On Fuzzy Strongly \( \alpha \)-Continuous Mappings

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Abstract

The concept of strongly \( \alpha \)-continuous mappings by Y. Beceren [2] has been extended in topological spaces. We introduce and study fuzzy strongly \( \alpha \)-continuous maps on fuzzy topological spaces. Some of its properties have also been investigated. Relation between fuzzy strongly \( \alpha \)-continuous and fuzzy \( \alpha \)-continuous map has also been established. The stronger form of fuzzy \( \alpha \)-continuous map is strongly \( \alpha \)-continuous map has been proved with an example. Equivalent conditions for a map from one fuzzy topological space to another to be fuzzy strongly \( \alpha \)-continuous map have been obtained. We establish significant properties of fuzzy strongly \( \alpha \)-continuous maps.

Keywords: Fuzzy topology, Fuzzy \( \alpha \)-open, Fuzzy semi-open, Fuzzy pre-open, Fuzzy \( \alpha \)-continuous, Fuzzy strongly \( \alpha \)-continuous and Fuzzy \( \alpha \)-irresolute.

Introduction

The concept of fuzzy sets introduced by Prof. L. A. Zadeh in his classical paper [10]. Chang [4] has introduced the concept of fuzzy topological spaces. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notation of fuzzy \( \alpha \)-open and \( \alpha \)-closed sets has introduced by A. S. Bin Shahana [3] in fuzzy topological spaces. Azad [1] has studied various generalized forms of fuzzy continuous map viz. fuzzy semi-continuous map, fuzzy almost continuous map, and fuzzy weakly continuous map.

In this paper we have studied fuzzy strongly \( \alpha \)-continuous maps and investigated necessary and sufficient condition for a map to be fuzzy strongly \( \alpha \)-continuous. Further we have established some important properties of fuzzy strongly \( \alpha \)-continuous maps.
Preliminaries
Let $X$ be a non empty set. A collection $\tau$ of fuzzy sets in $X$ is called a fuzzy topology on $X$ if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of $\tau$ and $\tau$ is closed with respect to any union and finite intersection. The members of $\tau$ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The closure of a fuzzy set $\lambda$ (denoted by $cl(\lambda)$) is the intersection of all fuzzy closed sets which contains $\lambda$. The interior of a fuzzy set $\lambda$ (denoted by $int(\lambda)$) is the union of all fuzzy open subsets of $\lambda$. A fuzzy set $\lambda$ in $X$ is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:
- Fuzzy semi-open [1] if $\lambda \leq cl(int(\lambda))$,
- Fuzzy pre-open [3] if $\lambda \leq int(cl(\lambda))$,
- Fuzzy $\alpha$-open [3] if $\lambda \leq int(cl(int(\lambda)))$,
- Fuzzy semipro-open [8] if $\lambda \leq cl(int(cl(\lambda)))$.

Remark 2.2: Every fuzzy open set is fuzzy $\alpha$-open, every fuzzy $\alpha$-open set is fuzzy semi open (resp. fuzzy pre-open) and every fuzzy semi open (resp. fuzzy pre-open) set is fuzzy semi pre-open. But the converses may not be true. The complement of a fuzzy semi-open (resp. fuzzy pre-open, fuzzy $\alpha$-open, fuzzy semi pre-open) set is called fuzzy semi-closed (resp. fuzzy pre-closed, fuzzy $\alpha$-closed, fuzzy semi pre-closed).

Definition 2.3: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:
- Fuzzy semi-closed [1] if $int(cl(\lambda)) \leq \lambda$,
- Fuzzy pre-closed [3] if $cl(int(\lambda)) \leq \lambda$,
- Fuzzy $\alpha$-closed [3] if $cl(int(cl(\lambda))) \leq \lambda$
- Fuzzy semipro-closed [8] if $int(cl(int(\lambda))) \leq \lambda$.

