

## Forecasting the Movement of Share Market Price using Fuzzy Time Series

**B.P. Joshi and Sanjay Kumar Ruhela**

*Department of Mathematics, Statistics & Computer Science  
G. B. Pant University of Agriculture & Technology,  
Pantnagar- 263 145(Uttarakhand) India  
E-mail: bpjoshi.13march@gmail.com*

### Abstract

There are various methods established on time series data having linguistic values for forecasting the future values with the help of fuzzy time series forecasting. However, the major problem in fuzzy time series forecasting is the accuracy in the forecasted values. The present paper proposes a new method of forecasting using fuzzy time series. The proposed method is based on highest membership grade and is a simplified computational approach for the forecasting. The proposed method is implemented to forecast the movement of share market price of State Bank of India (SBI) at Bombay Stock Exchange (BSE). The forecasted data have been compared with the results obtained by the method given by Singh [5] to show the superiority of proposed method.

**Keywords:** Fuzzy time series, Fuzzy membership grade, Fuzzy logical relations, movement of market price.

### Introduction

Fuzzy set theory and the concept of linguistic variables and its application to approximate reasoning developed by Zadeh [9] has been successfully employed by Song and Chissom [6-8] in fuzzy time series forecasting. Song and Chissom [6-8] developed the fuzzy time series models and implemented the developed models on the historical enrollment data of University of Alabama.

Chen [1-3] presented a simplified method for time series forecasting using the arithmetic operations rather than complicated max-min composition operations, there are many researchers used the concept of fuzzy time series in forecasting. Huarng [4] presented a heuristic model for time series forecasting using heuristic increasing and

decreasing relations to improve the forecast of enrollments and also implemented it for Taiwan Futures Exchange (TAIFEX) forecasting.

In this paper, we present a new method for forecasting the movements of market share price with fuzzy time series. It provides simple computational and minimizes the time of generating relational equations by using complex min-max composition operations. It overcomes the difficulty of searching a suitable defuzzification procedure providing crisp output of better accuracy. Further, the proposed method is implemented to forecast the market price of a State Bank of India's share at Bombay Stock Exchange (BSE).

### Some basic concepts of fuzzy time series

The various definitions and properties of fuzzy time series forecasting found, summarized and are presented as:

**Definition 1.** A fuzzy set is a class of objects with a continuum of grade of membership. Let  $U$  be the Universe of discourse with  $U = \{u_1, u_2, u_3, \dots, u_n\}$ , where  $u_i$  are possible linguistic values of  $U$ , then a fuzzy set of linguistic variables  $A_i$  of  $U$  is defined by

$$A_i = \mu_{A_i}(u_1)/u_1 + \mu_{A_i}(u_2)/u_2 + \mu_{A_i}(u_3)/u_3 + \dots + \mu_{A_i}(u_n)/u_n$$

Where  $\mu_{A_i}$  is the membership function of the fuzzy set  $A_i$ , such that  $\mu_{A_i} : U \rightarrow [0,1]$

**Definition 2.** Let  $Y(t)$  ( $t = \dots, 0, 1, 2, 3, \dots$ ), is a subset of  $R$ , be the Universe of discourse on which fuzzy sets  $f_i(t)$ , ( $i = 1, 2, 3, \dots$ ) are defined and  $F(t)$  is the collection of  $f_i$ , then  $F(t)$  is defined as fuzzy time series on  $Y(t)$ .

**Definition 3.** Suppose  $F(t)$  is caused only by  $F(t-1)$  and is denoted by  $F(t-1) \rightarrow F(t)$ ; then there is a fuzzy relationship between  $F(t)$  and  $F(t-1)$  and can be expressed as the fuzzy relational equation:

$$F(t) = F(t-1) \circ R(t, t-1)$$

here, "o" is Max-Min composition operator. The relation  $R$  is called first-order model of  $F(t)$ . Further if fuzzy relation  $R(t, t-1)$  of  $F(t)$  is independent of time  $t$ , that is to say for different times  $t_1$  and  $t_2$ ,  $R(t_1, t_1-1) = R(t_2, t_2-1)$ , then  $F(t)$  is called a time invariant fuzzy time series.

