On Fuzzy Strongly $\alpha$-Irresolute Mappings

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Abstract

The concept of strongly $\alpha$-irresolute mappings by G. Faro Lo [4] has been extended in topological spaces. We introduce and study fuzzy strongly $\alpha$-irresolute maps on fuzzy topological spaces. Some of its properties have also been investigated. Relation between fuzzy strongly $\alpha$-irresolute and fuzzy $\alpha$-continuous map has also been established. The stronger form of fuzzy $\alpha$-continuous map is strongly $\alpha$-irresolute map has been proved with an example. Equivalent conditions for a map from one fuzzy topological space to another to be fuzzy strongly $\alpha$-irresolute map have also been obtained. We establish some significant properties of fuzzy strongly $\alpha$-irresolute maps.

Keywords: Fuzzy topology, Fuzzy $\alpha$-open, Fuzzy pre-open, Fuzzy semi pre-open, Fuzzy $\alpha$-continuous and Fuzzy strongly $\alpha$-irresolute.

Introduction

The fuzzy set theory has been found in recent years very useful in Information Technology, knowledge based systems, risk analysis, control systems and other such applications. It has been developed as a well defined technique for dealing with complex phenomena which were not amenable by classical methods based on probability theory and bivalent logic. Zadeh [10] in 1965 introduced the theory of fuzzy sets which are a natural generalization of classical sets or crisp sets. Fuzzy set theory has a wider scope of solving various real world problems. Chang [3] has introduced the concept of fuzzy topological spaces.

Azad [1] has studied various generalized forms of fuzzy continuous map viz. fuzzy semi-continuous map, fuzzy almost continuous map, and fuzzy weakly continuous map. Bin Shahana [2] has defined the notion of fuzzy pre-continuous map, fuzzy $\alpha$-continuous map, and obtained significant results. In this paper we have
studied fuzzy strongly $\alpha$-irresolute maps and investigated necessary and sufficient condition for a map to be fuzzy strongly $\alpha$-irresolute. Further we have established some important properties of fuzzy strongly $\alpha$-irresolute maps.

Preliminaries

Let $X$ be a non empty set. A collection $\tau$ of fuzzy sets in $X$ is called a fuzzy topology on $X$ if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of $\tau$ and $\tau$ is closed with respect to any union and finite intersection. The members of $\tau$ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The closure of a fuzzy set $\lambda$ (denoted by $cl(\lambda)$ ) is the intersection of all fuzzy closed sets which contains $\lambda$. The interior of a fuzzy set $\lambda$ (denoted by $int(\lambda)$) is the union of all fuzzy open subsets of $\lambda$. A fuzzy set $\lambda$ in $X$ is fuzzy open (resp. fuzzy closed) if and only $int(\lambda) = \lambda$ (resp. $cl(\lambda) = \lambda$).

Definition 2.1: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:

- Fuzzy semi-open [1] if $\lambda \leq cl(int(\lambda))$,
- Fuzzy pre-open [2] if $\lambda \leq int(cl(\lambda))$,
- Fuzzy $\alpha$-open [2] if $\lambda \leq int(cl(int(\lambda)))$,
- Fuzzy semipreopen [8] if $\lambda \leq cl(int(cl(\lambda)))$.

Remark 2.2: Every fuzzy open set is fuzzy $\alpha$-open, every fuzzy $\alpha$-open set is fuzzy semi open (resp. fuzzy pre-open) and every fuzzy semi open (resp. fuzzy pre-open) set is fuzzy semi pre-open. But the converses may not be true. The complement of a fuzzy semi-open (resp. fuzzy pre-open, fuzzy $\alpha$-open, fuzzy semi pre-open) set is called fuzzy semi-closed (resp. fuzzy pre-closed, fuzzy $\alpha$-closed, fuzzy semi pre-closed).

Definition 2.3: Let $(X, \tau)$ be a fuzzy topological space. A fuzzy set $\lambda$ in the space $X$ is called:

- Fuzzy semi-closed [1] if $int(cl(\lambda)) \leq \lambda$,
- Fuzzy pre-closed [2] if $cl(int(\lambda)) \leq \lambda$,
- Fuzzy $\alpha$-closed [2] if $cl(int(cl(\lambda))) \leq \lambda$,
- Fuzzy semipro-closed [8] if $int(cl(int(\lambda))) \leq \lambda$.

