Intuitionistic Fuzzy Completely Semi-Generalized Continuous Mappings

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy completely semi-generalized continuous mappings in intuitionistic fuzzy topological space. Some interesting properties are investigated besides giving suitable examples.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy semi-generalized closed set, intuitionistic fuzzy semi-generalized continuous mapping and intuitionistic fuzzy completely semi-generalized continuous mapping.

AMS Mathematics Subject Classification: 54A40, 03F55.

Introduction


Continuing the work done in the paper [9], we define the notion of intuitionistic fuzzy completely semi-generalized continuous mappings in intuitionistic fuzzy
topological spaces. We discuss characterizations of intuitionistic fuzzy completely semi-generalized continuous mappings. We also established their properties and relationships with other classes of early defined forms of intuitionistic continuous mappings. In section 3, intuitionistic fuzzy completely semi-generalized continuous mappings were introduced via intuitionistic fuzzy regular open set. The section 4 deals with some of the applications of intuitionistic fuzzy completely semi-generalized continuous mappings.

Preliminaries

**Definition 2.1** [1] An intuitionistic fuzzy set (IFS, for short) \( A \) in \( X \) is an object having the form

\[
A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}
\]

where the functions \( \mu_A: X \rightarrow [0,1] \) and \( \gamma_A: X \rightarrow [0,1] \) denote the degree of the membership (namely \( \mu_A(x) \)) and the degree of non- membership (namely \( \gamma_A(x) \)) of each element \( x \in X \) to the set \( A \) respectively, \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for each \( x \in X \).

**Definition 2.2** [1] Let \( A \) and \( B \) be IFS’s of the forms

\[
A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\} \quad \text{and} \quad B = \{(x, \mu_B(x), \gamma_B(x)) / x \in X\}
\]

Then,

(a) \( A \subseteq B \) if and only if \( \mu_A(x) \leq \mu_B(x) \) and \( \gamma_A(x) \geq \gamma_B(x) \) for all \( x \in X \),

(b) \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \),

(c) \( \overline{A} = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\} \),

(d) \( A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x))) / x \in X\} \),

(e) \( A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))) / x \in X\} \),

(f) \( \overline{0} = \{(x, 0, 1) / x \in X\} \) and \( \overline{1} = \{(x, 1, 0) / x \in X\} \),

(g) \( \overline{\overline{A}} = \overline{A}, \overline{\overline{0}} = \overline{1}, \overline{\overline{\overline{0}}} = \overline{0} \).

**Definition 2.3** [5] An intuitionistic fuzzy topology (IFT for short) on \( X \) is a family \( \tau \) of IFS’s in \( X \) satisfying the following axioms:

i. \( \emptyset, X \in \tau \),

ii. \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \),

iii. \( \bigcup G_i \in \tau \) for any arbitrary family \( \{G_i | i \in J\} \subseteq \tau \).

In this case the pair \( (X, \tau) \) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS for short) in \( X \). The complement \( \overline{A} \) of an IFOS \( A \) in IFTS \( (X, \tau) \) is called an intuitionistic fuzzy closed set (IFCS for short) in \( X \).
Definition 2.4 [5] Let X and Y are two non empty sets and f: X→ Y be a function. If
\[ B = \{(y, \mu_B(y), \gamma_B(y)) / y \in Y\} \]
is an IFS in Y, then the preimage of B under f, denoted by f⁻¹(B), is the IFS in X defined by
\[ f^{-1}(B) = \{(x, f^{-1}(\mu_B(x)), f^{-1}(\gamma_B(x))) / x \in X\} \]

Definition 2.5 [5] Let (X, τ) be an IFTS and \( A = \{x, \mu_A(x), \gamma_A(x) / x \in X\} \) be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by
\[ \text{int}(A) = \bigcup \{G | G \text{ is an IFOS in } X \text{ and } G \subseteq A\}, \quad \text{cl}(A) = \bigcap \{K | K \text{ is an IFCS in } X \text{ and } A \subseteq K\}. \]