**Definition 4.** If  $F(t)$  is caused by more fuzzy sets,  $F(t-n), F(t-n+1), \dots, F(t-1)$ , the fuzzy relationship is represented by

$$A_{i_1}, A_{i_2}, A_{i_3}, \dots, A_{i_n} \rightarrow A_j$$

here,  $F(t - n) = A_{i_1}, F(t - n + 1) = A_{i_2}, \dots, F(t - 1) = A_{i_n}$ .  
 This relationship is called  $n^{th}$ -order fuzzy time series model.

**Methodology to implement proposed method**

In this section, we present the stepwise procedure of the proposed method of forecasting by fuzzy time series.

**Step 1.** Define the Universe of discourse,  $U$  based on the range of available time series data, by rule  $U = [D_{min}-D_1, D_{max}-D_2]$  where  $D_1$  and  $D_2$  are two proper positive numbers.

**Step 2.** Partition the Universe of discourse into equal length of intervals:  $u_1, u_2, \dots, u_m$ . The number of intervals will be in accordance with the number of linguistic variables (triangular fuzzy sets)  $A_1, A_2, \dots, A_m$ , to be considered.

**Step 3.** Construct the fuzzy sets  $A_i$  in accordance with the intervals in Step 2 and apply the triangular membership rule to each interval in each triangular fuzzy set so constructed.

**Step 4.** Fuzzify the data and establish the fuzzy logical relationships by the rule: If  $A_i$  is the fuzzy production of month  $n$  and  $A_j$  is the fuzzify production of month  $n + 1$ , then the fuzzy logical relation is denoted as  $A_i \rightarrow A_j$ . Here  $A_i$  is called current state and  $A_j$  is next state.

**Step 5.** Utilizing the fuzzy logical relationships, obtain the fuzzified output. After that defuzzified and get crisp output.

**Step 6.** In time series forecasting, the forecasting accuracy of a model is commonly measured in terms of mean square error (MSE) or in terms of average error. Lower the MSE or average error, better the forecasting method. The MSE and forecasting error are defined as

$$\text{Mean Square error} = \frac{\sum_{i=1}^n (\text{actual value}_i - \text{forecasted value}_i)^2}{n} \tag{1}$$

$$\text{Forecasting error (in percentage)} = \frac{|\text{forecasted} - \text{actual value}|}{\text{actual value}} \times 100. \tag{2}$$

$$\text{Average forecasting error (in percentage)} = \frac{\text{sum of forecasting error}}{\text{numbers of errors}} \times 100. \tag{3}$$

### Forecasting market price of SBI's share price with proposed model

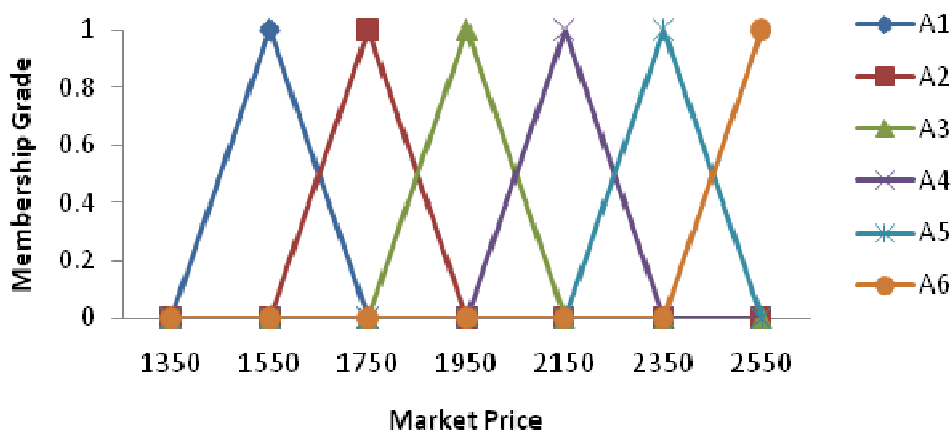
In this section, the proposed method is implemented to forecast market price data of SBI share at BSE.

