, x, t)$;
(IFM11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$;
(IFM12) $N(x, y,): [0, \infty) \rightarrow (0, 1]$ is continuous.
(IFM3) $\lim_{t\rightarrow\infty} N(x, y, t) = 0$

Then (M, N) is called an Intuitionistic Fuzzy metric on X.The function M(x, y, t) denote the degree of nearness between x and y with respect to t.And the function N(x, y, t) denote the degree of non nearness between x and y with respect to t.

Definition 2.4 A Class of Implicit Function
Let Φ be the set of all real continuous functions $F: (R^+)^5 \rightarrow R$ non decreasing in the first argument and the fourth argument satisfying the following conditions:
(Fa) For $u, v \geq 0$, $F(u, v, u, v, 1) \geq 0$ or $F(u, v, v, u, 1) \geq 0$ implies that $u \geq v$.
(Fb) $F\{u, 1, 1, u, 1\} \geq 0$ or $F\{u, u, 1, 1, u\} \geq 0$ implies that $u \geq 1$

Definition 2.5 E A property
Let A and B be self maps on an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$.
They are said to satisfy (EA) property if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Bx_n = x$ for some $x \in X$.

Definition 10. Reciprocally continuous
Let A and B be mappings from an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. Then the mappings are said to be reciprocally continuous if
$\lim_{n\rightarrow\infty} ABx_n = Az$, and $\lim_{n\rightarrow\infty} BAx_n = Bz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Bx_n = z$, for some $z \in X$.

Remark : If A and B are both continuous then they are obviously reciprocally continuous. But the converse need not be true.

Definition 2.6 ABSORBING MAP
Let A and B are two self maps on a fuzzy metric space.$(X, M, N, *, \diamond)$ then A is called B -absorbing if there exists a positive integer $R > 0$ such that $M(Bx, BAx, t) \geq M(Bx, Ax, t/R)$ and $N(Bx, BAx, t) \leq N(Bx, Ax, t/R)$ for all $x \in X$. Similarly B is called A -absorbing if there exists a positive integer $R > 0$ such That
$M(Ax, ABx, t) \geq M(Ax, Bx, t/R)$ and $N(Ax, ABx, t) \leq N(Ax, Bx, t/R)$ for all $x \in X$.

Definition 2.7 Semi compatible map

A pair (A, B) of self-maps of a intuitionistic fuzzy metric space (X, M, N, *, ◇.) is said to be semi-compatible if $\lim_{n\to\infty} ABx_n = Bx$ whenever there exist a sequence $x_n \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = x$ for some $x \in X$.

Lemma 2.1
In an intuitionistic fuzzy metric spaceX, M(x, y, *) is non-decreasing and N(x, y, ◇) is non-increasing for all x, y∈ X.

Lemma 2.2
Let (X, M, N, *., ◇) be an intuitionistic fuzzy metric space.If there exists a constant $k \in (0, 1)$ such that $M(x, y, kt) \leq M(x, y, t)$, $N(x, y, kt) \geq N(x, y, t)$, for $x, y \in X$. Then $x = y$.

Main Results

Theorem
Let A, B, C, D, E and F be a self map of an intuitionistic fuzzy metric space (X, M, N, *, ◇) with * as continuous t-norm and ◇.continuous t –co norm satisfying the conditions.
(1) E(X) C CD(X), F(X) C AB(X);
(2) AB = BA, CD = DC, EB = BE, FD = DF ;
(3) F is CD-absorbing ;
(4) There exists $k \in (0, 1)$ such that

$$\int_0^{M(Ex,Fy,kt)} \Phi(t)\,dt \geq \int_0^{m(x,y,t)} \Phi(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

$$\int_0^{N(Ex,Fy,kt)} \Phi(t)\,dt \leq \int_0^{n(x,y,t)} \Phi(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

Where $\Phi : R^+ \to R^+$ is a lebesque integerable mapping which is summable non negative such that

$\int_0^{\varepsilon} \Phi(t)\,dt > 0$ for each $\epsilon > 0$ And

$m(x, y, t) = \Psi\{M(ABx, CDy, t), M(Ex, ABx, t), M(Fy, CDy, t),$
$M(Ex, CDx, t), M(Fy, ABx, (2-\alpha)\, t)\}$

$n(x, y, t) = \theta\,\{N(ABx, CDy, t), N(Ex, ABx, t), N(Fy, CDy, t),$
$N(Ex, CDx, t), N(Fy, ABx, (2-\alpha)\, t)\}$
for every $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$, where Ψ and θ is an increasing function of class of implicit function.