Note that, for any IFS \( A \) in \((X, \tau)\), we have
\[ \text{cl}(\bar{A}) = \overline{\text{int}(A)} \quad \text{and} \quad \text{int}(\bar{A}) = \overline{\text{cl}(A)} \]

Definition 2.6 An IFS \( A = \{< x, \mu_A(x), \gamma_A(x) > / x \in X\} \) in an IFTS \((X, \tau)\) is called an
i. intuitionistic fuzzy semi-open set (IFSOS) if \( A \subseteq \text{cl}(\text{int}(A)) \) [6].
ii. intuitionistic fuzzy \( \alpha \)-open set (IF\( \alpha \)OS) if \( A \subseteq \text{int}(\text{cl}(\text{int}(A))) \) [6].
iii. intuitionistic fuzzy preopen set (IFPOS) if \( A \subseteq \text{int}(\text{cl}(A)) \) [6].
iv. intuitionistic fuzzy regular open set (IFROS) if \( \text{int}(\text{cl}(A)) = A \) [6].

An IFS \( A \) is called an intuitionistic fuzzy semi closed set, intuitionistic fuzzy \( \alpha \)-closed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi-preclosed set, respectively (IFSCS, IF\( \alpha \)CS, IFPCS, IFRCs and IFSPCS resp), if the complement \( \bar{A} \) is an IFSOS, IF\( \alpha \)OS, IFPOS, IFROS and IFSPPOS respectively.

The family of all intuitionistic fuzzy semi open (resp. intuitionistic fuzzy \( \alpha \)-open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semi-preopen) sets of an IFTS \((X, \tau)\) is denoted by IFSO(X) (resp IF\( \alpha \)(X), IFPO(X), IFRO(X) and IFSPO(X)).

Definition 2.7 [10] An IFS \( A \) of an IFTS \((X, \tau)\) is called an intuitionistic fuzzy semi-generalized closed (intuitionistic fuzzy sg-closed) set (IFSGCS) if \( \text{scl}(A) \subseteq U \), whenever \( A \subseteq U \) and U is an IFSOS.

The complement \( \bar{A} \) of an intuitionistic fuzzy semi-generalized closed set \( A \) is called an intuitionistic fuzzy semi-generalized open (intuitionistic fuzzy sg-open) set (IFSGOS).

Definition 2.8 [10] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy semi-\( T_{1/2} \) space, if every intuitionistic fuzzy sg-closed set in X is an intuitionistic fuzzy semi-closed in X.
**Definition 2.9** Let \( f: X \rightarrow Y \) be a mapping from an IFTS \( X \) into an IFTS \( Y \). The mapping \( f \) is called an

i. **intuitionistic fuzzy continuous**, if \( f^{-1}(B) \) is an IFOS in \( X \), for each IFOS \( B \) in \( Y \) [6].

ii. **intuitionistic fuzzy semi-continuous**, if \( f^{-1}(B) \) is an IFSOS in \( X \), for each IFOS \( B \) in \( Y \) [6].

iii. **intuitionistic fuzzy pre-continuous**, if \( f^{-1}(B) \) is an IFPOS in \( X \), for each IFOS \( B \) in \( Y \) [6].

iv. **intuitionistic fuzzy \( \alpha \)-continuous**, if \( f^{-1}(B) \) is an IF\( \alpha \)OS in \( X \), for each IFOS \( B \) in \( Y \) [6].

v. **intuitionistic fuzzy completely continuous**, if \( f^{-1}(B) \) is an IFROS in \( X \), for each IFOS \( B \) in \( Y \) [7].

vi. **intuitionistic fuzzy semi-generalized continuous**, if \( f^{-1}(B) \) is an IFSGOS in \( X \), for each IFOS \( B \) in \( Y \) [11].

vii. **intuitionistic fuzzy semi-generalized irresolute**, if \( f^{-1}(B) \) is an IFSGOS in \( X \), for each IFSGOS \( B \) in \( Y \) [11].

viii. **intuitionistic fuzzy almost semi-generalized continuous**, if \( f^{-1}(B) \) is an IFSGCS in \( X \), for each IFRCS \( B \) in \( Y \) [10].

ix. **intuitionistic fuzzy irresolute**, if \( f^{-1}(B) \) is an IFOS in \( X \), for each IFSOS \( B \) in \( Y \) [15].

