International Journal of Engineering Studies.
ISSN 0975-6469 Volume 9, Number 1 (2017), pp. 45-56

http://www.ripublication.com

Fuzzy Soft Topology

G. Kalpana and C. Kalaivani

Department of Mathematics, SSN College of Engineering,
Kalavakkam - 603110, Chennai, India.

Abstract

In the present paper we introduce the concept of fuzzy soft topological space in Šostak sense. Then the notions of fuzzy soft r-closure and r-interior are introduced and their basic properties are investigated. Furthermore, fuzzy soft r -continuous mapping, fuzzy soft r -open, fuzzy soft r -closed mappings and fuzzy soft r -homeomorphism for fuzzy soft topological spaces are given and structural characteristics are discussed.

1. INTRODUCTION

The notion of a fuzzy set was introduced by Zadeh [13] in his classical paper of 1965. In 1968, Chang [2] gave the definition of fuzzy topology, which is family of fuzzy sets satisfying the three classical axioms. Later, fuzzy topological space is generalized in different ways, one of which is developed by Šostak [10]. Then plenty of works on Šostak fuzzy topological spaces have been done in order to extend various concepts in classical topology.

In 1999, Molodtsov [7] introduced the concept of a soft set and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. Maji et al. [5,6] presented some new definitions on soft sets and discussed in detail the application of soft set theory in decision making problems.

Shabir and Naz [9] introduced the concept of soft topological space and studied neighborhoods and separation axioms. Tanay and Kandemir [11] introduced the definition of fuzzy soft topology over a subset of the initial universe set. Later, Roy and Samanta [8] gave the definition of fuzzy soft topology over the initial universe set. It was further studied by Varol and Ayg$\ddot{u}$n [12] and Cetkin and Aygun [3] etc.

The aim of the paper is to introduced fuzzy soft topology in Šostak sense. Also, we introduce the notion of fuzzy soft r-closure, fuzzy soft r-interior and discussed some of its basic properties. Furthermore, we also define fuzzy soft r-continuity, fuzzy soft r-open mapping, fuzzy soft r-closed mapping and fuzzy soft r-homeomorphism and investigate some characterization.

2. PRELIMINARIES

Throughout this paper, X refers to an initial universe, E is the set of all parameters for X, $\mathrm{I} = [0,1]$, $\mathrm{I}_0 = (0,1]$ and I^X is the set of all fuzzy sets on X $\widetilde{(\mathrm{X}, \mathrm{E})}$ denotes the collection of all fuzzy soft sets on X and is called a fuzzy soft universe.

Definition 2.1. [4] $\mathrm{f_A}$ is called a fuzzy soft set on X, where f is a mapping from the parameter set , E into I^X , i.e., $\mathrm{f_A(e)}$ is a fuzzy set on X, for each $e \in A \subseteq E$ and $\mathrm{f_A(e)} = 0_\mathrm{X}$ if $e \notin A$, where 0_X denotes empty fuzzy set on X.

Definition 2.2. [4] Two fuzzy soft sets $\mathrm{f_A}$ and $\mathrm{g_B}$ on X we say that $\mathrm{f_A}$ is called fuzzy soft subset of $\mathrm{g_B}$ and write $\mathrm{f_A} \sqsubseteq \mathrm{g_B}$ if $\mathrm{f_A(e)} \leq \mathrm{g_B(e)}$, for every $e \in E$.

Definition 2.3. [4] Two fuzzy soft sets $\mathrm{f_A}$ and $\mathrm{g_B}$ on X are called equal if $\mathrm{f_A} \sqsubseteq \mathrm{g_B}$ and $\mathrm{g_B} \sqsubseteq \mathrm{f_A}$.

Definition 2.4. [4] Let $\mathrm{f_A}, \mathrm{g_B} \in \widetilde{(\mathrm{X}, \mathrm{E})}$. Then the union of $\mathrm{f_A}$ and $\mathrm{g_B}$ is also a fuzzy soft set $\mathrm{h_C}$, defined by $\mathrm{h_C(e)} = \mathrm{f_A(e)} \vee \mathrm{g_B}\ \mathrm{(e)}$ for all $e \in E$, where $C = A \cup B$. Here we write $\mathrm{h_C} = \mathrm{f_A} \sqcup \mathrm{g_{B.}}$

Definition 2.5. [4] Let $\mathrm{f_A}, \mathrm{g_B} \in \widetilde{(\mathrm{X}, \mathrm{E})}$. Then the interasection of $\mathrm{f_A}$ and $\mathrm{g_B}$ is also a fuzzy soft set $\mathrm{h_C}$, defined by $\mathrm{h_C(e)} = \mathrm{f_A(e)} \wedge \mathrm{g_B}\ \mathrm{(e)}$ for all $e \in E$, where $C = A \cap B$. Here we write $\mathrm{h_C} = \mathrm{f_A} \sqcap \mathrm{g_{B.}}$

Definition 2.6. [4] A fuzzy soft set $\mathrm{f_E}$ on X is called a null fuzzy soft set denoted by 0_E if $\mathrm{f_E(e)} = 0_\mathrm{X}$ for each $e \in E$.

Definition 2.7. [4] A fuzzy soft set $\mathrm{f_E}$ on X is called a absolute fuzzy soft set denoted by 1_E if $\mathrm{f_E(e)} = 1_\mathrm{X}$ for each $e \in E$.

Definition 2.8. [4]

Let $\mathrm{f_A} \in \widetilde{(\mathrm{X}, \mathrm{E})}$. Then the complement of $\mathrm{f_A}$ is denoted by f_A^c and is defined by $f_A^c\ (e) = 1 - \mathrm{f_A(e)}$ for each $e \in E$.

