Edge-Magic Labeling of some Graphs

A. Solairaju¹ and R. Raziya Begam²

¹Associate Professor of Mathematics,
Jamal Mohamed College, Tiruchy-620 020, India
E-mail: solairama@yahoo.co.in
²Assistant Professor of Mathematics,
Periyar E.V.R. College, Tiruchy-620 023, Tamilnadu, India.

Abstract
An edge magic labeling f of a graph with p vertices and q edges is a bijection
f: V ∪ E → {1, 2, ..., p + q} such that there exists a constant s for any (x, y)
in E satisfying f(x) + f(x, y) + f(y) = s. In this paper, the edge-magic labelings
of ncm and some other graphs are discussed.

Keywords: Edge-magic, Super edge-magic labeling; Merge Graph

Introduction
In this paper, by a graph we mean only finite simple undirected graphs. The standard
notations and terminology are as in [2]. Labeled graphs form useful mathematical
models for a wide range of applications such as coding theory, X-ray crystallography,
radar, astronomy, circuit design, remote control, communication networks and
database management [1]. They also have theoretical application in various areas of
mathematics such as linear algebra, group theory and combinatorial number theory.
Some definitions used in this paper are given below.

Definition 1.1: A graph with p vertices and edges is called total edge-magic if there is
a bijection f: V ∪ E → {1, 2, ..., p + q} such that there exists a constant ‘s’ for any
(u, v) in E satisfying f(u) + f(u, v) + f(v) = s. The original concept of total edge-magic
graph is due to Kotzig and Rosa.[3]

Definition 1.2: A total edge magic graph is called a super edge magic
graph if f: (V(G)) → {1, 2, ..., p}. The concept was introduced by Hikoe Enomoto et al., in
1998 [3].

Definition 1.3: A merge graph G₁ * G₂ can be formed from two graphs G₁
and G₂ by merging a node of G₁ with a node of G₂. As an example let us consider T₃, a
tree with three vertices and $S_2$ a star on three vertices then $T_3 \ast S_2$ is formed as follows. Consider a vertex $b$ of $T_3$. Consider a vertex $v_1$ of $S_2$.

Merge $v_1$ and $b$ to get $T_3 \ast S_2$ as in Fig. This graph has 5 vertices and 4 edges.

**Edge-Magic Labeling of $ncm$ Graphs**

**Theorem 2.1:** The $ncm$ graph is edge-magic for $m=3$ and $n=2, 3$,

**Proof:** The $ncm$ graphs have edge-magic labeling for $m=3$ and $n=2, 3$ as shown below.
Super Edge-Magic Labelings of Merge Graphs

Theorem 3.1: \( T_5 \ast S_n \) is super edge magic, where \( T_5 \) is a tree on five vertices and \( S_n \) is a star with \( n+1 \) vertices for \( n \geq 2 \).

Proof: Consider \( a, b, c, d, e \) as the vertices of the tree \( T_5 \) with the label \( c \) denoting the center. Label the star on \( n+1 \) vertices \( S_n \), by giving the labels \( v_2, v_3, \ldots, v_{n-1}, v_{n+1} \) continuously starting from one pendant vertex till the last pendant vertex and putting the label \( v_1 \) for the center of \( S_n \). To form the graph \( T_5 \ast S_n \), merge the vertex \( v_1 \) of \( S_n \) with the vertex \( d \) of \( T_5 \) and label it \( v_1 \). There are \( n+5 \) vertices and \( n+4 \) edges. Change the names of the labels \( a, b, c, \) and \( e \) as \( v_{n+2}, v_{n+3}, v_{n+4}, v_{n+5} \) respectively. The labeling \( f : V(T_5 \ast S_n) \rightarrow \{1, 2, \ldots, n+5\} \) defined by

\[
\begin{align*}
  f(v_i) &= i+1 \text{ for } i = 1; \\
  f(v_i) &= i+3 \text{ for } 1 < i \leq n+1, \\
  f(v_i) &= 3 \text{ for } i = n + 2; \\
  f(v_i) &= 1 \text{ for } i = n + 3, \\
  f(v_i) &= 4 \text{ for } i = n + 4; \\
  f(v_i) &= i \text{ for } i = n + 5
\end{align*}
\]

is a super edge-magic labeling. Here \( s = 2n+13 \).

We demonstrate the super edge magic labeling for \( T_5 \ast S_2 \)

A super edge-magic labeling of \( T_5 \ast S_2 \)

Figure 5
Theorem 3.2: If $T_6$ is a tree on 6 vertices and $S_n$ is a star on $n+1$ vertices, $T_6 * S_n$ is super edge magic for $n \geq 2$.

Proof: Let $a, b, c, d, e, f$ be the vertices of the tree $T_6$ with the label $c$ denoting the center of $T_6$. Consider the star $S_n$ on $n+1$ vertices. Label the pendant vertices $v_2, v_3, \ldots, v_{n+1}$ continuously and label the center of $S_n$ as $v_1$. Merge the vertex $c$ of the graph $T_6$ with the vertex $v_1$ of $S_n$ and rename it as $v_1$. We get $T_6 * S_n$ with $n+6$ vertices and $n+5$ edges. Change the names of the labels of $a, b, d, e, f$ as $v_{n+2}, v_{n+3}, v_{n+4}, v_{n+5}, v_{n+6}$ respectively.

The labeling $f : V(T_6*S_n) \rightarrow \{1, 2, \ldots, n+6\}$ defined by

- $f(v_i) = 2$ for $i = 1$,
- $f(v_i) = i + 3$ for $1 < i \leq n, i = n+2$,
- $f(v_i) = 3$ for $i = n+1$,
- $f(v_i) = 4$ for $i = n+3$,
- $f(v_i) = i+2$ for $i = n+4$,
- $f(v_i) = 1$ for $i = n+5$

is a super edge-magic labeling. Here $s=2n+16$. A demonstration of the super edge magic labeling is as follows:

![Diagram of $T_6*S_2$]

A super edge-magic labeling of $T_6*S_2$

Figure 6

Concluding Observations
We have obtained results similar to theorem 3.1, theorem 3.2 for the merge graphs $T_3 * S_3$ [4] and $T_5 * S_4$ [5]. An interesting open problem is whether it is possible to find a super edge-magic labeling for a general merge graph $T_m * S_n$ for $m > 2$, $n > 1$. Another important open problem to look into is, whether there exists an edge magic labeling for a general $ncm$ graph for $m>3$ and $0 < n < m$.

References