Definition 2.4: A mapping $f$ from a fuzzy topological space $(X, \tau)$ to fuzzy topological space $(Y, \sigma)$ is called:

- Fuzzy continuous if $f^{-1}(\lambda)$ is fuzzy open in $X$ for every fuzzy open set $\lambda$ of $Y$ [3].
- Fuzzy semi-continuous if $f^{-1}(\lambda)$ is fuzzy semi-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [1].
Fuzzy pre-continuous if $f^{-1}(\lambda)$ is fuzzy pre-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [2].

- Fuzzy $\alpha$-continuous if $f^{-1}(\lambda)$ is fuzzy $\alpha$-open in $X$ for every fuzzy open set $\lambda$ of $Y$ [2].
- Fuzzy $\alpha$-irresolute if $f^{-1}(\lambda)$ is fuzzy $\alpha$-open in $X$ for every fuzzy $\alpha$-open set $\lambda$ of $Y$ [5].

Every fuzzy continuous map is fuzzy $\alpha$-continuous and every fuzzy $\alpha$-continuous map is fuzzy semi-continuous and fuzzy pre-continuous map. However converse may not be true. We have following example.

Example 3.4: Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$. Let $\eta_1, \eta_2, \eta_3$, and $\eta_4$ be fuzzy sets in $X$ defined as $\eta_1(x_1) = 0.1$, $\eta_1(x_2) = 0.2$, $\eta_2(x_1) = 0.3$, $\eta_2(x_2) = 0.3$, $\eta_3(x_1) = 0.3$, $\eta_3(x_2) = 0.6$, $\eta_4(x_1) = 0.6$, $\eta_4(x_2) = 0.7$. Let $y_1$ and $y_2$ be fuzzy sets in $Y$ defined as $y_1(y_1) = 0.2$, $y_1(y_2) = 0.3$, $y_2(y_1) = 0.4$, and $y_2(y_2) = 0.6$. Suppose

Fuzzy Strongly $\alpha$-Irresolute Maps

Definition 3.1: Let $X, Y$ be fuzzy topological spaces and $f: X \to Y$ be a map. Then the map $f$ is said to be fuzzy strongly $\alpha$-irresolute if for each fuzzy $\alpha$-open set $\lambda$ in $Y$, $f^{-1}(\lambda)$ is fuzzy open set in $X$.

Example 3.2: Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$. Let $\mu, \lambda$ be fuzzy sets in $X$ defined as $\mu(x_1) = 0.2$, $\mu(x_2) = 0.3$, $\lambda(x_1) = 0.4$ and $\lambda(x_2) = 0.6$. Let $\vartheta, \gamma$ be fuzzy sets in $Y$ defined as $\vartheta(y_1) = 0.2$, $\vartheta(y_2) = 0.3$, $\gamma(y_1) = 0.4$, and $\gamma(y_2) = 0.6$. Let $\tau = \{0, \mu, \lambda, 1\}$ and $\tau' = \{0, \vartheta, \gamma, 1\}$ be fuzzy topologies on sets $X$ and $Y$ respectively. The map $f: (X, \tau) \to (Y, \tau')$ defined as $f(x_i) = y_i$, $i = 1, 2$ is fuzzy strongly $\alpha$-irresolute map.

Since each fuzzy open set is fuzzy $\alpha$-open it follows that every fuzzy strongly $\alpha$-irresolute map is fuzzy continuous (and hence fuzzy $\alpha$-continuous). However converse may not be true. We have following example.

Example 3.3: Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$. Let $\lambda$ and $\mu$ be fuzzy sets in $X$ and $Y$ respectively, defined as $\lambda(x_1) = 0.5$, $\lambda(x_2) = 0.6$, $\mu(y_1) = 0.5$ and $\mu(y_2) = 0.6$. Let $\tau = \{0, \lambda, 1\}$ and $\tau' = \{0, \mu, 1\}$ be fuzzy topologies on sets $X$ and $Y$ respectively. The map $f: (X, \tau) \to (Y, \tau')$ defined as $f(x_i) = y_i$, $i = 1, 2$ is fuzzy continuous map. The fuzzy set $\eta$ in $Y$ defined as $\eta(y_1) = 0.6$, $\eta(y_2) = 0.7$ is fuzzy $\alpha$-open set in $Y$ but $f^{-1}(\eta)$ is not fuzzy open set in $X$. Hence the map $f: X \to Y$ is fuzzy continuous but not fuzzy strongly $\alpha$-irresolute.