Lemma 2.9. [1] Let Δ be an index set and $f_A, g_B, h_C, (f_A)_i, (g_B)_i \in \widetilde{(X,E)}$, $i \in \Delta$, then we have the following properties:

(1) $f_A \sqcap f_A = f_A,\ f_A \sqcup f_A = f_A$.

(2) $f_A \sqcap g_B = g_B \sqcap f_A,\ f_A \sqcup g_B = g_B \sqcup f_A$.

(3) $f_A \sqcup (g_B \sqcup h_C) = (f_A \sqcup g_B) \sqcup h_C,\ f_A \sqcap (g_B \sqcap h_C) = (f_A \sqcap g_B) \sqcap h_C$.

(4) $f_A = f_A \sqcup (g_B \sqcap h_C),\ f_A = f_A \sqcap (g_B \sqcup h_C)$.

(5) $f_A \sqcap (\sqcup_{i\in\Delta} (g_B)_i) = \sqcup_{i\in\Delta} (f_A \sqcap (g_B)_i)$.

(6) $f_A \sqcup (\sqcap_{i\in\Delta} (g_B)_i) = \sqcap_{i\in\Delta} (f_A \sqcup (g_B)_i)$.

(7) $0_E \sqsubseteq f_A \sqsubseteq 1_E$.

(8) $(f_A^c)^c = f_A$.

(9) $(\sqcap_{i\in\Delta} (f_A)_i)^c = \sqcup_{i\in\Delta} (f_A)_i^c$.

(10) $(\sqcup_{i\in\Delta} (f_A)_i)^c = \sqcap_{i\in\Delta} (f_A)_i^c$.

(11) If $f_A \sqsubseteq g_B$, then $g_A^c \sqsubseteq f_A^c$.

Definition 2.10. [1] Let $\varphi: X \to Y$ and $\psi: E \to F$ be two mappings, where E and F are parameter sets for the crisp sets X and Y, respectively. Then φ_ψ is called a fuzzy soft mapping from $\widetilde{(X,E)}$ into $\widetilde{(Y,F)}$ and denoted by $\varphi_\psi: \widetilde{(X,E)} \to \widetilde{(Y,F)}$.

Definition 2.11. [1] Let f_A and g_B be two fuzzy soft sets over X and Y, respectively and let φ_ψ be a fuzzy soft mapping from $\widetilde{(X,E)}$ into $\widetilde{(Y,F)}$.

(1) The image of f_A under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi(f_A)$ and is defined as, $\varphi_\psi(f_A)_k(y) =$

$$\begin{cases} \vee_{\varphi(x)=y} \vee_{\psi(e)=k} f_A(e)(x), & \text{if } \varphi^{-1}(y) \neq \emptyset,\ \psi^{-1}(k) \neq \emptyset; \\ 0, & otherwise, \end{cases}$$

for all $k \in F$, for all $y \in Y$.

(2) The preimage of g_B under the fuzzy soft mapping φ_ψ, denoted by $\varphi_\psi^{-1}(g_B)$ and is defined as,

$$\varphi_\psi^{-1}(g_B)(e)(x) = g_B(\psi(e))(\varphi(x)),\ \text{for all } e \in E,\ \text{for all } x \in.$$

Definition 2.12. [1] If φ and ψ are injective (surjective) then the fuzzy soft mapping φ_ψ is injective (surjective). If φ_ψ is both injective and surjective, then it is called bijective.

Lemma 2.13. [1] Let X and Y be two Universes, f_A, $(f_A)_i \in \widetilde{(X,E)}$, g_B, $(g_B)_i \in \widetilde{(Y,F)}$, for all $i \in \Delta$ where Δ is an index set, and φ_ψ is a soft mapping from $\widetilde{(X,E)}$ into $\widetilde{(Y,F)}$. Then,

(1) If $(f_A)_1 \sqsubseteq (f_A)_2$, then $\varphi_\psi((f_A)_1) \sqsubseteq \varphi_\psi((f_A)_2))$.

(2) If $(g_B)_1 \sqsubseteq (g_B)_2$, then $\varphi_\psi^{-1}((g_B)_1) \sqsubseteq \varphi_\psi^{-1}(g_B)_2)$.

(3) $f_A \sqsubseteq \varphi_\psi^{-1}\left(\varphi_\psi(f_A)\right)$, the equality holds if φ_ψ is injective.

(4) $\varphi_\psi(\varphi_\psi^{-1}(g_B)) \sqsubseteq g_B$, the equality holds if φ_ψ is surjective.

(5) $\varphi_\psi(\sqcup_{i\in\Delta}(f_A)_i) = \sqcup_{i\in\Delta}\varphi_\psi((f_A)_i)$.

(6) $\varphi_\psi(\sqcap_{i\in\Delta}(f_A)_i) \sqsubseteq \sqcap_{i\in\Delta}\varphi_\psi((f_A)_i)$, the equality holds if φ_ψ is injective.

(7) $\varphi_\psi^{-1}(\sqcup_{i\in\Delta}(g_B)_i) = \sqcup_{i\in\Delta}\varphi_\psi^{-1}((g_B)_i)$.

(8) $\varphi_\psi^{-1}(\sqcap_{i\in\Delta}(g_B)_i) = \sqcap_{i\in\Delta}\varphi_\psi^{-1}((g_B)_i)$.

(9) $\varphi_\psi^{-1}(g_B^c) = (\varphi_\psi^{-1}(g_B))^c$.

(10) $\varphi_\psi^{-1}(0_F) = 0_E$, $\varphi_\psi^{-1}(1_F) = 1_E$.

(11) $\varphi_\psi(1_E) = 1_F$ if φ_ψ is surjective and $\varphi_\psi(0_E) = 0_F$.