Definition 2.4: A mapping $f$ from a fuzzy topological space $(X, \tau)$ to fuzzy topological space $(Y, \sigma)$ is called:
- Fuzzy continuous if $f^{-1}(\lambda)$ is fuzzy open in $X$ for every fuzzy open set $\lambda$ of $Y$ [4].
- Fuzzy semi-continuous if $f^{-1}(\lambda)$ is fuzzy semi-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [1].
- Fuzzy pre-continuous if $f^{-1}(\lambda)$ is fuzzy pre-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [3].
- Fuzzy $\alpha$-continuous if $f^{-1}(\lambda)$ is fuzzy $\alpha$-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [3].
• Fuzzy $\alpha$-irresolute if $f^{-1}(\lambda)$ is fuzzy $\alpha$-open in $X$ for every fuzzy $\alpha$-open set $\lambda$ of $Y$.[5]

Every fuzzy continuous map is fuzzy $\alpha$-continuous map is fuzzy $\alpha$-continuous and every fuzzy $\alpha$-continuous map is fuzzy semi-continuous and fuzzy pre-continuous map. However converse may not be true. Since each fuzzy open set is fuzzy $\alpha$-open, it follows that every fuzzy $\alpha$-irresolute map is fuzzy $\alpha$-continuous. However the converse may not be true.

**Fuzzy Strongly $\alpha$-Continuous Maps**

**Definition 3.1:** Let $X, Y$ be fuzzy topological spaces and $f: X \to Y$ be a map. Then the map $f$ is said to be fuzzy strongly $\alpha$-continuous if for each fuzzy semi-open set $\lambda$ in $Y$, $f^{-1}(\lambda)$ is fuzzy $\alpha$-open set in $X$.

**Example 3.2:** Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$. Let $\mu$ and $\lambda$ be fuzzy sets in $X$ and $Y$ respectively defined as $\mu(x_1) = 0.5$, $\mu(x_2) = 0.3$, $\lambda(x_1) = 0.5$, $\lambda(x_2) = 0.6$, $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \lambda, 1\}$ be the fuzzy topologies on sets $X$ and $Y$ respectively. The map $f: X \to Y$ defined as $f(x_i) = y_i$, $i = 1, 2$ is fuzzy strongly $\alpha$-continuous map.

Since each fuzzy $\alpha$-open set is fuzzy semi-open, it follows that every fuzzy strongly $\alpha$-continuous map is fuzzy $\alpha$-irresolute. However converse may not be true. And since each fuzzy open set is fuzzy semi-open, it follows that every fuzzy strongly $\alpha$-continuous map is fuzzy $\alpha$-continuous. However, a fuzzy $\alpha$-continuous map may not be fuzzy strongly $\alpha$-continuous. We have following example.

**Example 3.3:** Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$. Let $\mu, \lambda, \vartheta$ be fuzzy sets in $X$ and $\gamma$ be a fuzzy set in $Y$ defined as $\mu(x_1) = 0.2$, $\mu(x_2) = 0.3$, $\lambda(x_1) = 0.5$, $\lambda(x_2) = 0.6$, $\vartheta(x_1) = 0.7$, $\vartheta(x_2) = 0.7$, $\gamma(y_1) = 0.6$ and $\gamma(y_2) = 0.7$. Let $\tau = \{0, \mu, \lambda, \vartheta, \gamma\}$ and $\tau' = \{0, \gamma, 1\}$ be the fuzzy topologies on sets $X$ and $Y$ respectively. The map $f: X \to Y$ defined as $f(x_i) = y_i$, $i = 1, 2$ is fuzzy $\alpha$-continuous map. The fuzzy set $\eta$ in $Y$ defined as $\eta(y_1) = 0.7$, $\eta(y_2) = 0.8$ is fuzzy semi-open set in $Y$ but $f^{-1}(\eta)$ is not fuzzy $\alpha$-open set in $X$. Hence the map $f: X \to Y$ is fuzzy $\alpha$-continuous but not fuzzy strongly $\alpha$-continuous.

The concepts of fuzzy continuity and fuzzy strongly $\alpha$-continuity are independent of each other. The map $f: X \to Y$ mentioned in the Example 3.2, is fuzzy strongly $\alpha$-continuous but not fuzzy continuous. In the following example we see that a fuzzy continuous map $f: X \to Y$ may not be fuzzy strongly $\alpha$-continuous.