(5) If (E, AB) is reciprocally continuous and compatible maps and(CD, F) satisfy EA property
Then A, B, C, D, E and F have a unique common fixed point in X.

Proof :

Let $\{x_n\}$ be a sequence in X such that $Ex_n \to z$ and $AB\,x_n \to z$ for some $z \in X$.

Since the pair (CD, F) satisfy EA property then the sequence $\{x_n\}$ in X have $\lim_{n\to\infty} F\,x_n = z = \lim_{n\to\infty} CD\,x_n$.

Since the pair of (E, AB) is reciprocal continuous; we have
$\lim_{n\to\infty} EAB\,x_n = Ez$ and $\lim_{n\to\infty} ABE\,x_n = ABz$ and their compatibility yield
$\lim_{n\to\infty} M(EABx_n, ABEx_n, t)=1$ and $\lim_{n\to\infty} N(EABx_n, ABEx_n, t)=0$
taking limit $n\to\infty$ we have
$M(Ez, ABz, t)=1$ and $N(E\,z, ABz_n, t)=0$

$Ez = ABz$.

Step 1 : We claim that $Ez = ABz = z$.

For $\alpha = 1$ put $x = z$ and $y = x_n$ in (4.1) and (4.2) we have,

$$\int_0^{M(Ez,Fx_n,kt)} \Phi(t)dt \geq \int_0^{m(z,x_n,t)} \Phi(t)\,dt$$

$$\int_0^{N(Ez,Fx_n,kt)} \Phi(t)dt \geq \int_0^{n(z,x_n,t)} \Phi(t)\,dt$$

Where $m(z, x_n, t) = \Psi\,\{M(ABz, CD\,x_n, t), M(Ez, AB\,z, t), M(Fx_n, CD\,x_n, t), M(E\,z, CDx_n, t), M(Fx_n, ABz, t)\}$

$n(z, x_n, t) = \theta\,\{N(ABz, CD\,x_n, t), N(Ez, ABz, t), N(Fx_n, CD\,x_n, t), N(Ez, CD\,x_n, t), N(F\,x_n, ABz, t)\}$

i.e

$$\int_0^{M(Ez,Fx_n,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(ABz,CD\,x_n,t),\,M(Ez,ABz,t),\,M(F\,x_n,CD\,x_n,t),\,M(Ez,CD\,x_n,t),\,M(Fx_n,ABz,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Ez,Fx_n,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(ABz,CD\,x_n,t),\,N(Ez,ABz,t),\,N(F\,x_n,CD\,x_n,t),\,N(Ez,CD\,x_n,t),\,N(Fx_n,ABz,t)\}} \Phi(t)\,dt$$

Taking limit $n\to\infty$ we have,

$$\int_0^{M(Ez,z,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(Ez,z,t),\,M(E\,z,Ez,t),\,M(z,z\,t),\,M(Ez,z,t),\,M(z,Ez,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Ez,z,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(Ez,z,t),\,N(Ez,Ez,t),\,N(z,z,t),\,N(Ez,z,t),\,N(z,Ez,t)\}} \Phi(t)\,dt$$

Since Ψ is increasing in each of its coordinates and $\Psi(t, t, t, t, t)>t$ and $\theta(t, t, t, t, t) < t$, for all $t \in [0, 1)$ implies that

$$\int_0^{M(Ez,z,kt)} \Phi(t)\,dt \geq \int_0^{M(Ez,z,t)} \Phi(t)\,dt$$
$$\int_0^{N(Ez,z,kt)} \Phi(t)\,dt \leq \int_0^{N(Ez,z,t)} \Phi(t)\,dt$$