Since every fuzzy open sets is fuzzy $\alpha$-open it follows that every fuzzy strongly $\alpha$-irresolute map is fuzzy $\alpha$-irresolute. However the converse may not be true. We have following example.

Example 3.4: Let $X = \{x_1, x_2\}, Y = \{y_1, y_2\}$. Let $\eta_1, \eta_2, \eta_3$, and $\eta_4$ be fuzzy sets in $X$ defined as $\eta_1(x_1) = 0.1$, $\eta_1(x_2) = 0.2$, $\eta_2(x_1) = 0.3$, $\eta_2(x_2) = 0.3$, $\eta_3(x_1) = 0.3$, $\eta_3(x_2) = 0.6$, $\eta_4(x_1) = 0.6$, $\eta_4(x_2) = 0.7$. Let $y_1$ and $y_2$ be fuzzy sets in $Y$ defined as $y_1(y_1) = 0.2$, $y_1(y_2) = 0.3$, $y_2(y_1) = 0.4$, and $y_2(y_2) = 0.6$. Suppose
Theorem 3.5: Let $X$ and $Y$ be fuzzy topological spaces and $f : X \to Y$ be a map. Then following conditions are equivalent:

- $f$ is fuzzy strongly $\alpha$-irresolute.
- For each fuzzy point $p_x^\beta$ in $X$ and each fuzzy $\alpha$-open set $\lambda$ in $Y$ containing $f(p_x^\beta)$, there exists a fuzzy open set $\mu$ in $X$ containing $p_x^\beta$ such that $f(\mu) \subseteq \lambda$.
- For each fuzzy $\alpha$-closed set $\lambda$ in $Y$, $f^{-1}(\lambda)$ is fuzzy closed set in $X$.
- For each fuzzy set $\mu$ in $X$, $f(cl(\mu)) \leq \alpha - cl(f(\mu))$.
- For each fuzzy set $\lambda$ in $Y$, $cl(f^{-1}(\lambda)) \leq f^{-1}(\alpha - cl(\lambda))$.
- For each fuzzy set $\lambda$ in $Y$, $f^{-1}(\alpha - int(\lambda)) \leq int(f^{-1}(\lambda))$.

Proof: (i) $\implies$ (ii). Let $f : X \to Y$ be a fuzzy strongly $\alpha$-irresolute map. Let $p_x^\beta$, where $x \in X$ and $0 < \beta \leq 1$, be a fuzzy point in $X$ and let $\lambda$ be a fuzzy $\alpha$-open set in $Y$ containing the fuzzy point $f(p_x^\beta)$. Since $f(p_x^\beta)(f(x)) = \beta \leq \lambda(f(x))$, we have $\beta \leq f^{-1}(\lambda)(x)$, i.e. $\mu = f^{-1}(\lambda)$ contains the fuzzy point $p_x^\beta$. Moreover, since $f$ is fuzzy strongly $\alpha$-irresolute map, $\mu$ is fuzzy open set in $X$ containing the fuzzy point $p_x^\beta$ and $f(\mu) = f(f^{-1}(\lambda)) \leq \lambda$.

(ii) $\implies$ (i). Let $\lambda$ be a fuzzy $\alpha$-open set in $Y$. For $x \in X$ and $0 < \beta \leq 1$, let $p_x^\beta$ be a fuzzy point in $f^{-1}(\lambda)$. Then $\lambda$ contains $f(p_x^\beta)$ and so by given condition (ii) there exists a fuzzy open set say $\mu(p_x^\beta)$ in $X$ containing the fuzzy point $p_x^\beta$ and $f(\mu(p_x^\beta)) \leq \lambda$. Since $f^{-1}(\lambda)$ is the union of all fuzzy points belonging to $f^{-1}(\lambda)$, we have, $f^{-1}(\lambda) = \bigcup_{p_x^\beta \in f^{-1}(\lambda)} p_x^\beta \leq \bigcup_{p_x^\beta \in f^{-1}(\lambda)} \mu(p_x^\beta)$. This implies $f^{-1}(\lambda) \subseteq \bigcup_{p_x^\beta \in f^{-1}(\lambda)} \mu(p_x^\beta)$.

