Fuzzy Matrix Theory and its Application for Recognizing the Qualities of Effective Teacher

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Abstract
Teacher quality matters a great deal in terms of student learning. Therefore, teacher quality measurement is important. Researchers who study teacher quality put a great interest in investigating the quality of effective teacher. A number of policy makers and researchers have proposed that effectiveness, as determined by teachers’ contribution to student learning, should be an important component of assessing teacher quality. The purposes of this research are to determine the components of teacher quality and to apply the Teacher Quality Index (TQI) for recognizing the qualities of effective teacher, using FRM (Fuzzy relational maps) for the developments of educational institutes.

Keywords: Fuzzy Matrix, Fuzzy Set, Fuzzy logic.

Introduction
Fuzzy set theory which initiated about 40 years ago by Zadeh in the scientific community. Fuzzy set theory is a generalization of classical set theory, in the sense that a given universe $\chi$ and a subset $A$ of it, any element $x$ of $\chi$, instead of having a degree of membership either 0 or 1 in $A$ as postulated under the classical set theory, can have a membership value $\mu_A(x) \in [0,1]$ in a set $A$ which represents the degree of its belonging to $A$. In other words, $A$ is a fuzzy subset of universe $\chi$, characterized by the membership function $\mu_A(x)$, $x \in \chi$.

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth -- truth-values between "completely true" and "completely false". Dr. Lotfi Zadeh of UC/Berkeley introduced it in the 1960's as a means to model the uncertainty of natural language.

Vasanta Kandaswamy introduced the notion of Fuzzy Relational Maps (FRMs). It
is analogous to Fuzzy Control Maps (FCMs) described and discussed earlier. In FCMs he promotes the correlations between causal associations among concurrently active units. But in FRMs he divide the very causal associations into two disjoint units, for example, the relation between a teacher and a student or relation between an employee or employer or a relation between doctor and patient and so on.

Using the model described by Kandaswamy, we will try to Recognizing the Qualities of Effective Teacher for the management of educational institute of Indore District. Our work will be helpful to describe the quality of teacher, which gives the best results for students as well good management gives bright future too.

The purpose of this study was to research educators’ perception of teacher quality and whether it derives meaning from a social construct of both policy—No Child Left Behind— and environmental factors.

The purpose of this study is three fold
1. to review how the terms excellence and quality are shaped by socially constructed realities
2. to identify how educators perceive teaching quality, and
3. to review how school districts develop teacher quality.

The research questions include
1. How does the social construction of reality interplay with the terms quality/excellence?
2. How does the No Child Left Behind law shape a definition of quality teaching?
3. How is student achievement connected to teacher quality?
4. What are administrators’ and teachers’ perceptions of quality teaching attributes?
5. How do educators who are identified by administrators as quality teachers identify and Develop attributes of teacher quality in themselves?
6. How do school districts cultivate quality?

Preliminary

Definition 2.1: A FRM is a directed graph or a map from D to R with concepts like policies or events etc as nodes and causalities as edges. It represents causal relation between spaces D and R.

Definition 2.2: Let $D_1, \ldots, D_n$ be the nodes of the domain space D of an FRM and $R_1, \ldots, R_m$ be the nodes of the range space R of an FRM. Let the matrix $E$ be defined as $E = (e_{ij})$ where $e_{ij}$ is the weight of the directed edge $D_iR_j$ (or $R_jD_i$). $E$ is called the relational matrix of the FRM.

Definition 2.3: Let $D_1, \ldots, D_n$ and $R_1, \ldots, R_m$ denote the nodes of the FRM. Let $D_iR_j$ (or $R_jD_i$) be the edges of an FRM, $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, n$. Let the edges form a directed cycle.
An FRM is said to be a cycle if it posses a directed cycle.
An FRM is said to be acyclic if it does not posses any directed cycle.
An FRM with cycles is said to be an FRM with feedback.

**Definition 2.4:** When there is a feedback in the FRM, i.e when the causal relations flow through a cycle in a revolutionary manner, the FRM is called a dynamical system,

Let $D_iR_j \,(or\, R_jD_i), 1 \leq j \leq m, 1 \leq i \leq n$. When $R_i \,(or\, D_i)$ is switched on and if causality flows through edges of the cycle and if it again causes $R_i \,(or\, D_i)$, we say that the dynamical system goes round and round. This is true for any node $R_j \,(or\, D_j)$ for $1 \leq i \leq n$, (or $1 \leq j \leq m$). The equilibrium state of this dynamical system is called the hidden pattern.