**Definition 2.10** Let \( f: X \rightarrow Y \) be a mapping from an IFTS \( X \) into an IFTS \( Y \). The mapping \( f \) is called an

i. **intuitionistic fuzzy open**, if \( f(B) \) is an IFOS in \( Y \), for each IFOS \( B \) in \( X \) [9].

ii. **intuitionistic fuzzy semi-generalized open**, if \( f(B) \) is an IFSGOS in \( Y \), for each IFOS \( B \) in \( X \) [12].

iii. **intuitionistic fuzzy quasi semi-generalized open**, if \( f(B) \) is an IFOS in \( Y \), for each IFSGOS \( B \) in \( X \) [13].

iv. **intuitionistic fuzzy quasi sg*-irresolute**, if \( f(B) \) is an IFSGOS in \( Y \), for each IFSGOS \( B \) in \( X \) [12].

**Lemma 2.10** [7] Let \( g: X \times Y \rightarrow X \times Y \) be the graph of a function \( f: X \rightarrow Y \). If \( A \) is an IFS of \( X \) and \( B \) is an IFS of \( Y \), then \( g^{-1}(A \times B)(x) = (A \cap f^{-1}(B))(x) \).

**Intuitionistic Fuzzy Completely Semi-Generalized Continuous Mappings**

**Definition 3.1** A mapping \( f: X \rightarrow Y \) from an IFTS \( X \) into an IFTS \( Y \) is called an intuitionistic fuzzy completely semi-generalized continuous (intuitionistic fuzzy completely sg-continuous) mapping if \( f^{-1}(B) \) is an IFROS of \( X \) for each IFSGOS \( B \) of \( Y \).

**Theorem 3.2** Let \( f: X \rightarrow Y \) be an intuitionistic fuzzy completely sg-continuous mapping and then \( f \) is an intuitionistic fuzzy completely continuous mapping.
Proof: Let $B$ be an IFOS in $Y$. Since every IFOS is an IFSGOS, $B$ is an IFSGOS in $Y$. Since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(B)$ is an IFROS in $X$. Hence $f$ is an intuitionistic fuzzy completely continuous mapping.

Example 3.3 Let $X = \{a, b\}$.
Let $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right) \rangle$.

Then $\tau = \{0_\sim, 1_\sim, A\}$ is an IFTS on $X$. Define a mapping $f: (X, \tau) \rightarrow (X, \tau)$ by $f(a) = a, f(b) = b$. Clearly $f$ is an intuitionistic fuzzy completely continuous map.

Let $B = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.5}\right) \rangle$ be an IFSGOS in $Y$.

Then $f^{-1}(B) = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.5}\right) \rangle, \text{cl}(f^{-1}(B)) = 1_\sim, \text{int}(\text{cl}(f^{-1}(B))) = \text{int}(1_\sim) = 1_\sim \neq f^{-1}(B)$.

Therefore $f^{-1}(B)$ is not an IFROS in $X$. Hence $f$ is not an intuitionistic fuzzy completely sg-continuous map.

Theorem 3.4 Let $f: X \rightarrow Y$ be an intuitionistic fuzzy completely sg-continuous mapping and then $f$ is an intuitionistic fuzzy sg-irresolute mapping.

Proof: Let $B$ be an IFSGOS in $Y$. Since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(B)$ is an IFROS in $X$. Since every IFROS is an IFOS, also since every IFOS is an IFSGOS, $f^{-1}(B)$ is an IFSGOS in $X$. Hence $f$ is an intuitionistic fuzzy sg-irresolute mapping.

The converse of above theorem is not true as seen from the following example.

Example 3.5 Let $X = \{a, b\}$.
Let $A = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}\right) \rangle$.