(iii) $\implies$ (iv). Let $\mu$ be a fuzzy $\alpha$-closed set in $X$. Since $f : X \to Y$ is fuzzy strongly $\alpha$-irresolute, $f^{-1}(\lambda^c) = f^{-1}(1 - \lambda)$ is fuzzy open set in $X$. This implies, $f^{-1}(\lambda) = f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda^c)$ is fuzzy closed set in $X$.

(iv) $\implies$ (i). Let $\mu$ be a fuzzy $\alpha$-closed set in $Y$. Then $\lambda^c = 1 - \lambda$ is fuzzy $\alpha$-open set in $Y$. Therefore by given condition (iii), $f^{-1}(\lambda^c) = f^{-1}(1 - \lambda)$ is fuzzy closed set in $X$. Hence $f^{-1}(\lambda) = f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda^c)$ is fuzzy open set in $X$. Thus $f : X \to Y$ is fuzzy strongly $\alpha$-irresolute.
On Fuzzy Strongly $\alpha$-Irresolute Mappings

Condition (iii), $f^{-1}(\alpha - cl(f(\mu)))$ is fuzzy closed set in $X$ containing $\mu$. Since $cl(\mu)$ is the smallest fuzzy closed set containing $\mu$, it follows that $cl(\mu) \leq f^{-1}(\alpha - cl(f(\mu)))$. This implies $f(cl(\mu)) \leq \alpha - cl(f(\mu))$.

(iv)$\Rightarrow$(iii). Let $\lambda$ be a fuzzy $\alpha$-closed set in $Y$. Then by given condition (iv), we have $f(cl(f^{-1}(\lambda))) \leq \alpha - cl(f(f^{-1}(\lambda))) \leq \alpha - cl(\lambda) = \lambda$. This implies, $cl(f^{-1}(\lambda)) \leq f^{-1}(\lambda)$. Since $f^{-1}(\lambda) \subseteq cl(f^{-1}(\lambda))$, we deduce that $cl(f^{-1}(\lambda)) = f^{-1}(\lambda)$. Hence $f^{-1}(\lambda)$ is fuzzy closed set in $X$.

(iv)$\Rightarrow$(v). Let $\lambda$ be a fuzzy set in $Y$. Then by given condition (iv), we have, $f(cl(f^{-1}(\lambda))) \leq \alpha - cl(f(f^{-1}(\lambda))) \leq \alpha cl(\lambda)$. This implies $cl(f^{-1}(\lambda)) \leq f^{-1}(\alpha - cl(\lambda))$.

(v)$\Rightarrow$(iv). Let $\mu$ be a fuzzy set in $X$. Then by given condition (v), $cl(f^{-1}(f(\mu))) \leq f^{-1}(\alpha - cl(f(\mu)))$. This implies, $cl(\mu) \leq f^{-1}(\alpha - cl(f(\mu)))$, and hence $f(cl(\mu)) \leq \alpha - cl(f(\mu))$.

(i)$\Rightarrow$(vi). Let $\lambda$ be a fuzzy set in $Y$. Since $\alpha -\text{int}(\lambda)$ is a fuzzy $\alpha$-open set in $Y$, from given condition (i), $f^{-1}(\alpha -\text{int}(\lambda))$ is fuzzy open set in $X$. Hence we have $f^{-1}(\alpha -\text{int}(\lambda)) = \text{int}(f^{-1}(\alpha -\text{int}(\lambda)))$. Since $\text{int}(f^{-1}(\alpha -\text{int}(\lambda))) \leq \text{int}(f^{-1}(\lambda))$ we deduce that $f^{-1}(\alpha -\text{int}(\lambda)) \leq \text{int}(f^{-1}(\lambda))$.