**Example 3.4:** Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$. Let $\mu, \lambda$ be fuzzy sets in $X$ and $\vartheta$ be a fuzzy set in $Y$ defined as $\mu(x_1) = 0.3$, $\mu(x_2) = 0.4$, $\lambda(x_1) = 0.5$, $\lambda(x_2) = 0.6$, $\vartheta(y_1) = 0.5$, and $\vartheta(y_2) = 0.6$. Let $\tau = \{0, \mu, \lambda, \vartheta\}$ and $\tau' = \{0, \vartheta, \gamma\}$ be the fuzzy topologies on sets $X$ and $Y$ respectively. The map $f: X \to Y$ defined as $f(x_i) = y_i$, $i = 1, 2$ is fuzzy continuous map. The fuzzy set $\eta$ in $X$ defined as $\eta(y_1) = 0.6$, and $\eta(y_2) = 0.5$.
Hence the map \( f: X \to Y \) is fuzzy continuous but not fuzzy strongly \( \alpha \)-continuous.

**Theorem 3.5:** Let \( X \) and \( Y \) be fuzzy topological spaces and \( f: X \to Y \) be a map. Then following conditions are equivalent:

- \( f \) is fuzzy strongly \( \alpha \)-continuous.
- For each fuzzy point \( p^\beta_x \) in \( X \) and each fuzzy semi-open set \( \lambda \) in \( Y \) containing \( f(p^\beta_x) \), there exists a fuzzy \( \alpha \)-open set \( \mu \) in \( X \) containing \( p^\beta_x \) such that \( f(\mu) \leq \lambda \).
- For each fuzzy semi-closed set \( \lambda \) in \( Y \), \( f^{-1}(\lambda) \) is fuzzy \( \alpha \)-closed set in \( X \).
- For each fuzzy set \( \mu \) in \( X \), \( f(\mu - cl(\mu)) \leq scl(f(\mu)) \).
- For each fuzzy set \( \lambda \) in \( Y \), \( \alpha - cl(f^{-1}(\lambda)) \leq f^{-1}(scl(\lambda)) \).
- For each fuzzy set \( \lambda \) in \( Y \), \( cl(int(cl(f^{-1}(\lambda)))) \leq f^{-1}(scl(\lambda)) \).
- For each fuzzy set \( \mu \) in \( X \), \( f(cl(int(cl(\mu)))) \leq scl(f(\mu)) \).

**Proof:** (i) \( \Rightarrow \) (ii). Let \( f: X \to Y \) be a fuzzy strongly \( \alpha \)-continuous map. Let \( p^\beta_x \), where \( x \in X \) and \( 0 < \beta \leq 1 \), be a fuzzy point in \( X \) and \( \lambda \) be a fuzzy semi-open set in \( Y \) containing the fuzzy point \( f(p^\beta_x) \). Since \( f(p^\beta_x)f(x)) = \beta \leq \lambda(f(x)) \), we have \( \beta \leq f^{-1}(\lambda)(x) \), i.e., \( \mu = f^{-1}(\lambda) \) contains the fuzzy point \( p^\beta_x \). Moreover, since \( f \) is fuzzy strongly \( \alpha \)-continuous map, \( \mu \) is fuzzy \( \alpha \)-open set in \( X \), containing the fuzzy point \( p^\beta_x \) and \( f(\mu) \leq \lambda \).

(ii) \( \Rightarrow \) (i). Let \( \lambda \) be a fuzzy semi-open set in \( Y \). For \( x \in X \) and \( 0 < \beta \leq 1 \), let \( p^\beta_x \) be a fuzzy point in \( f^{-1}(\lambda) \). Then \( \lambda \) contains \( f(p^\beta_x) \) and so by given condition (ii) there exists a fuzzy \( \alpha \)-open set \( \mu \) in \( X \) containing the fuzzy point \( p^\beta_x \) and \( f(\mu) \leq \lambda \). This implies \( \mu \leq int(cl(int(\mu))) \leq int(cl(int(f^{-1}(\lambda)))) \) and therefore \( int(cl(int(f^{-1}(\lambda)))) \) contains the fuzzy point \( p^\beta_x \). Thus each fuzzy point of \( f^{-1}(\lambda) \) is also a fuzzy point of \( int(cl(int(f^{-1}(\lambda)))) \). This shows that \( f^{-1}(\lambda) \leq int(cl(int(f^{-1}(\lambda)))) \), i.e., \( f^{-1}(\lambda) \) is a fuzzy \( \alpha \)-open set in \( X \). Hence \( f: X \to Y \) is fuzzy strongly \( \alpha \)-continuous map.