Then $\tau = \{0_\sim, 1_\sim, A\}$ is an IFTS on $X$. Define a mapping $f: (X, \tau) \rightarrow (X, \tau)$ by $f(a) = a, f(b) = b$. Clearly $f$ is an intuitionistic fuzzy sg-irresolute map.

Let $B = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.5}\right) \rangle$ be an IFSGOS in $Y$.

Then $f^{-1}(B) = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.1}, \frac{b}{0.5}\right) \rangle, \text{cl}(f^{-1}(B)) = 1_\sim, \text{int}(\text{cl}(f^{-1}(B))) = \text{int}(1_\sim) = 1_\sim \neq f^{-1}(B)$.

Therefore $f^{-1}(B)$ is not an IFROS in $X$. Hence $f$ is not an intuitionistic fuzzy completely sg-continuous map.

Theorem 3.6 Let $f: X \rightarrow Y$ be an intuitionistic fuzzy completely sg-continuous mapping, then $f$ is an intuitionistic fuzzy almost sg-continuous mapping.
Proof: Let B be an IFROS in Y. Since every IFROS is an IFSGOS, B is an IFSGOS in Y. Since f is an intuitionistic fuzzy completely sg-continuous mapping, \( f^{-1}(B) \) is an IFROS in X. Thus \( f^{-1}(B) \) is an IFSGOS in X. Hence f is an intuitionistic fuzzy almost sg-continuous mapping.

The converse of above theorem3.6 is not true in general as seen from the following example.

**Example 3.7** Let \( X = \{a, b\} \), \( Y = \{u, v\} \)

Let \( A = \langle x, \left( \frac{a}{0.3}, \frac{b}{0.4} \right), \left( \frac{a}{0.7}, \frac{b}{0.6} \right) \rangle \)

\( B = \langle x, \left( \frac{u}{0.7}, \frac{v}{0.8} \right), \left( \frac{u}{0.3}, \frac{v}{0.2} \right) \rangle \)

Then \( \tau = \{0\}, \{1\}, \{A\} \) and \( \kappa = \{0\}, \{1\}, \{B\} \) is an IFTS on X. Define a mapping \( f: (X, \tau) \rightarrow (X, \tau) \) by \( f(a) = a, f(b) = b \). Clearly f is an intuitionistic fuzzy almost sg-continuous mapping.

Let \( C = \langle x, \left( \frac{a}{0.8}, \frac{b}{0.8} \right), \left( \frac{a}{0.2}, \frac{b}{0.2} \right) \rangle \) be an IFS in Y. Since \( \text{ sint}(C) = C \), it is an IFSGOS and hence C is an IFSGOS in Y.

Now \( \text{ cl}(f^{-1}(C)) = 1\), \( \text{ int}(\text{ cl}(f^{-1}(C))) = \text{ int}(1) = 1. \neq f^{-1}(C) \). Therefore \( f^{-1}(C) \) is an IFROS in X. Hence f is not an intuitionistic fuzzy completely sg-continuous map.

**Theorem 4.8** The following are equivalent for a function \( f: X \rightarrow Y \):

i. f is an intuitionistic fuzzy completely sg-continuous mapping.

ii. the inverse image of each IFSGCS V of Y is an IFRCS in X.

**Proof:** Obvious.

**Theorem 3.9** A mapping \( f: X \rightarrow Y \) from an IFTS X into an IFTS Y is an intuitionistic fuzzy completely sg-continuous if and only if \( f^{-1}(B) \) is an IFRCS of X, for each IFSGCS B of Y.

**Proof:** Let B be an IFSGCS of Y. Then \( B \) is an IFSGOS of Y. Since f is an intuitionistic fuzzy completely sg-continuous mapping, \( f^{-1}(B) \) is an IFROS of X. But \( f^{-1}(\overline{B}) = \overline{f^{-1}(B)} \), hence \( f^{-1}(B) \) is an IFRCS of X.