Using Lemma 2 we have
Ez = z = ABz

Step 2: We claim that Az = Bz = Ez = z
For α = 1 put x =Bz and y = x_n in (4.1) and (4.2) we have,

$$\int_0^{M(EBz,Fx_n,kt)} \Phi(t)dt \geq \int_0^{m(Bz,x_n,t)} \Phi(t)\,dt$$
$$\int_0^{N(EBz,Fx_n,kt)} \Phi(t)dt \leq \int_0^{n(Bz,x_n,t)} \Phi(t)\,dt$$

Where $m(Bz, x_n, t) = \Psi\{M(ABBz, CDx_n, t), M(EBz, ABBz, t), M(Fx_n, CDx_n, t), M(EBz, CDx_n, t), M(Fx_n, ABBz, t)\}$

$n(Bz, x_n, t) = \theta\{N(ABBz, CDx_n, t), N(EBz, ABBz, t), N(Fx_n, CDx_n, t), N(EBz, CDx_n, t), N(Fx_n, ABBz, t)\}$

$$\int_0^{M(EBz,Fx_n,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(ABBz,CDx_n,t),M(EBz,ABBz,t),M(Fx_n,CDx_n,t),M(EBz,CDx_n,t),M(Fx_n,ABBz,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(EBz,Fx_n,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(ABBz,CDx_n,t),N(EBz,ABBz,t),N(Fx_n,CDx_n,t),N(EBz,CDx_n,t),N(Fx_n,ABBz,t)\}} \Phi(t)\,dt$$

Since EB=BE it implies E(Bz)=B(Ez) = Bz also AB=BA it implies that ABBz = BABz = B(ABz) = Bz

Therefore

$$\int_0^{M(Bz,Fx_n,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(Bz,CDx_n,t),M(Bz,Bz,t),M(Fx_n,CDx_n,t),M(Bz,CDx_n,t),M(Fx_n,Bz,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(EBz,Fx_n,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(Bz,CDx_n,t),N(Bz,Bz,t),N(Fx_n,CDx_n,t),N(Bz,CDx_n,t),N(Fx_n,Bz,t)\}} \Phi(t)\,dt$$

Taking limit n→∞ we have,

$$\int_0^{M(Bz,z,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(Bz,z,t),M(z,Bz,t),M(z,z,t),M(Bz,z,t),M(z,Bz,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Bz,z,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(Bz,z,t),N(z,Bz,t),N(z,z,t),N(Bz,z,t),N(z,Bz,t)\}} \Phi(t)\,dt$$

Since Ψ is increasing in each of its coordinates and Ψ(t, t, t, t, t)>t and θ(t, t, t, t, t) < t, for all t ϵ [0, 1) implies that

$$\int_0^{M(Bz,z,kt)} \Phi(t)\,dt \geq \int_0^{M(Bz,z,t)} \Phi(t)\,dt$$

$$\int_0^{N(Bz,z,kt)} \Phi(t)\,dt \leq \int_0^{N(Bz,z,t)} \Phi(t)\,dt$$

using Lemma 2 we have

Bz = z = Ez = Az ..... (1)

Step 3 : We claim that Fz = z = CDz

Since Ex ⊂ CDx therefore there exist u ϵ X such that z = Ez = CDu.