**Step 1:** Annual report of SBI [10] for the year 2009-10 is used to define  $D_{min}$  and  $D_{max}$ . Then for this we define universe of discourse  $U = [1350, 2550]$ .

**Step 2:** The Universe of discourse is partitioned into six intervals of linguistic values:

$$u_1 = [1350, 1550], u_2 = [1550, 1750], u_3 = [1750, 1950], \\ u_4 = [1950, 2150], u_5 = [2150, 2350], u_6 = [2350, 2550].$$

**Step 3:** Six fuzzy sets  $A_1, A_2, \dots, A_6$ , are defined on the universe of discourse  $U$  and given in Fig. 1.



**Figure 1:** Fuzzy sets  $A_1, A_2, \dots, A_6$ ,

**Table 1:** Actual market price of SBI share at BSE.

Months	SBI's Share Price at BSE (Rs.)
April 2009	1355.00
May 2009	1891.00
June 2009	1935.00
July 2009	1840.00
August 2009	1886.90
September 2009	2235.00
October 2009	2500.00
November 2009	2394.00
December 2009	2374.75
January 2010	2315.25
February 2010	2059.95
March 2010	2120.05

**Step 4:** The time series data is fuzzified with triangular membership function and are placed in Table 2.

**Table 2:** Fuzzification of actual market price of SBI’s share.

Months	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	Fuzzified value
April 2009	0.25	0	0	0	0	0	A <sub>1</sub>
May 2009	0	.045	.0955	0	0	0	A <sub>3</sub>
June 2009	0	0	0.825	0.175	0	0	A <sub>3</sub>
July 2009	0	0.3	0.7	0	0	0	A <sub>3</sub>
August 2009	0	0.07	0.93	0	0	0	A <sub>3</sub>
September 2009	0	0	0	0.325	0.675	0	A <sub>5</sub>
October 2009	0	0	0	0	0.25	0.75	A <sub>6</sub>
November 2009	0	0	0	0	0.78	0.22	A <sub>5</sub>
December 2009	0	0	0	0	0.88	.12	A <sub>5</sub>
January 2010	0	0	0	0.18	0.82	0	A <sub>5</sub>
February 2010	0	0	0.46	0.54	0	0	A <sub>4</sub>
March 2010	0	0	0.15	0.85	0	0	A <sub>4</sub>

**Step 5:** Fuzzy logical relationships and fuzzy logical relationship groups of market price obtained and are as (Table 3 and 4).

**Table 3:** Fuzzy logical relationships of market price.

$A_1 \rightarrow A_3$	$A_3 \rightarrow A_3$	$A_3 \rightarrow A_3$	$A_3 \rightarrow A_3$
$A_3 \rightarrow A_5$	$A_5 \rightarrow A_6$	$A_6 \rightarrow A_5$	$A_5 \rightarrow A_5$
$A_5 \rightarrow A_5$	$A_5 \rightarrow A_4$	$A_4 \rightarrow A_4$	

**Table 4:** Fuzzy logical relationship groups.

$A_1 \rightarrow A_3$		
$A_3 \rightarrow A_3$	$A_3 \rightarrow A_5$	
$A_4 \rightarrow A_4$		
$A_5 \rightarrow A_5$	$A_5 \rightarrow A_4$	$A_5 \rightarrow A_6$
$A_6 \rightarrow A_5$		

**Step 5:** The following table shows the forecasted market price of SBI share at BSE obtained by proposed model. Results obtained by the method given by Singh [5] are also included for the comparison.