(i) \( \Rightarrow \) (iii). Let \( \lambda \) be a fuzzy semi-closed set in \( Y \). Then \( \lambda^c = 1 - \lambda \) is fuzzy semi-open set in \( Y \). Since \( f: X \to Y \) is fuzzy strongly \( \alpha \)-continuous, \( f^{-1}(\lambda^c) = f^{-1}(1 - \lambda^c) \) is fuzzy \( \alpha \)-open set in \( X \). This implies, \( f^{-1}(\lambda) = f^{-1}(1 - \lambda^c) = f^{-1}(1 - \lambda^c) \) is fuzzy \( \alpha \)-closed set in \( X \).

(iii) \( \Rightarrow \) (i). Let \( \lambda \) be a fuzzy semi-open set in \( Y \). Then \( \lambda^c = 1 - \lambda \) is fuzzy semi-closed set in \( Y \). Therefore by given condition (iii), \( f^{-1}(\lambda^c) = f^{-1}(1 - \lambda) \) is fuzzy \( \alpha \)-closed set in \( X \).

Hence \( f^{-1}(\lambda) = f^{-1}(1 - \lambda^c) = 1 - f^{-1}(\lambda^c) \) is fuzzy \( \alpha \)-open set in \( X \). Thus \( f: X \to Y \) is fuzzy strongly \( \alpha \)-continuous map.

(iii) \( \Rightarrow \) (iv). Let \( \mu \) be a fuzzy set in \( X \). Since \( \mu \leq f^{-1}(f(\mu)) \) we have \( \mu \leq f^{-1}(scl(f(\mu))) \). Now \( scl(f(\mu)) \) is a fuzzy semi-closed set in \( Y \). Hence by given condition (iii), \( f^{-1}(scl(f(\mu))) \) is fuzzy \( \alpha \)-closed set in \( X \) containing \( \mu \). Since \( \alpha - cl(\mu) \) is the smallest fuzzy \( \alpha \)-closed set containing \( \mu \), it follows that \( \alpha - cl(\mu) \)
Let $\lambda$ be a fuzzy semi-closed set in $Y$. Then by given condition (iv), we have $f(\alpha - cl(f^{-1}(\lambda))) \leq scl(f(f^{-1}(\lambda))) \leq scl(\lambda) = \lambda$. This implies, $\alpha - cl(f^{-1}(\lambda)) \leq f^{-1}(\lambda)$. Since $f^{-1}(\lambda) \leq \alpha - cl(f^{-1}(\lambda))$, we deduce that $f^{-1}(\lambda) = \alpha - cl(f^{-1}(\lambda))$. Now $\alpha - cl(f^{-1}(\lambda))$ is fuzzy $\alpha$-closed set in $X$, it follows that $f^{-1}(\lambda)$ is fuzzy $\alpha$-closed set in $X$.

Let $\lambda$ be a fuzzy set in $Y$. Then by given condition (iv), $f(\alpha - cl(f^{-1}(\lambda))) \leq scl(f(f^{-1}(\lambda))) \leq scl(\lambda)$ which implies $\alpha - cl(f^{-1}(\lambda)) \leq f^{-1}(scl(\lambda))$. Since $\alpha - int(f^{-1}(scl(\lambda))) \leq \alpha - int(f^{-1}(\lambda))$ we find that $f^{-1}(scl(\lambda)) \leq \alpha - int(f^{-1}(\lambda))$.