3. FUZZY SOFT r-CLOSURE AND r-INTERIOR

In this section, we generalized the concept of fuzzy topological space (X,τ,E) in the sense of Šostak. Here, $\tau: \widetilde{(X,E)} \to I$ is a mapping which assigns to every fuzzy soft set of X the real number. The value of $\tau(f_A)$ is interpreted as the degree of openness of a fuzzy soft set f_A.

Definition 3.1. A fuzzy soft topology is a map $\tau: \widetilde{(X,E)} \to I$, satisfying the following three axioms:

(1) $\tau(0_E) = \tau(1_E) = 1$,

(2) $\tau(f_A \sqcap g_B) \geq \tau(f_A) \wedge \tau(g_B)$, for all $f_A, g_B \in \widetilde{(X,E)}$,

(3) $\tau(\sqcup_{i\in\Delta}(f_A)_i) \geq \wedge_{i\in J}\tau((f_A)_i)$, for all $((f_A)_i)_{i\in J} \in \widetilde{(X,E)}$.

Then (X, τ, E) is called a fuzzy soft topological space.

Definition 3.2. Let (X, τ, E) be a fuzzy soft topological space and $\eta: \widetilde{(X,E)} \to I$ defined by $\eta(f_A) = \tau(f_A^c)$, satisfying the following three axioms:

(1) $\eta(0_E) = \eta(1_E) = 1$,

(2) $\eta(f_A \sqcup g_B) \geq \eta(f_A) \wedge \tau(g_B)$, for all $f_A, g_B \in \widetilde{(X,E)}$,

(3) $\eta(\sqcap_{i \in \Delta} (f_A)_i) \geq \wedge_{i \in J} \eta((f_A)_i)$, for all $((f_A)_i)_{i \in J} \in \widetilde{(X,E)}$.

Then (X, τ, E) is called a fuzzy soft cotopological space.

Counterexample 3.3. Let $X = \{x, y\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2,\}$, $B = \{e_2, e_3\}$, $C = \{e_2\}$. We define $f_A, g_B, h_C, i_E \in \widetilde{(X,E)}$ by $f_A = \left\{f_A(e_1)_{[x,y]}^{[0.6,0.5]}, f_A(e_2)_{[x,y]}^{[0.5,0.4]}\right\}$, $g_B = \left\{g_B(e_2)_{[x,y]}^{[0.5,0.4]}, g_B(e_3)_{[x,y]}^{[0.7,0.2]}\right\}$, $h_C = \left\{h_C(e_2)_{[x,y]}^{[0.5,0.4]}\right\}$ and $i_E = \left\{i_E(e_1)_{[x,y]}^{[0.5,0.4]}, i_E(e_2)_{[x,y]}^{[0.5,0.4]}, i_E(e_3)_{[x,y]}^{[0.7,0.2]}\right\}$.

We define $\tau: \widetilde{(X,E)} \to I$ by

$$\tau(U) = \begin{cases} 1, & U = 0_E \text{ or } 1_E, \\ 0.6, & U = f_A, \\ 0.5, & U = g_B, \\ 0.7, & U = h_C, \\ 0.5, & U = i_E, \\ 0, & otherwise. \end{cases}$$

(1) Clearly, $\tau(0_E) = \tau(1_E) = 1$,

(2) Clearly, we have $f_A \sqcap g_B = h_C$, $f_A \sqcap h_C = h_C$, $g_B \sqcap h_C = h_C$, $f_A \sqcap i_E = f_A$, $g_B \sqcap i_E = g_B$, $h_C \sqcap i_E = h_C$. Hence, $\tau(f_A \sqcap g_B) \geq \tau(f_A) \wedge \tau(g_B)$, for all $f_A, g_B \in \widetilde{(X,E)}$,

(3) Clearly we verified that $\tau(\sqcup_{i \in \Delta} (f_A)_i) \geq \wedge_{i \in J} \tau((f_A)_i)$, for all $((f_A)_i)_{i \in J} \in \widetilde{(X,E)}$.

Hence, (X, τ, E) is called a fuzzy soft topological space.

Definition 3.4. Let (X, τ, E) be a fuzzy soft topological space and $f_A \in \widetilde{(X,E)}$. Then f_A is called fuzzy soft r-open if $\tau(f_A) \geq r$, where $r \in I_0$.

Definition 3.5. Let (X, τ, E) be a fuzzy soft topological space and $f_A \in \widetilde{(X,E)}$. Then f_A is called fuzzy soft r-closed if $\tau(f_A^c) \geq r$, where $r \in I_0$.

Definition 3.6. Let (X,τ,E) be a fuzzy soft topological space and $f_A \in \widetilde{(X,E)}$, $r \in I_0$. Then, fuzzy soft r -closure of f_A is denoted by $Cl(f_A, r)$ and is defined by $Cl(f_A, r) = \sqcap \{ f_C : \tau(f_C^c) \geq r,\ f_A \sqsubseteq f_C \}$.

Clearly, $Cl(f_A, r)$ is the smallest fuzzy soft r-closed set which contains f_A.

Definition 3.7. Let (X,τ,E) be a fuzzy soft topological space and $f_A \in \widetilde{(X,E)}$, $r \in I_0$. Then, fuzzy soft r -interior of f_A is denoted by $Int(f_A, r)$ and is defined by $Int(f_A, r) = \sqcap \{ f_B : \tau(f_B) \geq r,\ f_B \sqsubseteq f_A \}$.

Clearly, $Int(f_A, r)$ is the largest fuzzy soft r-open set which contained in f_A.