If the FRM settles down with a state vector repeating in the form,

$$A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow \ldots \rightarrow A_i \rightarrow A_1 \,(or\, B_1 \rightarrow B_2 \rightarrow \ldots \rightarrow B_i \rightarrow B_1)$$

Then this equilibrium is called a limit cycle.

**Fuzzy Matrix**

Here we recall some of the basic properties about fuzzy matrices and operations using them. Throughout this article $[0, 1]$ denotes the unit interval.

We say $x \in [0, 1]$ if $0 \leq x \leq 1$. We also call the unit interval as a fuzzy interval.

A fuzzy associative matrix express fuzzy logic rules in matrix form this rules usually takes to variables as input, mapping clearly to a 2-dimensional matrix, although theoretically a matrix of any number of dimensions is possible.

We have just defined fuzzy sets and the need of it’s in our study. Now we proceed on to define various types of fuzzy matrices without going deep into their structure.

We call

$$A = \begin{bmatrix}
0.3 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.6 & 0.5 & 1.0 & 0.9 & 0.0 \\
0.0 & 0.4 & 0.1 & 0.0 & 1.0
\end{bmatrix}$$

to be fuzzy matrix of order $3 \times 5$. We see $A$ is also a $3 \times 5$ matrix. Thus we can still see that all fuzzy matrices are matrices but every matrix in general is not a fuzzy matrix.

**Multiplication of Fuzzy Matrix**

The product of two fuzzy matrices under usual matrix multiplication is not a fuzzy matrix. So we need to define a compatible operation analogous to product that the product again happens to be a fuzzy matrix. However even for this new operation if the product $XY$ is to be defined we need the number of columns of $X$ is equal to the number of rows of $Y$. The two types of operations which we can have are max-min operation and min-max operation.
Let
\[ X = \begin{bmatrix}
0.3 & 1 & 0.7 & 0.2 & 0.5 \\
1 & 0.9 & 0 & 0.8 & 0.1 \\
0.8 & 0.2 & 0.3 & 1 & 0.4 \\
0.5 & 1 & 0.6 & 0.7 & 0.8 \\
\end{bmatrix} \]
be a $4 \times 5$ fuzzy matrix and let
\[ Y = \begin{bmatrix}
0.8 & 0.3 & 1 \\
0.7 & 0 & 0.2 \\
1 & 0.7 & 1 \\
0.5 & 0.4 & 0.5 \\
0.4 & 0 & 0.7 \\
\end{bmatrix} \]
be a $5 \times 3$ fuzzy matrix. \(X \times Y\) defined using max. min function.
\[ X \times Y = \begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33} \\
C_{41} & C_{42} & C_{43} \\
\end{bmatrix} \]
where,
\[ C_{11} = \max \{\min (0.3, 0.8), \min (1, 0.7), \min (0.7, 1), \min (0.2, 0.5), \min (0.5, 0.4)\} = \max \{0.3, 0.7, 0.7, 0.2, 0.4\} = 0.7. \]
\[ C_{12} = \max \{\min (0.3, 0.3), \min (1, 0), \min (0.7, 0.7), \min (0.2, 0.4), \min (0.5, 0)\} = \max \{0.3, 0, 0.7, 0.2, 0\} = 0.7 \]
and so on.
\[ X \times Y = \begin{bmatrix}
0.7 & 0.7 & 0.7 \\
0.8 & 0.4 & 1.0 \\
0.8 & 0.4 & 0.8 \\
0.7 & 0.6 & 0.7 \\
\end{bmatrix} \]
is a $4 \times 3$ matrix.

Similarly for the same \(X\) and \(Y\) we can adopt the operation as min. max operation.

**Teacher**: One who teaches or instruct, one whose business is or occupation is to instruct others.

**Student**: A person who studies, one who is devoted to learning.

**Method**
Here we are giving a simple example to understand the FRM method as suggested by Kandasamy.

Suppose the domain space as the concepts belonging to the teacher say \(D_1, \ldots, D_3\) and the range space denote the concepts belonging to the student say \(R_1, R_2\) and \(R_3\), describe as follows.