**Converse:** Let B be any IFSGCS of Y. Then \( B \) is an IFSGOS of Y. By our assumption \( f^{-1}(B) \) is an IFRCS of X. From \( f^{-1}(B) = \overline{f^{-1}(B)} \), it follows that, \( f^{-1}(B) \) is an IFROS of X. Therefore f is an intuitionistic fuzzy completely sg-continuous mapping.

**Theorem 3.10** Let \( f_1: (X, \tau) \rightarrow (Y, \kappa) \) and \( f_2: (X, \tau) \rightarrow (Y, \kappa) \) be any two intuitionistic fuzzy completely sg-continuous mappings. Then the function \( (f_1, f_2): (X, \tau) \rightarrow (Y \times Y, \kappa \times \kappa) \) is also an intuitionistic fuzzy completely sg-continuous mapping.
**Proof:** Let $A \times B$ be any IFSGOS of $Y \times Y$. Then
\[
I(f_1, f_2)^{-1}((A \times B)(x)) = (A \times B)((f_1(x), f_2(x))
= \langle x, \min \left( \mu_A(f_1(x)), \mu_B(f_2(x)) \right), \max \left( \gamma_A(f_1(x)), \gamma_B(f_2(x)) \right) \rangle
= \langle x, \min \left( f_1^{-1}(\mu_A(x)), f_2^{-1}(\mu_B(x)) \right), \max \left( f_1^{-1}(\gamma_A(x)), f_2^{-1}(\gamma_B(x)) \right) \rangle
= \left( f_1^{-1}(A) \cap f_1^{-1}(A) \right)(x).
\]

Since $f_1$ and $f_2$ are intuitionistic fuzzy completely sg-continuous mapping $f^{-1}(A)$ and $f^{-1}(B)$ are IFROS in $Y$. In [6], it has been proved that intersection of two IFROS is an IFROS. Therefore $f^{-1}(A) \cap f^{-1}(A)$ is an IFROS of $X$. Hence $(f_1, f_2)$ is an intuitionistic fuzzy completely sg-continuous mapping.

**Theorem 3.11** Let $f: (X, \tau) \rightarrow (Y, \kappa)$ be a function and $g: X \rightarrow X \times Y$ the graph of the function $f$. Then $f$ is an intuitionistic fuzzy completely sg-continuous, if $g$ is so.

**Proof:** Let $B$ be an IFSGOS in $Y$. Then $f^{-1}(B) = f^{-1}(1_\times \times B) = 1_\times \cap f^{-1}(B) = g^{-1}(1_\times \times B)$. Since $B$ is an IFSGOS in $Y$, $1_\times \times B$ is an IFROS in $Y \times X$. Also, since $g$ is an intuitionistic fuzzy completely sg-continuous mapping, $g^{-1}(1_\times \times B)$ is an IFROS in $X$. Hence $f^{-1}(B)$ is an IFROS in $X$. Therefore $f$ is an intuitionistic fuzzy completely sg-continuous mapping.

**Lemma 3.12** Let $X$ is an intuitionistic fuzzy semi- $T_{1/2}$ and $A, B$ is an IFSGOS in IFTSs $X$ and $Y$ respectively. Then $A \times B$ is an IFSGOS in the IFPTS of $X \times Y$.

**Proof:** Let $A$ and $B$ be any two IFSGOS of $X$ and $Y$ respectively. $A \times B = (A \times 1_\times) \cap (1_\times \times B)$, where $A \times 1_\times$ and $1_\times \times B$ are IFSGOS in $X$ and $Y$. Since $X$ is an intuitionistic fuzzy semi-$T_{1/2}$ space $A \times 1_\times$ and $1_\times \times B$ are IFOS in $X$ and $Y$. Also in [6], it has been proved that intersection of two IFOS is also an IFOS. Therefore $(A \times 1_\times) \cap (1_\times \times B)$ is an IFOS which implies $A \times B$ is an IFOS in $X \times Y$. Hence $\times B$ is an IFOS in $X \times Y$.