Theorem 3.8. Let (X,τ,E) be a fuzzy soft topological space. For each f_A, $g_B \in \widetilde{(X,E)}$ and $r \in I_0$. Then,

(1) $Cl(0_E, r) = 0_E$.

(2) $f_A \sqsubseteq Cl(f_A, r)$.

(3) $Cl(Cl(f_A, r), r) = Cl(f_A, r)$.

(4) If $f_A \sqsubseteq g_B$, then $Cl(f_A, r) \sqsubseteq Cl(g_B, r)$.

(5) f_A is fuzzy soft r -closed if and only if $f_A = Cl(f_A, r)$.

(6) $Cl(f_A \sqcup g_B, r) = Cl(f_A, r) \sqcup Cl(g_B, r)$.

(7) $Cl(f_A \sqcap g_B, r) \sqsubseteq Cl(f_A, r) \sqcap Cl(g_B, r)$.

Proof. (1) Clearly, $0_E \sqsubseteq Cl(0_E, r)$. Since, 0_E is fuzzy soft r-closed set and $Cl(0_E, r)$ is the smallest r-closed set containing 0_E , $Cl(0_E, r) \sqsubseteq 0_E$. Therefore, $Cl(0_E, r) = 0_E$.

(2) By the definition of fuzzy soft r -closure, $f_A \sqsubseteq Cl(f_A, r)$.

(3) By (2), $Cl(f_A, r) \sqsubseteq Cl(Cl(f_A, r), r)$. Here $Cl(f_A, r)$ is fuzzy soft r-closed set. Since, $Cl(Cl(f_A, r), r)$ is the smallest fuzzy soft r -closed set containing $Cl(f_A, r)$, $Cl(Cl(f_A, r), r) \sqsubseteq Cl(f_A, r)$.Therefore, $Cl(Cl(f_A, r), r) = Cl(f_A, r)$.

(4) Since $f_A \sqsubseteq g_B$, we have $\{ f_C : \tau(f_C^c) \geq r,\ f_A \sqsubseteq f_C \} \sqsupseteq \{ f_C : \tau(f_C^c) \geq r,\ g_B \sqsubseteq f_C \}$. Therefore,

$$\begin{aligned} Cl(f_A, r) &= \sqcap \{ f_C : \tau(f_C^c) \geq r, f_A \sqsubseteq f_C \} \\ &\sqsubseteq \sqcap \{ f_C : \tau(f_C^c) \geq r, g_B \sqsubseteq f_C \} \\ &= Cl(g_B, r). \end{aligned}$$

(5) Assume that f_A is fuzzy soft r-closed set. (2), we have $f_A \sqsubseteq Cl(f_A, r)$. Since f_A is fuzzy soft r-closed set and $Cl(f_A, r)$ is the smallest fuzzy soft r-closed set containing

f_A, $Cl(f_A, r) \sqsubseteq f_A$. Hence, $Cl(f_A, r) = f_A$.

Conversely, assume that $Cl(f_A, r) = f_A$. Since, $Cl(f_A, r)$ is the smallest fuzzy soft r-closed set. Hence, f_A is fuzzy soft r-closed set.

(6) Clearly, $f_A \sqcup g_B \sqsubseteq Cl(f_A, r) \sqcup Cl(g_B, r)$. Since, $Cl(f_A \sqcup g_B, r)$is the smallest fuzzy soft r-closed containing $f_A \sqcup g_B$, $Cl(f_A \sqcup g_B, r) \sqsubseteq Cl(f_A, r) \sqcup Cl(g_B, r)$.

Clearly, $Cl(f_A, r) \sqsubseteq Cl(f_A \sqcup g_B, r)$ and $Cl(g_B, r) \sqsubseteq Cl(f_A \sqcup g_B, r)$. Therefore, $Cl(f_A, r) \sqcup Cl(g_B, r) \sqsubseteq Cl(f_A \sqcup g_B, r)$. Hence, $Cl(f_A \sqcup g_B, r) = Cl(f_A, r) \sqcup Cl(g_B, r)$.

(7) Clearly, $f_A \sqcap g_B \sqsubseteq f_A$ and $f_A \sqcap g_B \sqsubseteq g_B$. By (4), $Cl(f_A \sqcap g_B, r) \sqsubseteq Cl(f_A, r)$ and $Cl(f_A \sqcap g_B, r) \sqsubseteq Cl(g_B, r)$. Therefore, $Cl(f_A \sqcap g_B, r) \sqsubseteq Cl(f_A, r) \sqcap Cl(g_B, r)$.

Theorem 3.9. Let (X, τ, E) be a fuzzy soft topological space. For each f_A, $g_B \in \widetilde{(X, E)}$ and $r \in I_0$. Then,

(1) $Int(1_E, r) = 1_E$.

(2) $Int(f_A, r) \sqsubseteq f_A$.

(3) $Int(Int(f_{A,}, r), r) = Int(f_A, r)$.

(4) If $f_A \sqsubseteq g_B$, then $Int(f_A, r) \sqsubseteq Int(g_B, r)$.

(5) f_A is fuzzy soft r-open if and only if $f_A = Int(f_A, r)$.

(6) $Int(f_A \sqcap g_B, r) = Int(f_A, r) \sqcap Int(g_B, r)$.

(7) $Int(f_A, r) \sqcup Int(g_B, r) \sqsubseteq Int(f_A \sqcup g_B, r)$.

Proof. (1) Clearly, $Int(1_E, r) \sqsubseteq 1_E$. Since, 1_E is fuzzy soft r-open set and $Int(1_E, r)$ is the largest r-open set contained in 1_E, $1_E \sqsubseteq Int(1_E, r)$. Therefore, $Int(1_E, r) = 1_E$.