**Domain Space**
\(D_1 – \text{Teaching is good}\)
D₂ – Teaching is average  
D₃ – Teaching is poor  

**Range Space**  
R₁ – Good Student  
R₂ – Average Student  
R₃ – Bad Student.

\[
\begin{array}{ccc}
D1 & D2 & D3 \\
R1 & R2 & R3 \\
\end{array}
\]

The relational matrix \( E \) got from the above map is

\[
E = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

If \( A = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \) is passed on in the relational matrix \( E \), the instantaneous vector ,

\[
AE = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\( AE = (0 \ 0 \ 1) \) implies that the student considered is a bad student.

Considering  
\( AE = A_1 \)

Then

\[
A_1 E^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\( = (0 \ 0 \ 1) \)

Gives the outcome is that the student is bad student.

**Conclusion and Results**  
Teacher quality matters a great deal in terms of student learning. Therefore, teacher quality measurement is important. *Regulation of Ministry of National Education No.1,*
2007 article 1 states that every teacher must fulfill the academic qualification standard and teacher competency. Teacher competency includes pedagogical, personal, social, and professional competencies while the qualifications are achieved by getting higher education certificate. The qualities of effective teachers were investigated through the series of statements in each category:

- teacher as a person,
- planning for instruction,
- classroom management and organization,
- implementing instruction, as well as monitoring student progress and potential.

Senior high school students were surveyed across Indore City; including Government and Private Schools. Following result analyze the quality of students and teachers using the methods as described in 2.7

**Result**

Suppose the domain space as the concepts belonging to the teacher say D₁,…, D₁₂ and the range space denote the concepts belonging to the student say R₁, R₂ and R₃, as follows:

**Domain space**

D₁ Teacher has quality of passionate about teaching, and tries to motivate his student.
D₂ He has uses the latest technology in teaching, uses creativity and variety.
D₃ The fulfillment of the prerequisite to get to know his student individually.
D₄ Expression and affection towards his student.
D₅ Influencing his student towards positive change.
D₆ Teaching is good.
D₇ Teaching style by traditional methods.
D₈ Problem solution are average.
D₉ Teaching is poor.
D₁₀ Doesn’t spend time thinking about student.
D₁₁ Often gets angry.
D₁₂ Doesn’t listen to students.

**Range Space**

R₁ Excellent student (Creative in nature, consistently doing something forms a habit, willing to learn, Pays attention in class, help other students about their difficulties )
R₂ Good Student (Interactive in class, ask question about his queries, complete his home work.)
R₃ Average student.( Often misunderstands original thoughts of a speaker or writer and derives a wrong conclusion).
R₄ Bad Student. (not interested to learn. , Angry on others )
The relational directed graph of the teacher-student model is given in Figure.

The relational matrix (E) is given by:

\[
E = \begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]

If \( A = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \)
Now let $AE = A_1$ then

$$AE = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$AE = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Excellent Student.}$$

Here we find five best qualities of Teachers which implies we got Excellent Student and the quality as follows:

1. Teacher has quality of passionate about teaching and tries to motivate his student.
2. He has uses the latest technology in teaching, uses creativity and Varity.
3. The fulfillment of the prerequisite to get to know his student individually.
4. Expression and affection to words his student.
5. Influencing his student to words his student.

Now again we put $A_1 E^T = A_2$

Then
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\[
A_2 E = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
5 & 0 & 0 & 0
\end{bmatrix}
\]

After threshold

\[= \begin{bmatrix}
1 & 0 & 0 & 0
\end{bmatrix}\]

Which implies that the student is excellent student.

Let

\[\begin{align*}
A_2 E &= A_3 \\
A_3 E &= \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Again we found best five qualities of teachers.

After updating and threshold the instantaneous vector at each stage we obtain the following chain

\[A_1 \rightarrow A_2 \rightarrow A_3 \ldots \rightarrow A_i \rightarrow A_1\]

Thus the equilibrium a limit cycle and \(A_1\) is a fixed point. We can say if the teacher having the best qualities, then the result has come out with excellent students and subsequently institutes get benefited for better results of student’s future.
References