**Theorem 3.13** Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then the following properties hold:
- If $f$ and $g$ are intuitionistic fuzzy completely sg-continuous mapping, then $g \circ f$ is also an intuitionistic fuzzy completely sg-continuous mapping.
- If $f$ is an intuitionistic fuzzy completely sg-continuous mapping and $g$ is an intuitionistic fuzzy sg-irresolute mapping, then $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping.
- If $f$ is an intuitionistic fuzzy completely sg-continuous mapping and $g$ is an intuitionistic fuzzy sg-continuous mapping, then $g \circ f$ is an intuitionistic fuzzy completely continuous mapping.
Proof:
i. Let $B$ be an IFSGOS in $Z$. Since $g$ is an intuitionistic fuzzy completely sg-continuous mapping, $g^{-1}(B)$ is an IFROS in $Y$. Since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFROS in $X$. Hence $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping.

iii. Let $B$ be an IFSGOS in $Z$. Since $g$ is an intuitionistic fuzzy sg- irresolute mapping, $g^{-1}(B)$ is an IFSGOS in $Y$. Also, since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in $X$. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping.

iv. Let $B$ be an IFOS in $Z$. Since $g$ is an intuitionistic fuzzy sg-continuous mapping, $g^{-1}(B)$ is an IFSGOS in $Y$. Also, since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in $X$. From the fact that $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, it follows that $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping.

Theorem 3.14 Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two functions. Then the following properties hold:

- If $f$ is an intuitionistic fuzzy irresolute mapping and $g$ is an intuitionistic fuzzy completely sg-continuous mapping, then $g \circ f$ is an intuitionistic fuzzy almost sg-continuous mapping.
- If $f$ is an intuitionistic fuzzy sg*-open mapping and $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping, then $g$ is an intuitionistic fuzzy completely sg-continuous mapping.
- If $f$ is an intuitionistic fuzzy sg-open mapping and $g \circ f$ is an intuitionistic fuzzy completely sg-continuous mapping, then $g$ is an intuitionistic fuzzy sg*-irresolute mapping.
- If $f$ is an intuitionistic fuzzy completely sg-continuous mapping and $g \circ f$ is an intuitionistic fuzzy sg*-open mapping, then $g$ is an intuitionistic fuzzy sg*-open mapping.
- If $f$ is an intuitionistic fuzzy completely sg-continuous mapping and $g \circ f$ is an intuitionistic fuzzy open mapping, then $g$ is an intuitionistic fuzzy quasi sg-open mapping.

Proof: Obvious.

Corollary 3.15 Let $X_i$, $X_{i1}$, $X_{i2}$ be IFTSs and $p_i: X_1 \times X_2 \rightarrow X_i$, $i = 1,2$ are the projections of $X_1 \times X_2$ onto $X_i$. If $f: X \rightarrow X_1 \times X_2$ is an intuitionistic fuzzy sg-continuous mapping, then $p_i f$ is an intuitionistic fuzzy completely continuous mapping.
Proof: Since projections are intuitionistic fuzzy continuous mappings and each intuitionistic fuzzy continuous mapping is an intuitionistic fuzzy sg-continuous mapping, the proof follows immediately.

The following diagram shows the relationships of intuitionistic fuzzy completely sg-continuous mapping with other classes of intuitionistic fuzzy continuous and intuitionistic fuzzy irresolute mappings. In the diagram IF stands for intuitionistic fuzzy, C stands for continuous mapping and I stands for irresolute mapping.

Applications

Definition 4.1 An IFTS X is said to be fuzzy almost regular (FAR) if for each IFP \( p(\alpha, \beta) \) and each IFRCS F in X, there exists an IFROSs U and V, such that \( p(\alpha, \beta) U, F V \) and \( U V \).

Definition 4.2 An IFTS X is said to be fuzzy semi-generalized regular (FSGR) if for each IFP \( p(\alpha, \beta) \) and each IFRCS F in X, there exists an IFSGOSs U and V, such that \( p(\alpha, \beta) U, F V \) and \( U V \).

Definition 4.3 An IFTS X is said to be fuzzy almost normal (FAN) if for each IFCS \( F_1 \) and each IFRCS \( F_2 \) such that \( F_1 F_2 \), there exists an IFROSs U and V, such that \( U F_1 V \).