(2) By the definition of fuzzy soft r-interior, $Int(f_A, r) \sqsubseteq f_A$.

(3) By (2), $Int(Int(f_A, r), r) \sqsubseteq Int(f_A, r)$. Here $Int(f_A, r)$ is fuzzy soft r-open set. Since, $Int(Int(f_A, r), r)$ is the largest fuzzy soft r-open set contained in $Int(f_A, r)$, $Int(f_A, r) \sqsubseteq Int(Int(f_A, r), r)$. Therefore, $Int(Int(f_{A,}, r), r) = Int(f_A, r)$.

(4) Assume that $f_A \sqsubseteq g_B$. Therefore,

$$
\begin{aligned}
Int(f_A, r) &= \sqcup \{ f_C : \tau(f_C) \geq r, f_C \sqsubseteq f_A \} \\
&\sqsubseteq \sqcup \{ f_C : \tau(f_C) \geq r, f_C \sqsubseteq g_B \} \\
&= Int(g_B, r).
\end{aligned}
$$

(5) Assume that f_A is fuzzy soft r-open set. By (2), we have $\mathrm{Int}(f_A, r) \sqsubseteq f_A$. Since f_A is fuzzy soft r-open set and $\mathrm{Int}(f_A, r)$ is the largest fuzzy soft r-open set contained in f_A, $f_A \sqsubseteq \mathrm{Int}(f_A, r)$. Hence, $f_A = \mathrm{Int}(f_A, r)$.

Conversely, assume that $f_A = \mathrm{Int}(f_A, r)$. Since, $\mathrm{Int}(f_A, r)$ is the largest fuzzy soft r-open set. Hence, f_A is fuzzy soft r open set.

(6) Clearly, $\mathrm{Int}(f_A, r) \sqcap \mathrm{Int}(g_B, r) \sqsubseteq f_A \sqcap g_B$. Since, $\mathrm{Int}(f_A \sqcap g_B, r)$ is the largest fuzzy soft r-open contained in $f_A \sqcap g_B$, $\mathrm{Int}(f_A, r) \sqcap \mathrm{Int}(g_B, r) \sqsubseteq \mathrm{Int}(f_A \sqcap g_B, r)$.

Clearly, $\mathrm{Int}(f_A \sqcap g_B, r) \sqsubseteq \mathrm{Int}(f_A, r)$ and $\mathrm{Int}(f_A \sqcap g_B, r) \sqsubseteq \mathrm{Int}(g_B, r)$. Therefore, $\mathrm{Int}(f_A \sqcap g_B, r) \sqsubseteq \mathrm{Int}(f_A, r) \sqcap \mathrm{Int}(g_B, r)$. Hence, $\mathrm{Int}(f_A \sqcap g_B, r) = \mathrm{Int}(f_A, r) \sqcap \mathrm{Int}(g_B, r)$.

(7) Clearly, $f_A \sqsubseteq f_A \sqcup g_B$ and $g_B \sqsubseteq f_A \sqcup g_B$. By (4), $\mathrm{Int}(f_A, r) \sqsubseteq \mathrm{Int}(f_A \sqcup g_B, r)$ and $\mathrm{Int}(g_B, r) \sqsubseteq \mathrm{Int}(f_A \sqcup g_B, r)$. Therefore, $\mathrm{Int}(f_A, r) \sqcup \mathrm{Int}(g_B, r) \sqsubseteq \mathrm{Int}(f_A \sqcup g_B, r)$.

Theorem 3.10.. Let (X, τ, E) be a fuzzy soft topological space. For each $f_A \in \widetilde{(X, E)}$. Then,

(1) $(\mathrm{Int}(f_A, r))^c = \mathrm{Cl}(f_A^c, r)$.

(2) $(\mathrm{Cl}(f_A, r))^c = \mathrm{Int}(f_A^c, r)$.

Proof. (1) Clearly, $\mathrm{Int}(f_A, r) \sqsubseteq f_A$ By Theorem 2.13(11), $f_A^c \sqsubseteq (\mathrm{Int}(f_A, r))^c$. Since $(\mathrm{Int}(f_A, r))^c$ is fuzzy soft r-closed and by Theorem 3.8 (2) and (5), $\mathrm{Cl}(f_A^c, r) \sqsubseteq \mathrm{Cl}((\mathrm{Int}(f_A, r))^c) = (\mathrm{Int}(f_A, r))^c$.

Now, $f_A^c \sqsubseteq \mathrm{Cl}(f_A^c, r)$ By Theorem 2.13(11), $(\mathrm{Cl}(f_A^c, r))^c \sqsubseteq (f_A^c)^c = f_A$. Clearly $(\mathrm{Cl}(f_A^c, r))^c$ fuzzy soft r-open. Since $\mathrm{Int}(f_A, r)$ is the largest fuzzy soft r-open set contained in f_A, $(\mathrm{Cl}(f_A^c, r))^c \sqsubseteq \mathrm{Int}(f_A, r)$. By Theorem 2.13(11), $(\mathrm{Int}(f_A, r))^c \sqsubseteq (\mathrm{Cl}(f_A^c, r))^c = \mathrm{Cl}(f_A^c, r)$.

(2) Similar to (1).

4. FUZZY SOFT r-CONTINUITY

Definition 4.1. Let $\varphi_\psi : (X, \tau_1, E) \rightarrow : (Y, \tau_2, F)$ be a fuzzy soft mapping and $r \in I_0$. If φ_ψ is called fuzzy soft r-continuous if $\tau_1(\varphi_\psi^{-1}(g_B)) \geq r$ whenever $g_B \in \widetilde{(Y, F)}$ and $\tau_2(g_B) \geq r$.