**Table 5:** Forecasted market price of SBI share.

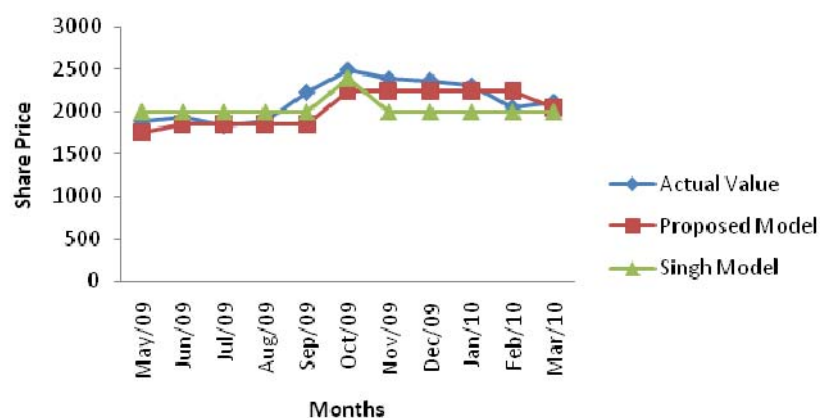
Months	Market price of SBI's Share at BSE (Rs.)		
	Actual	Forecasted	Singh's method
April 2009	1355.00		
May 2009	1891.00	1750.00	2000.00
June 2009	1935.00	1850.00	2000.00
July 2009	1840.00	1850.00	2000.00
August 2009	1886.90	1850.00	2000.00
September 2009	2235.00	1850.00	2000.00
October 2009	2500.00	2250.00	2400.00
November 2009	2394.00	2250.00	2000.00
December 2009	2374.75	2250.00	2000.00
January 2010	2315.25	2250.00	2000.00
February 2010	2059.95	2250.00	2000.00
March 2010	2120.05	2050.00	2000.00

**Step 6:** To have a comparison of accuracy in forecasted values of our proposed model with Singh model, the mean square error (MSE) and average error of forecast have been computed and have been compared with Singh method in Table 6.

The MSE or average error of forecasting of a method is measure of accuracy of that forecasting method. Lower the MSE or average error, better the forecasting method.

**Table 6:** Comparison of MSE and average error.

Model	Proposed Model	Singh Model
Mean Square Error(MSE)	29170.43	48434.98
Average Forecasting Error	6.199161	8.432273

**Figure 2:** Actual versus Proposed model and Singh Model of SBI's at BSE.

## **Conclusions**

In this paper we have proposed different method of fuzzification, which gives different fuzzy logic relations. The algorithm of the proposed method is simple. It minimizes the complicated computations of fuzzy relational matrices and search for a suitable defuzzification process and provides the forecasted values of better accuracy. Further the method has also been implemented to forecast the market price of State Bank of India's share at Bombay Stock Exchange (BSE). The proposed method is also compared with the method given by Singh [5]. The comparison of MSE and average forecasted error given in Table 6 shows the superiority of the proposed model over the Singh's model as it provides forecast of higher accuracy.

## **References**

- [1] Chen, S.M., Hsu, C.C.: A new Method to Forecast Enrollments using Fuzzy Time Series, *International Journal of Applied Sciences and Engineering*. 2 (3), 234–244(2004)
- [2] Chen, S.M.: Forecasting Enrollments based on Fuzzy Time Series, *Fuzzy Sets and Systems*. 81, 311–319(1996)
- [3] Chen, S.M.: Forecasting Enrollments based on High-Order Fuzzy Time Series, *Cybernetics and Systems: An International Journal*. 33, 1–16(2002)
- [4] Huarng, K.: Heuristic Models of Fuzzy Time Series for Forecasting, *Fuzzy Sets and Systems*. 123, 369–386(2001)
- [5] Singh, S.R.: A simple Method of Forecasting based on Fuzzy Time Series, *Applied Mathematics and Computation*. 186, 330–339(2007)
- [6] Song, Q., Chissom, B.: Fuzzy Time Series and Its Models, *Fuzzy Sets and Systems*. 54, 269–277(1993)
- [7] Song, Q., Chissom, B.S.: Forecasting Enrollments with Fuzzy Time Series – Part I, *Fuzzy Sets and Systems*. 54, 1–9(1993)
- [8] Song, Q., Leland, R.P.: Learning Defuzzification Techniques and Applications, *Fuzzy Sets and Systems*. 81 (3), 321–329(1996)
- [9] Zadeh, L.A.: Fuzzy set, *Information and Control*. 8, 338–353(1965)
- [10] Annual Report of State Bank of India, (2009-10)