For α = 1 put x = x_n and y = u in (4.1) and (4.2) we have,

$$\int_0^{M(Exn,Fu,kt)} \Phi(t)\,dt \geq \int_0^{m(xn,u,t)} \Phi(t)\,dt$$

$$\int_0^{N(Exn,Fu,kt)} \Phi(t)\,dt \leq \int_0^{n(xn,u,t)} \Phi(t)\,dt$$

Where $m(x_n, u, t) = \Psi\{M(ABx_n, CDu, t), M(Ex_n, ABx_n, t), M(Fu, CDu, t), M(Ex_n, CDu, t), M(Fu, ABx_n, t)\}$

$n(x_n, u, t) = \theta\{N(ABx_n, CDu, t), N(Ex_n, ABx_n, t), N(Fu, CDu, t), N(Ex_n, CDu, t), N(Fu, ABx_n, t)\}$

i.e

$$\int_0^{M(Exn,Fu,kt)} \Phi(t)\,dt \leq \int_0^{\Psi\{M(ABx_n,CDu,t),M(Ex_n,ABx_n,t),M(Fu,CDu,t),M(Ex_n,CDu,t),M(Fu,ABx_n,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Exn,Fu,kt)} \Phi(t)\,dt \leq \int_0^{\theta\{N(ABx_n,CDu,t),N(Ex_n,ABx_n,t),N(Fu,CDu,t),N(Ex_n,CDu,t),N(Fu,ABx_n,t)\}} \Phi(t)\,dt$$

Taking limit n→∞ we have,

$$\int_0^{M(z,Fu,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(z,z,t),M(z,z,t),M(Fu,z,t),M(z,z,t),M(Fu,z,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(z,Fu,kt)} \Phi(t)dt \le \int_0^{\theta\{N(z,z,t),N(z,z,t),N(Fu,z,t),N(z,z,t),N(Fu,z,t)\}} \Phi(t)\,dt$$

Since Ψ and θ is increasing in each of its coordinates and Ψ(t, t, t, t, t)>t and θ(t, t, t, t, t) < t, for all t ϵ [0, 1) implies that

$$\int_0^{M(z,Fu,kt)} \Phi(t)dt \ge \int_0^{M(z,Fu,t)} \Phi(t)\,dt$$

$$\int_0^{N(z,Fu,kt)} \Phi(t)dt \le \int_0^{N(z,Fu,t)} \Phi(t)\,dt$$

Using lemma 2 we have
z = Fu
Fu = CDu = z
Since F is CD absorbing
M(CDu, CDFu, t) ≥ M(CDu, Fu, t/R)
N(CDu, CDFu, t) ≤ N(CDu, Fu, t/R)
CDu = Fu
Therefore M(CDu, CDFu, t) ≥ 1 & N(CDu, CDFu, t) ≤ 0
CDu=CDFu
CDz = z

Again For α = 1 put x = x_n and y =z in (4.1) and (4.2) we have,

$$\int_0^{M(Ex_n,Fz,kt)} \Phi(t)dt \ge \int_0^{m(x_n,z,t)} \Phi(t)\,dt$$

$$\int_0^{N(Ex_n,Fz,kt)} \Phi(t)dt \le \int_0^{n(x_n,z,t)} \Phi(t)\,dt$$

Where $m(x_n, z, t) = \Psi\{M(ABx_n, CDz, t), M(Ex_n, ABx_n, t), M(Fz, CDz, t), M(Ex_n, CDz, t), M(Fz, ABx_n, t)\}$

$n(x_n, z, t) = \theta\{N(ABx_n, CDz, t), N(Ex_n, ABx_n, t), N(Fz, CDz, t), N(Ex_n, CDz, t), N(Fz, ABx_n, t)\}$

i.e

$$\int_0^{M(Ex_n,Fz,kt)} \Phi(t)dt \ge \int_0^{\Psi\{M(ABx_n,CDz,t),M(Ex_n,ABx_n,t),M(Fz,CDz,t),M(Ex_n,CDz,t),M(Fz,ABx_n,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Ex_n,Fz,kt)} \Phi(t)dt \le \int_0^{\theta\{M(ABx_n,CDz,t),M(Ex_n,ABx_n,t),M(Fz,CDz,t),M(Ex_n,CDz,t),M(Fz,ABx_n,t)\}} \Phi(t)\,dt$$

Taking limit n→∞ and CDz = z we have,

$$\int_0^{M(z,Fz,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(z,z,t),M(z,z,t),M(Fz,z,t),M(z,z,t),M(Fz,z,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(z,Fz,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(z,z,t),N(z,z,t),N(Fz,z,t),N(z,z,t),N(Fz,z,t)\}} \Phi(t)\,dt$$