Definition 4.2. Let $\varphi_\psi : (X, \tau_1, E) \rightarrow : (Y, \tau_2, F)$ be a fuzzy soft mapping and $r \in I_0$.

If φ_ψ is called fuzzy soft r-open if $\tau_2(\varphi_\psi(\mathrm{f_A})) \geq r$ whenever $\mathrm{f_A} \in \widetilde{(X,E)}$ and $\tau_1(\mathrm{f_A}) \geq r$.

Definition 4.3. Let $\varphi_\psi: (X, \tau_1, E) \rightarrow: (Y, \tau_2, F)$ be a fuzzy soft mapping and $\mathrm{r} \in \mathrm{I_0}$. If φ_ψ is called fuzzy soft r-closed if $\eta_2(\varphi_\psi(\mathrm{f_A})) \geq r$ whenever $\mathrm{f_A} \in \widetilde{(X,E)}$ and $\eta_1(\mathrm{f_A}) \geq r$.

Theorem 4.4. Let $\varphi_\psi: (X, \tau_1, E) \rightarrow: (Y, \tau_2, F)$ be a fuzzy soft mapping and $\mathrm{r} \in \mathrm{I_0}$. Then the following are equivalent:

(1) φ_ψ is fuzzy soft r-continuous.

(2) $\eta_1(\varphi_\psi^{-1}(\mathrm{g_B})) \geq r$ whenever $\mathrm{g_B} \in \widetilde{(Y,F)}$ and $\eta_2(\mathrm{g_B}) \geq r$.

(3) $\varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r})) \sqsubseteq \mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), \mathrm{r})$, for all $\mathrm{f_A} \in \widetilde{(X,E)}$.

(4) $\mathrm{Cl}\,(\varphi_\psi^{-1}(\mathrm{g_B}), r) \sqsubseteq \varphi_\psi^{-1}\big(\mathrm{Cl}(\mathrm{g_B}, r)\big)$ for all $\mathrm{g_B} \in \widetilde{(Y,F)}$.

(5) $\varphi_\psi^{-1}\,(\mathrm{Int}(\mathrm{g_B}, \mathrm{r})) \sqsubseteq \mathrm{Int}\,(\varphi_\psi^{-1}(\mathrm{g_B}), r)$, for all $\mathrm{g_B} \in \widetilde{(Y,F)}$.

Proof. (1) → (2).

Assume that φ_ψ is fuzzy soft r-continuous. Let $\mathrm{g_B} \in \widetilde{(Y,F)}$ and $\eta_2(\mathrm{g_B}) \geq r$. Clearly $\eta_2(\mathrm{g_B}) = \tau_2(g_B^c) \geq r$. Since φ_ψ is fuzzy soft r-continuous, $\tau_1(\varphi_\psi^{-1}(g_B^c)) \geq r$. By Lemma 2.13(9), $\varphi_\psi^{-1}(g_B^c) = (\varphi_\psi^{-1}(\mathrm{g_B}))^c$. Hence, $\tau_1((\varphi_\psi^{-1}(\mathrm{g_B}))^c) \geq r$. Therefore, $\eta_1(\varphi_\psi^{-1}(\mathrm{g_B})) \geq r$.

(2) → (3).

Assume $\eta_1(\varphi_\psi^{-1}(\mathrm{g_B})) \geq r$ whenever $\mathrm{g_B} \in \widetilde{(Y,F)}$ and $\eta_2(\mathrm{g_B}) \geq r$. Let $\mathrm{f_A} \in \widetilde{(X,E)}$. By Lemma 2.13(3) and by Theorem 3.8 (2), $\mathrm{f_A} \sqsubseteq \varphi_\psi{}^{-1}\left(\varphi_\psi(\mathrm{f_A})\right) \sqsubseteq \varphi_\psi{}^{-1}\big(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r)\big)$. Since $\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r)$ is fuzzy soft r-closed set, $\eta_2(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r)) \geq r$. By our assumption $\eta_1(\varphi_\psi^{-1}(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r))) \geq r$. Hence, $\varphi_\psi^{-1}(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r))$ is a fuzzy soft r-closed set such that $\mathrm{f_A} \sqsubseteq \varphi_\psi^{-1}(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r))$. Since $\mathrm{Cl}(\mathrm{f_A}, r)$ is the smallest fuzzy soft r-closed set containing $\mathrm{f_A}$, $\mathrm{Cl}(\mathrm{f_A}, r) \sqsubseteq \varphi_\psi^{-1}(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r))$. By Lemma 2.13(4),

$$(\mathrm{Cl}(\mathrm{f_A}, r)) \sqsubseteq \varphi_\psi(\varphi_\psi^{-1}(\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r))) \sqsubseteq \mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), \mathrm{r}).$$

(3) → (4).

Assume that $\varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r})) \sqsubseteq \mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), \mathrm{r})$, for all $\mathrm{f_A} \in \widetilde{(X,E)}$. Let $\mathrm{g_B} \in$

$(\widetilde{Y,F})$. Then, $\varphi_\psi^{-1}(\mathrm{g_B}) \in (\widetilde{X,E})$. By our assumption and by Lemma 2.13 (4), $\varphi_\psi(\mathrm{Cl}(\varphi_\psi^{-1}(\mathrm{g_B}),\mathrm{r})) \sqsubseteq \mathrm{Cl}(\varphi_\psi\left(\varphi_\psi^{-1}(\mathrm{g_B})\right),\mathrm{r}) \sqsubseteq \mathrm{Cl}(\mathrm{g_B},\mathrm{r})$. By Lemma 2.13 (3), $\mathrm{Cl}(\varphi_\psi^{-1}(\mathrm{g_B}),\mathrm{r}) \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(\mathrm{Cl}(\varphi_\psi^{-1}(\mathrm{g_B}),\mathrm{r})) \sqsubseteq \varphi_\psi^{-1}(\mathrm{Cl}(\mathrm{g_B},\mathrm{r}))$.