(vi)$\Rightarrow$(i). Let $\lambda$ be a fuzzy $\alpha$-open set in $Y$. Then we have $\alpha - \text{int}(\lambda) = \lambda$. Therefore from given condition (vi), $f^{-1}(\lambda) = f^{-1}(\alpha -\text{int}(\lambda)) \leq \text{int}(f^{-1}(\lambda))$, i.e., $f^{-1}(\lambda) \leq \text{int}(f^{-1}(\lambda))$. Since $\text{int}(f^{-1}(\lambda)) \subseteq f^{-1}(\lambda)$, we have, $f^{-1}(\lambda) = \text{int}(f^{-1}(\lambda))$. Hence $f^{-1}(\lambda)$ is a fuzzy open set in $X$. Thus $f : X \to Y$ is a fuzzy strongly $\alpha$-irresolute map.

**Theorem 3.6:** Let $X$ and $Y$ be fuzzy topological spaces and $f : X \to Y$ be a bijective map. Then $f$ is fuzzy strongly $\alpha$-irresolute if and only if for each fuzzy set $\mu$ in $X$, $\alpha - \text{int}(f(\mu)) \leq f(\text{int}(\mu))$.

**Proof:** Let $f : X \to Y$ be a bijective map. Suppose $f$ is fuzzy strongly $\alpha$-irresolute. If $\mu$ is a fuzzy set in $X$ then $f(\mu)$ is fuzzy set in $Y$. Since $f$ is fuzzy strongly $\alpha$-irresolute, from Theorem 3.5, we have, $f^{-1}(\alpha - \text{int}(f(\mu))) \leq \text{int}(f^{-1}(f(\mu)))$. Since $f$ is one-one, $\text{int}(f^{-1}(f(\mu))) = \text{int}(\mu)$. This shows that $f^{-1}(\alpha - \text{int}(f(\mu))) \leq \text{int}(\mu)$.

Further since $f$ is onto we have, $\alpha - \text{int}(f(\mu)) = f(f^{-1}(\alpha - \text{int}(f(\mu)))) \leq f(\text{int}(\mu))$. Thus $\alpha - \text{int}(f(\mu)) \leq f(\text{int}(\mu))$.

Conversely let $\lambda$ be a fuzzy $\alpha$-open set in $Y$. Then $\alpha -\text{int}(\lambda) = \lambda$. Now $f^{-1}(\lambda)$ is a fuzzy set in $X$, from hypothesis, $f(\text{int}(f^{-1}(\lambda))) \geq \alpha - f(f^{-1}(\lambda))$. Since $f$ is onto, $\alpha - f(f^{-1}(\lambda)) = \alpha - \text{int}(\lambda) = \lambda$. Therefore $f(\text{int}(f^{-1}(\lambda))) \geq \lambda$. Further since $f$ is one-one, $\text{int}(f^{-1}(\lambda)) = f^{-1}(f(\text{int}(f^{-1}(\lambda)))) \geq f^{-1}(\lambda)$. As $\text{int}(f^{-1}(\lambda)) \subseteq f^{-1}(\lambda)$, we deduce that $f^{-1}(\lambda) = \text{int}(f^{-1}(\lambda))$. Thus $f^{-1}(\lambda)$ is a fuzzy open set in $X$. Hence $f : X \to Y$ is fuzzy strongly $\alpha$-irresolute.

**Theorem 3.7:** Let $X, Y$, and $Z$ be fuzzy topological spaces. Let $f : X \to Y$ and $g : Y \to Z$ be maps. If $f : X \to Y$ is fuzzy strongly $\alpha$-irresolute and $g : Y \to Z$ is fuzzy $\alpha$-continuous then $gof : X \to Z$ is fuzzy continuous.
Proof: Let \( \lambda \) be a fuzzy open set in \( Z \). Since \( g: Y \rightarrow Z \) is fuzzy \( \alpha \)-continuous, \( g^{-1}(\lambda) \) is fuzzy \( \alpha \)-open set in \( Y \). Further, since \( f:X \rightarrow Y \) is fuzzy strongly \( \alpha \)-irresolute map, 
\[
(f^{-1}(g^{-1}(\lambda))) = (gof)^{-1}(\lambda) \text{ is fuzzy open set in } X.
\]
Hence \( (gof)^{-1}(\lambda) \) is fuzzy open set in \( X \), for each fuzzy open set \( \lambda \) in \( Z \). Thus \( gof: X \rightarrow Z \) is fuzzy continuous map.