Since Ψ is increasing in each of its coordinates and $\Psi(t, t, t, t, t)>$tand $\theta(t, t, t, t, t)<t$ for all $t \in [0, 1)$ implies that

$$\int_0^{M(z,Fz,kt)} \Phi(t)dt \geq \int_0^{M(z,Fz,t)} \Phi(t)\,dt$$

$$\int_0^{N(z,Fz,kt)} \Phi(t)dt \leq \int_0^{N(z,Fz,t)} \Phi(t)\,dt$$

Using lemma 2 we have
$Fz = z = CDz$ ……(2)

Step 4 : We claim that $Dz = z = Cz = Fz$

For $\alpha = 1$ put $x = x_n$ and $y = Dz$ in (4.1) and (4.2) we have,

$$\int_0^{M(Ex_n,FDz,kt)} \Phi(t)dt \geq \int_0^{m(x_n,Dz,t)} \Phi(t)\,dt$$

$$\int_0^{N(Ex_n,FDz,kt)} \Phi(t)dt \leq \int_0^{n(x_n,,Dz,t)} \Phi(t)\,dt$$

Where $m(x_n, Dz, t) = \Psi\{M(ABx_n, CDDz, t), M(Ex_n, ABx_n, t), M(FDz, CDDz, t), M(Ex_n, CDDz, t), M(FDz, ABx_n, t)\}$

$n(x_n, Dz, t) = \theta\{N(ABx_n, CDDz, t), N(Ex_n, ABx_n, t), N(FDz, CDDz, t), N(Ex_n, CDDz, t), N(FDz, ABx_n, t)\}$

$$\int_0^{M(Ex_n,FDz,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(ABx_n,CDDz,t),M(Ex_n,ABx_n,t),(FDz,CDDz,t),M(Ex_n,CDDz,t),M(FDz,ABx_n,t)\}} \Phi(t)\,dt$$

$$\int_0^{N(Ex_n,FDz,kt)} \Phi(t)dt \leq$$

$$\int_0^{\theta\{N(ABx_n, CDDz, t), N(Ex_n, ABx_n, t), N(Dz, CDDz, t), N(Ex_n, CDDz, t), N(FDz, ABx_n, t)\}} \Phi(t)\,dt$$

Since FD=DF it implies that FDz =DFz=Dz also CD=DC it implies that CDDz=D(CDz)=Dz

Taking limit n→∞ and CDDz =Dz and FDz=Dz we have,

$$\int_0^{M(z,Dz,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(z, Dz, t), M(z, z, t), M(Dz, Dz, t), M(z, Dz, t), M(Dz, z, t)\}} \Phi(t)\,dt$$

$$\int_0^{N(z,Dz,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(z, Dz, t), N(z, z, t), N(Dz, Dz, t), N(z, Dz, t), N(Dz, z, t)\}} \Phi(t)\,dt$$

Since Ψ and θ is increasing in each of its coordinates and Ψ(t, t, t, t, t)>t and θ(t, t, t, t, t) < t, for all t ϵ [0, 1) implies that

$$\int_0^{M(z,Dz,kt)} \Phi(t)dt \geq \int_0^{M(z,Dz,t)} \Phi(t)\,dt$$

$$\int_0^{N(z,Dz,kt)} \Phi(t)dt \leq \int_0^{N(z,Dz,t)} \Phi(t)\,dt$$

Using lemma 2 we have
Dz = z
Dz = Cz = Fz = z

Hence Az = Bz = Ez = Dz = Cz = Fz = z
Therefore z is the common fixed point of A, B, C, D, E and F.
Uniqueness
Let u be any other fixed point of A, B, C, D, E and F then,