(4) $\rightarrow$ (5).

Assume that Cl $(\varphi_\psi^{-1}(\mathrm{g_B}),r) \sqsubseteq \varphi_\psi^{-1}\big(\mathrm{Cl}(\mathrm{g_B},r)\big)$ for all $\mathrm{g_B} \in (\widetilde{Y,F})$. Let $\mathrm{g_B} \in (\widetilde{Y,F})$. Clearly, $\mathrm{g_B^c} \in (\widetilde{Y,F})$. By our assumption,

Cl $(\varphi_\psi^{-1}(\mathrm{g_B^c}),r) \sqsubseteq \varphi_\psi^{-1}\big(\mathrm{Cl}(\mathrm{g_B^c},r)\big)$. By Theorem 3.10 (1) and by Lemma 2.13 (9),

$$\begin{aligned}(\mathrm{Int}\big((\varphi_\psi^{-1}(\mathrm{g_B}),\mathrm{r})\big)^{\mathrm{c}} &= \mathrm{Cl}\ ((\varphi_\psi^{-1}(\mathrm{g_B}))^c,\mathrm{r}) \\ &= \mathrm{Cl}\ (\varphi_\psi^{-1}(\mathrm{g_B^c}),r) \\ &\sqsubseteq \varphi_\psi^{-1}\big(\mathrm{Cl}(\mathrm{g_B^c},r)\big) \\ &= \varphi_\psi^{-1}(\mathrm{Int}(\mathrm{g_B},\mathrm{r})^{\mathrm{c}}) \\ &= (\varphi_\psi^{-1}(\mathrm{Int}(\mathrm{g_B},\mathrm{r})))^c.\end{aligned}$$

Therefore, φ_ψ^{-1} $(\mathrm{Int}(\mathrm{g_B},\mathrm{r})) \sqsubseteq \mathrm{Int}\ (\varphi_\psi^{-1}(\mathrm{g_B}),r)$.

(5) $\rightarrow$ (1).

Assume that φ_ψ^{-1} $(\mathrm{Int}(\mathrm{g_B},\mathrm{r})) \sqsubseteq \mathrm{Int}\ (\varphi_\psi^{-1}(\mathrm{g_B}),r)$, for all $\mathrm{g_B} \in (\widetilde{Y,F})$.

Let $\mathrm{g_B} \in (\widetilde{Y,F})$ and $\tau_2(\mathrm{g_B}) \geq r$. Therefore, $\mathrm{g_B} = \mathrm{Int}(\mathrm{g_B},\mathrm{r})$. By our assumption and by Theorem 3.9 (2),

$$\varphi_\psi^{-1}(\mathrm{g_B}) = \varphi_\psi^{-1}(\mathrm{Int}(\mathrm{g_B},\mathrm{r})) \sqsubseteq \mathrm{Int}\ (\varphi_\psi^{-1}(\mathrm{g_B}),r) \sqsubseteq \varphi_\psi^{-1}(\mathrm{g_B}).$$

Hence, $\varphi_\psi^{-1}(\mathrm{g_B}) = \mathrm{Int}\ (\varphi_\psi^{-1}(\mathrm{g_B}),r)$. By Theorem 3.9(5), $(\varphi_\psi^{-1}(\mathrm{g_B})$ is fuzzy soft r-open. Therefore, $\tau_1\left(\varphi_\psi^{-1}(\mathrm{g_B})\right) \geq r$. Thus, φ_ψ is fuzzy soft r-continuous.

Theorem 4.5. Let $\varphi_\psi : (X,\tau_1,E) \rightarrow : (Y,\tau_2,F)$ be a fuzzy soft mapping and $\mathrm{r} \in \mathrm{I_0}$. Then,

(1) φ_ψ is fuzzy soft r-open mapping iff for each fuzzy soft set $\mathrm{f_A}$ over X,

$$\varphi_\psi(\mathrm{Int}(\mathrm{f_A},\mathrm{r})) \sqsubseteq \mathrm{Int}(\varphi_\psi(\mathrm{f_A}),\mathrm{r}).$$

(2) φ_ψ is fuzzy soft r-closed mapping iff for each fuzzy soft set $\mathrm{f_A}$ over X,

$$\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \sqsubseteq \varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r})).$$

Proof. (1) Let φ_ψ is fuzzy soft r-open mapping and $\mathrm{f_A}$ be a fuzzy soft set over X. Clearly, $\mathrm{Int}(\mathrm{f_A}, \mathrm{r})$ is fuzzy soft r-open set. Since φ_ψ is fuzzy soft r -open and $\mathrm{Int}(\mathrm{f_A}, \mathrm{r}) \sqsubseteq \mathrm{f_A}$, $\varphi_\psi(\mathrm{Int}(\mathrm{f_A}, \mathrm{r}))$ is fuzzy soft r- open set in Y and $\varphi_\psi(\mathrm{Int}(\mathrm{f_A}, \mathrm{r})) \sqsubseteq \varphi_\psi(\mathrm{f_A})$. Since $\mathrm{Int}(\varphi_\psi(\mathrm{f_A}), r)$ is the largest r-open set contained in $\varphi_\psi(\mathrm{f_A})$, $\varphi_\psi(\mathrm{Int}(\mathrm{f_A}, \mathrm{r})) \sqsubseteq \mathrm{Int}(\varphi_\psi(\mathrm{f_A}), r)$.