Let $\lambda$ be a fuzzy semi-open set in $Y$. Then we have $scl(\lambda) = \lambda$. Therefore by given condition (vi), $f^{-1}(scl(\lambda)) = f^{-1}(scl(\lambda)) \leq \alpha - int(f^{-1}(\lambda))$, i.e., $f^{-1}(\lambda) \leq \alpha - int(f^{-1}(\lambda))$. Since $\alpha - int(f^{-1}(\lambda)) \leq \alpha - int(f^{-1}(\lambda))$, we find that $f^{-1}(\lambda) = \alpha - int(f^{-1}(\lambda))$. Hence $f^{-1}(\lambda)$ is a fuzzy $\alpha$-open set in $X$. Thus $f:X \rightarrow Y$ is a fuzzy strongly $\alpha$-continuous map.

Let $\lambda$ be a fuzzy set in $Y$. Then $scl(\lambda)$ is a fuzzy semi-closed set in $Y$. From given condition (iii), $f^{-1}(scl(\lambda))$ is a fuzzy $\alpha$-closed set in $X$. This implies, $f^{-1}(scl(\lambda)) \geq cl(int(cl(f^{-1}(\lambda)))) \geq cl(cl(cl(f^{-1}(\lambda))))$, i.e., $cl(cl(cl(f^{-1}(\lambda)))) \leq f^{-1}(scl(\lambda))$.

Let $\lambda$ be a fuzzy set in $X$. Then $f(\lambda)$ is a fuzzy set in $Y$. From given condition (vii) we have, $f^{-1}(f(\lambda)) \geq cl(int(cl(cl(f(\lambda))))) \geq cl(cl(cl(\lambda)))$. This implies, $cl(cl(\lambda)) \geq f^{-1}(f(\lambda)) \geq cl(cl(cl(\lambda)))$.

Thus we have, $f(cl(cl(\lambda))) \leq scl(f(\lambda))$. Therefore by given condition (viii) we have, $f(cl(cl(f^{-1}(\lambda)))) \leq scl(f(f^{-1}(\lambda)))$. Since $\alpha - int(f^{-1}(\lambda)) \leq \alpha - int(f^{-1}(\lambda))$, we find that $f^{-1}(\lambda) = \alpha - int(f^{-1}(\lambda))$. Hence $f^{-1}(\lambda)$ is a fuzzy $\alpha$-closed set in $X$.

**Theorem 3.6:** Let $X$ and $Y$ be fuzzy topological spaces and $f:X \rightarrow Y$ be a bijective map. Then $f$ is fuzzy strongly $\alpha$-continuous iff for each fuzzy set $\mu$ in $X$, $s-int(f(\mu)) \leq f(\alpha - int(\mu))$.

**Proof:** Let $f:X \rightarrow Y$ be a bijective map. Suppose $f$ is fuzzy strongly $\alpha$-continuous. If $\mu$ is a fuzzy set in $X$ then $f(\mu)$ is a fuzzy set in $Y$. Since $f$ is fuzzy strongly $\alpha$-continuous, from Theorem 3.5 we have, $f^{-1}(s-int(f(\mu))) \leq \alpha - int(\mu)$.
int \left(f^{-1}(f(\mu)) \right). Since f is one-one, \( \alpha - \text{int}(f^{-1}(f(\mu))) = \alpha - \text{int}(\mu) \). This shows that \( f^{-1}\left( s - \text{int}(f(\mu)) \right) \leq \alpha - \text{int}(\mu) \). Further since f is onto we have, \( s-\text{int}(f(\mu)) = f(\alpha - \text{int}(\mu)) \). Thus \( s-\text{int}(f(\mu)) \leq f(\alpha - \text{int}(\mu)) \).