Definition 4.4 An IFTS X is said to be fuzzy semi-generalized normal (FSGN) if for each IFCS \( F_1 \) and each IFRCS \( F_2 \) such that \( F_1 F_2 \), there exists an IFSGOSs U and V, such that \( F_1 U, F_2 V \) and \( U V \).
Theorem 4.5 Let \( f: X \to Y \) be an intuitionistic fuzzy completely sg-continuous bijective and intuitionistic fuzzy open mapping from an IFTS \( X \) into an IFTS \( Y \). If \( Y \) is a FSGR, then \( X \) is a FAR.

Proof: Let \( p_{(\alpha, \beta)} \) be an IFP and let \( B \) be an IFRCS in \( X \) such that \( p_{(\alpha, \beta)} \in B \). Then \( f(p_{(\alpha, \beta)}) \) is an IFP in \( Y \) and \( f(B) \) is an IFCS in \( Y \) such that \( f(p_{(\alpha, \beta)}) \in f(B) \). Since \( Y \) is a FSGR, there exists IFSGOSs \( U \) and \( V \) such that \( f(p_{(\alpha, \beta)}) \subseteq U, f(B) \subseteq V \) and \( U \subseteq V \). Then \( p_{(\alpha, \beta)} \in f^{-1}(U), B \subseteq f^{-1}(V) \) and \( f^{-1}(U) \subseteq f^{-1}(V) \). Hence \( X \) is FAR.

Theorem 4.6 Let \( f: X \to Y \) be an intuitionistic fuzzy completely sg-continuous, bijective and intuitionistic fuzzy open mapping. If \( Y \) is a FSGN, then \( X \) is a FAN.

Proof: Let \( F_1 \) be an IFCS and let \( F_2 \) be an IFRCS in \( X \) such that \( F_1 \subseteq \overline{F_2} \). Then \( f(F_1) \) and \( f(F_2) \) are IFCSs in \( Y \) such that \( f(F_1) \subseteq f(F_2) \). Since \( Y \) is a FSGN, there exists IFSGOSs \( U \) and \( V \) such that \( f(F_1) \subseteq U, f(F_2) \subseteq V \) and \( U \subseteq V \). By our assumption \( f^{-1}(U), f^{-1}(V) \) are IFROSs in \( X \). Then \( F_1 \subseteq f^{-1}(U), F_2 \subseteq f^{-1}(V) \) and \( f^{-1}(U) \subseteq f^{-1}(V) \). Hence \( X \) is FAN.

Definition 4.7 An IFTS \( X \) is called fuzzy nearly compact if each fuzzy regular open cover of \( A \) has a finite subcover for \( A \).

Definition 4.8 An IFTS \( X \) is called fuzzy semi-generalized compact (fuzzy sg-compact) if each fuzzy sg-open cover of \( X \) has a finite subcover for \( X \).

Definition 4.9 Let \( X \) be an IFTS. A family \( \{(x, \mu_{G_i}, Y_{G_i})/i \in I\} \) of IFSGOSs in \( X \) satisfies the condition \( 1_c = \cup \{(x, \mu_{G_i}, Y_{G_i})/i \in I\} \) is called a fuzzy semi-generalized open (fuzzy sg-open) cover of \( X \).

A finite subfamily of a fuzzy sg-open cover \( \{(x, \mu_{G_i}, Y_{G_i})/i \in I\} \) of \( X \) which is also a fuzzy sg-open cover of \( X \) is called a finite subcover of \( \{(x, \mu_{G_i}, Y_{G_i})/i \in I\} \).

Definition 4.10 An IFTS \( X \) is called fuzzy nearly Lindelof if each fuzzy regular open cover of \( A \) has a finite subcover for \( X \).

Definition 4.11 An IFTS \( X \) is called fuzzy semi-generalized Lindelof (fuzzy sg-Lindelof) if each fuzzy sg-open cover of \( A \) has a countable subcover for \( X \).