**Theorem 3.8:** Let \( X, Y \) and \( Z \) be fuzzy topological spaces. Let \( f:X \rightarrow Y \) and \( g:Y \rightarrow Z \) be maps. If \( f:X \rightarrow Y \) is fuzzy continuous and \( g:Y \rightarrow Z \) is fuzzy strongly \( \alpha \)-irresolute then \( gof:X \rightarrow Z \) is fuzzy strongly \( \alpha \)-irresolute.

**Corollary 3.9:** Composition of fuzzy strongly \( \alpha \)-irresolute maps is fuzzy strongly \( \alpha \)-irresolute.

**Corollary 3.10:** Let \( X, Y \) and \( Z \) be fuzzy topological spaces. Let \( p:X \times Y \rightarrow X \) and \( q:X \times Y \rightarrow Y \) be projection maps. If \( f:X \rightarrow Z \) and \( g:Y \rightarrow Z \) are fuzzy strongly \( \alpha \)-irresolute maps then \( fop:X \times Y \rightarrow Z \) and \( goq:X \times Y \rightarrow Z \) are also fuzzy strongly \( \alpha \)-irresolute maps.

**Theorem 3.11:** Let \( X \) and \( Y \) be fuzzy topological spaces such that \( X \) is product related to \( Y \). Suppose \( f:X \rightarrow Y \) is a map and \( g:X \rightarrow X \times Y \) is the graph of map \( f \). If \( g \) is fuzzy strongly \( \alpha \)-irresolute then \( f \) is fuzzy strongly \( \alpha \)-irresolute.

**Theorem 3.12:** Let \( X_i \) and \( Y_i, i = 1,2 \) be fuzzy topological spaces such that \( X_1 \) is product related to \( X_2 \) and \( Y_1 \) is product related to \( Y_2 \). If \( f_1 \times f_2:X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) is fuzzy strongly \( \alpha \)-irresolute map then \( f_1:X_1 \rightarrow Y_1 \) and \( f_2:X_2 \rightarrow Y_2 \) are also fuzzy strongly \( \alpha \)-irresolute maps.

**Proof:** Let \( \lambda \) be a fuzzy \( \alpha \) -open set in \( Y \). Since \( X \) is product related to \( Y \), we have, 
\[
int(cl(int(1 \times \lambda))) = 1 \times int(cl(int(\lambda))) \geq 1 \times \lambda \text{ in } X \times Y.
\]
This implies, \( 1 \times \lambda \) is a fuzzy \( \alpha \)-open set in \( X \times Y \). Since \( g:X \rightarrow X \times Y \) is fuzzy strongly \( \alpha \)-irresolute map, \( g^{-1}(1 \times \lambda) = f^{-1}(\lambda) \) is a fuzzy open set in \( X \). Hence \( f:X \rightarrow Y \) is fuzzy strongly \( \alpha \)-irresolute map.

**Proof:** Let \( \lambda \) be a fuzzy \( \alpha \)-open set in \( Y_1 \). Since \( Y_1 \) is product related to \( Y_2 \), \( \lambda \times 1 \) is a fuzzy \( \alpha \)-open set in \( Y_1 \times Y_2\). Since \( f_1 \times f_2:X_1 \times X_2 \rightarrow Y_1 \times Y_2 \) is fuzzy strongly \( \alpha \)-irresolute, we have \((f_1 \times f_2)^{-1}(\lambda \times 1) = f_1^{-1}(\lambda) \times 1\) is fuzzy open set in \( X_1 \times X_2\). This means \( int(f_1^{-1}(\lambda) \times 1) = f_1^{-1}(\lambda) \times 1 \). Further since \( X_1 \) is product related to \( X_2 \), we have \( int(f_1^{-1}(\lambda) \times 1) = int(f_1^{-1}(\lambda)) \times 1 \). This implies, \( int(f_1^{-1}(\lambda)) \times 1 = f_1^{-1}(\lambda) \times 1 \), and therefore \( int(f_1^{-1}(\lambda)) = f_1^{-1}(\lambda) \). Thus \( f_1^{-1}(\lambda) \) is fuzzy open set in \( X_1 \). Hence \( f_1:X_1 \rightarrow Y_1 \) is fuzzy strongly \( \alpha \)-irresolute map. By a similar argument we can show that \( f_2:X_2 \rightarrow Y_2 \) is also fuzzy strongly \( \alpha \)-irresolute map.
Bibliography