For α = 1 put x =z and y = u in (4.1) and (4.2) we have,

$$\int_0^{M(Ez,Fu,kt)} \Phi(t)\,dt \geq \int_0^{m(z,u,t)} \Phi(t)\,dt$$

$$\int_0^{N(Ez,Fu,kt)} \Phi(t)\,dt \leq \int_0^{n(z,u,t)} \Phi(t)\,dt$$

Where m(z, u, t) = Ψ{M(ABz, CD u, t), M(Ez, ABz, t), M(Fu, CDu, t), M(Ez, CDu, t), M(Fu, ABz, t) }

n(z, u, t) = θ { N(ABz, CD u, t), N(Ez, ABz, t), N(Fu, CD u, t),

$N(Ez, CDu, t), N(Fu, ABz, t)\}$

i.e

$$\int_0^{M(Ez,Fu,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(ABz,CDu,t),M(Ez,ABz,t),M(Fu,CDu,t),\ M(Ez,CDu,t),M(Fu,ABz,t)\}} \Phi(t)dt$$

$$\int_0^{N(Ez,Fu,kt)} \Phi(t)\,dt \leq \int_0^{\theta\{N(ABz,CDu,t),N(Ez,ABz,t),N(Fu,CDu,t),\ N(Ez,CDu,t),N(Fu,ABz,t)\}} \Phi(t)\,dt$$

Taking limit $n\to\infty$ we have,

$$\int_0^{M(z,u,kt)} \Phi(t)dt \geq \int_0^{\Psi\{M(z,u,t),M(z,z,t),M(u,u,t),M(z,z,t),\ M(u,z,t)\})\}} \Phi(t)\,dt$$

$$\int_0^{N(z,Dz,kt)} \Phi(t)dt \leq \int_0^{\theta\{N(z,u,t),N(z,z,t),N(u,u,t),N(z,z,t),\ N(u,z,t)\}} \Phi(t)\,dt$$

Since Ψ and θ is increasing in each of its coordinates and $\Psi(t, t, t, t, t)>t$ and $\theta(t, t, t, t, t) < t$, for all $t \in [0, 1)$ implies that

$$\int_0^{M(z,u,kt)} \Phi(t)\ dt \geq \int_0^{M(z,u,t)} \Phi(t)\ dt$$

$$\int_0^{N(z,u,kt)} \Phi(t)\ dt \leq \int_0^{N(z,u,t)} \Phi(t)\ dt$$

Using lemma 2 we have $z = u$.

Therefore z is the unique common fixed point of A, B, C, D, E and F.

Corollary

Let A, B, C and D be a self map of an intuitionistic fuzzy metric space (X, M, N, *, ◇.) with continuous t-norm and continuous t –conorm satisfying the conditions.

(1) AB = BA, CD = DC,

(2) there exists $k \in (0, 1)$ such that

$$\int_0^{M(x,y,kt)} \Phi(t)\,dt \geq \int_0^{m(x,y,t)} \Phi(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

$$\int_0^{N(x,y,kt)} \Phi(t)\,dt \leq \int_0^{n(x,y,t)} \Phi(t)\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

Where $\Phi : R+ \rightarrow R+$ is a lebesque integerable mapping which is summable non negative such that

$$\int_0^{\epsilon} \Phi(t)\, dt > 0 \text{ for each } \epsilon>0$$

$$m(x, y, t) = \Psi\{M(ABx, CDy, t), M(x, ABx, t), M(y, CDy, t), M(x, CDx, \alpha t), M(y, ABx, (2-\alpha) t)\}$$

$$n(x, y, t) = \theta\{N(ABx, CDy, t), N(x, ABx, t), N(y, CDy, t), N(x, CDx, \alpha t), N(y, ABx, (2-\alpha) t)\}$$

for every $x, y \epsilon X$, $\alpha \epsilon (0, 2)$ and $t > 0$.Where Ψ and θ is an increasing function of class of implicit function

(5) If (E, AB) is reciprocally continuous and compatible maps and(CD, F) satisfy EA property
Then A, B, C and D have a unique common fixed point in X.

Proof
In theorem 1 Take E and F as identity map, Then we have the conclusion.

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