Definition 4.12 An IFTS \( X \) is called fuzzy countable semi-generalized compact (fuzzy countable sg-compact) if each countable fuzzy sg-open cover of \( X \) has a finite subcover.

Definition 4.13 An IFTS \( X \) is called countably fuzzy nearly compact if each countable fuzzy regular open cover of \( A \) has a finite subcover for \( A \).
Theorem 4.14 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping from an IFTS $X$ onto an IFTS $Y$. If $X$ is fuzzy nearly compact, then $Y$ is fuzzy sg-compact (fuzzy compact).

Proof: Let $\{G_i / i \in I\}$ be any fuzzy sg-open cover of $Y$. Then $Y \subseteq \bigcup_{i \in I} G_i$. From the relation $Y \subseteq f^{-1}(\bigcup_{i \in I} G_i)$, follows that $Y \subseteq \bigcup_{i \in I} f^{-1}(G_i)$, so $\{f^{-1}(G_i) / i \in I\}$ is a fuzzy regular open cover of $X$. Since $X$ is fuzzy nearly compact, there exists a finite subcover $\{f^{-1}(G_i) / i \in 1,2,3,\ldots, n\}$. Therefore $Y \subseteq \bigcup_{i=1}^{n} f^{-1}(G_i)$. Hence $Y \subseteq f(\bigcup_{i=1}^{n} f^{-1}(G_i)) = \bigcup_{i=1}^{n} G_i$.

Theorem 4.15 If a function $f: X \to Y$ is an intuitionistic fuzzy sg-continuous mapping and $A$ is an IFOS in $X$, then the restriction $f|_A : (A, \tau_A) \to (Y, \kappa)$ is also an intuitionistic fuzzy completely continuous mapping.

Proof: Let $B$ be an IFSGOS of $Y$. Since $f$ is an intuitionistic fuzzy completely sg-continuous mapping, $f^{-1}(B)$ is an IFROS in $X$. Then $f^{-1}(B) \cap A$ is an IFROS in $(A, \tau_A)$ and $f^{-1}(B) \cap A = (f_A)^{-1}(B)$. This shows that $f_A$ is an intuitionistic fuzzy completely sg-continuous mapping.

Theorem 4.16 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping. If $X$ is fuzzy compact, $Y$ is fuzzy sg-compact.

Proof: Let $\{G_i / i \in I\}$ be any fuzzy sg-open cover of $Y$. Then $1_\sim = \bigcup_{i \in I} G_i$. From the relation $1_\sim = f^{-1}(\bigcup_{i \in I} G_i)$ follows that $1_\sim = \bigcup_{i \in I} f^{-1}(G_i)$, so $\{f^{-1}(G_i) / i \in I\}$ is a fuzzy regular open cover of $X$. Since $X$ is fuzzy compact, there exists a finite subcover $\{f^{-1}(G_i) / i \in I\}$. Therefore $1_\sim = \bigcup_{i=1}^{n} f^{-1}(G_i)$. Hence $1_\sim = f(\bigcup_{i=1}^{n} f^{-1}(G_i)) = \bigcup_{i=1}^{n} f(f^{-1}(G_i)) = \bigcup_{i=1}^{n} G_i$. Therefore there exists a finite sg-open subcover of $\{G_i / i \in I\}$ which covers $Y$. Thus $Y$ is fuzzy sg-compact.

Corollary 4.17 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping. If $X$ is fuzzy nearly Lindelof, then $Y$ is fuzzy sg-Lindelof (fuzzy Lindelof).

Corollary 4.18 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping. If $X$ is fuzzy Lindelof, then $Y$ is fuzzy sg-Lindelof (fuzzy Lindelof).

Corollary 4.19 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping. If $X$ is countably fuzzy nearly compact, then $Y$ is countably fuzzy sg-compact (countably fuzzy compact).

Corollary 4.20 Let $f: X \to Y$ be an intuitionistic fuzzy completely sg-continuous mapping. If $X$ is countably fuzzy compact, then $Y$ is countably fuzzy sg-compact (countably fuzzy compact).
References