Conversely let \( \lambda \) be a fuzzy semi-open set in Y. Then \( s-\text{int}(\lambda) = \lambda \). Now \( f^{-1}(\lambda) \) is a fuzzy set in X, from hypothesis, \( f(\alpha - \text{int}(f^{-1}(\lambda))) \geq s-\text{int}(f^{-1}(\lambda)) \). Since f is onto, \( s-\text{int}(f^{-1}(\lambda)) = s-\text{int}(\lambda) = \lambda \). Therefore \( f^{-1}\left( \alpha - \text{int}(f^{-1}(\lambda)) \right) \geq \lambda \). Further since f is one-one, \( \alpha - \text{int}(f^{-1}(\lambda)) = f^{-1}\left( f\left( \alpha - \text{int}(f^{-1}(\lambda)) \right) \right) \geq f^{-1}(\lambda) \). As \( \alpha - \text{int}(f^{-1}(\lambda)) \leq f^{-1}(\lambda) \), we deduce that \( f^{-1}(\lambda) \) is a fuzzy \( \alpha \)-open set in X. Hence \( f: X \to Y \) is fuzzy strongly \( \alpha \)-continuous map.

**Theorem 3.7:** Let \( X, Y \), and \( Z \) be fuzzy topological spaces and let \( f: X \to Y \) and \( g: Y \to Z \) be maps. If \( f \) is fuzzy strongly \( \alpha \)-continuous and \( g \) is fuzzy semi-continuous then \( gof: X \to Z \) is fuzzy \( \alpha \)-continuous.

**Corollary 3.8:** Let \( X, Y \), and \( Z \) be fuzzy topological spaces and let \( f: X \to Y \) and \( g: Y \to Z \) be maps. If \( f \) is fuzzy strongly \( \alpha \)-continuous and \( g \) is fuzzy continuous then \( gof: X \to Z \) is fuzzy \( \alpha \)-continuous.

**Corollary 3.9:** Let \( X, Y \), and \( Z \) be fuzzy topological spaces. Let \( p: Y \times Z \to Y \) and \( q: Y \times Z \to Z \) be projection maps. If \( f: X \to Y \times Z \) is fuzzy strongly \( \alpha \)-continuous then \( pof: X \to Y \) and \( qof: X \to Z \) are fuzzy \( \alpha \)-continuous.

**Proposition 3.10:** Let \( X \) and \( Y \) be fuzzy topological spaces such that \( X \) is product related to \( Y \). Suppose \( f: X \to Y \) is a map and \( g: X \to X \times Y \) is the graph of map \( f \). If \( g \) is fuzzy strongly \( \alpha \)-continuous then \( f \) is fuzzy strongly \( \alpha \)-continuous.

**Theorem 3.11:** Let \( X_i, i = 1,2 \) be fuzzy topological spaces such that \( X_1 \) is product related to \( X_2 \) and \( Y_i \) is product related to \( Y_2 \). If \( f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2 \) is fuzzy strongly \( \alpha \)-continuous, then \( f_1: X_1 \to Y_1 \) and \( f_2: X_2 \to Y_2 \) are fuzzy strongly \( \alpha \)-continuous maps.

**Proof:** Let \( \lambda \) be a fuzzy semi-open set in \( Y_i \). Since \( Y_1 \) is product related to \( Y_2, \lambda \times 1 \) is fuzzy semi-open set in \( Y_1 \times Y_2 \). Since \( f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2 \) is fuzzy strongly \( \alpha \)-continuous, we have \( (f_1 \times f_2)^{-1}(\lambda \times 1) = f_1^{-1}(\lambda) \times 1 \) is fuzzy \( \alpha \)-open set in \( X_1 \times X_2 \). Further since \( X_2 \) is product related to \( X_2 \), we have \( \text{int}(\text{cl}(\text{int}(f_1^{-1}(\lambda) \times 1))) = \text{int}(\text{cl}(\text{int}(f_1^{-1}(\lambda)))) \). This implies, \( \text{int}(\text{cl}(\text{int}(f_1^{-1}(\lambda)))) \geq f_1^{-1}(\lambda) \). Thus \( f_1^{-1}(\lambda) \) is fuzzy \( \alpha \)-open set in \( X_1 \). Hence \( f_1: X_1 \to Y_1 \) is fuzzy strongly \( \alpha \)-continuous map. By a similarly argument we can show that \( f_2: X_2 \to Y_2 \) is fuzzy strongly \( \alpha \)-continuous map.
Bibliography