Conversely $\tau_1(\mathrm{f_A}) \geq r$, where $\mathrm{f_A} \in \widetilde{(X, E)}$. Hence, $\mathrm{Int}(\mathrm{f_A}, \mathrm{r}) = \mathrm{f_A}$.

By our assumption and by Theorem 3.9 (2),

$$\begin{aligned} \varphi_\psi(\mathrm{f_A}) &= \varphi_\psi(\mathrm{Int}(\mathrm{f_A}, \mathrm{r}) \\ &\sqsubseteq \mathrm{Int}(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \sqsubseteq \varphi_\psi(\mathrm{f_A}). \end{aligned}$$

Hence, $\varphi_\psi(\mathrm{f_A}) = \mathrm{Int}\big(\varphi_\psi(\mathrm{f_A}), \mathrm{r}\big)$. Therefore, $\tau_2(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \geq r$. Thus φ_ψ is a fuzzy soft r -open.

(2) Let φ_ψ is fuzzy soft r-closed mapping and $\mathrm{f_A}$ be a fuzzy soft set over X. Clearly, $\mathrm{Cl}(\mathrm{f_A}, \mathrm{r})$ is fuzzy soft r-closed set. Since φ_ψ is fuzzy soft r -closed and $\mathrm{f_A} \sqsubseteq \mathrm{Cl}(\mathrm{f_A}, \mathrm{r})$, $\varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r}))$ is fuzzy soft r - closed set in Y and $\varphi_\psi(\mathrm{f_A}) \sqsubseteq \varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r}))$. Since $\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), r)$ is the smallest r-closed set containing $\varphi_\psi(\mathrm{f_A})$, $\mathrm{Cl}(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \sqsubseteq \varphi_\psi(\mathrm{Cl}(\mathrm{f_A}, \mathrm{r}))$.

Conversely $\mathrm{f_A}$ is fuzzy r-closed set over X. By our assumption and by Theorem 3.8 (2) and (5),

$$\begin{aligned} \mathrm{Cl}\big(\varphi_\psi(\mathrm{f_A}), \mathrm{r}\big)\, \varphi_\psi(\mathrm{f_A}) &= \varphi_\psi(\mathrm{Int}(\mathrm{f_A}, \mathrm{r}) \\ &\sqsubseteq \mathrm{Int}(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \sqsubseteq \varphi_\psi(\mathrm{f_A}). \end{aligned}$$

Hence, $\varphi_\psi(\mathrm{f_A}) = \mathrm{Int}\big(\varphi_\psi(\mathrm{f_A}), \mathrm{r}\big)$. Therefore, $\tau_2(\varphi_\psi(\mathrm{f_A}), \mathrm{r}) \geq r$. Thus $\varphi_\psi(\mathrm{f_A})$ is a fuzzy soft r -open.

Definition 4.6. Let (X, τ_1, E) and (Y, τ_2, F) be two fuzzy soft topological spaces and $\varphi_\psi: (X, \tau_1, E) \to (Y, \tau_2, F)$ be a fuzzy soft mapping. If φ_ψ is a bijection, fuzzy soft r -continuous and φ_ψ is fuzzy soft r -open mapping, then φ_ψ is said to be fuzzy soft r -homeomorphism from X to Y.

Theorem 4.7. Let (X, τ_1, E) and (Y, τ_2, F) be two fuzzy soft topological spaces and $\varphi_\psi: (X, \tau_1, E) \to (Y, \tau_2, F)$ be a bijective and fuzzy soft r-continuous mapping.

Then the following conditions are equivalent:

[1] φ_ψ is a fuzzy soft r-homeomorphism,

[2] φ_ψ is fuzzy soft r-closed mapping,

[3] φ_ψ is fuzzy soft r-open mapping.

Proof. It is easily obtained.

ACKNOWLEDGEMENT

The authors thank the management SSN Institutions for their constant support towards the successful completion of this work.

REFERENCES

[1] Aygunoglu, A. and Aygün, H., *Introduction to fuzzy soft groups*, Computers and Mathematics with Applications 58 (2009).

[2] Chang, C. L., *Fuzzy topological spaces*, J. Math. Anal. Appl., 24(1) (1968), 182-189.

[3] Cetkin, V. and Aygün, H., *A note on fuzzy soft topological spaces*, Proceedings of the 8th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2013), 56-60.

[4] Maji, P.K., Biswas, R. and Roy, A.R., *fuzzy soft sets*, Journal of fuzzy Mathematics, 9 (3) (2001), 589-602.

[5] Maji, P.K., Biswas, R. and Roy, A.R., *Soft set theory*, Computers Math. Appl., 45 (2003), 555-562.

[6] Maji, P.K. and Roy, A.R., *An application of soft set in decision making problem*, Computers Math. Appl., 44 (2002), 1077-1083.

[7] Molodtsov, D., *Soft set theory-First results*, Computers Math. Appl., 37 (4/5) (1999), 19-31.

[8] Roy, S. and Samanta, T. K., *A note on fuzzy soft topological spaces*, Ann. Fuzzy Math. Inform., 3(2) (2011), 305-311.

[9] Shabir, M. and Naz, M., *On soft topological spaces*, Computers and Mathematics with Applications, 61(7) (2011), 1786-1799.

[10] Šostak, A. P., *On some modification of fuzzy topologies*, Mat. Vesnik, 14 (1989), 51-64.

[11] Tanay, B., Kandemir, M. B., *Topological structures of fuzzy soft sets*, Comput. Math. Appl., 61 (2011) 412-418.

[12] Varol, B.P. and Aygün, H., *Fuzzy soft topology*, Hacettepe Journal of Mathematics and Statistics, 41(3) (2012), 407-419.

[13] Zadeh, L. A., *Fuzzy sets*, Information and Control, 8 (1965), 338-353.